Scalar Pair Production In Near Extremal Charged Black Holes

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We review the spontaneous pair production of charged scalar particles in the near horizon region of near extremal charged black holes. The pair production rate is analytically computed and its thermal interpretation is discussed. Moreover, the associated holographical correspondence has been checked at the 2-point function level by comparing the absorption cross section ratio as well as the pair production rate both from the gravity and the conformal field theories (CFTs) sides.

There are two independent processes for the spontaneous pair production occurring in charged black holes—the Schwinger mechanism [1] and the Hawking radiation [2]. The dominated contribution is expected coming from the near horizon region. We investigate the scalar particle emissions in the spacetime of the near-horizon region of the near extremal charged black holes. A simple case is the scalar production in the Reissner-Nordström (RN) black holes [3] (see [4] for spinor production). In such case the considered spacetime has an $\mathrm{AdS}_2 \times S^2$ structure and the electric field is constant. One can analytically solve the Klein-Gordon (KG) equation and give an exact expression for the production rate. The analysis can be generalized to the Kerr-Newman (KN) black holes [5] and also to include a magnetic charge [6]. The angular momentum deforms the near horizon spacetime geometry to be a warped AdS_3, but the KG equation still can be solved analytically. The pair production have a remarkable thermal interpretation based on the discussion in [7–9]. Moreover, the scalar production has a nice conformal field theory (CFT) dual picture supporting the KN/CFTs correspondence [10, 11].

The geometry of the near horizon of a near extreme dyonic KN black hole has the structure of a warped AdS_3 as [6]

$$
\begin{aligned}
ds^2 &= \Gamma(\theta)\left[-(\rho^2 - B^2)d\tau^2 + \frac{d\rho^2}{\rho^2 - B^2} + d\theta^2\right] + \gamma(\theta)(d\varphi + b\rho d\tau)^2, && (1)\\
A_{[1]} &= -\frac{Q(r_0^2 - a^2\cos^2\theta) - 2Pr_0 a\cos\theta}{\Gamma(\theta)}\rho d\tau \\
&\quad - \frac{Qr_0 a\sin^2\theta - P(r_0^2 + a^2)\cos\theta \pm P\Gamma(\theta)}{\Gamma(\theta)}d\varphi, && (2)
\end{aligned}
$$

where

$$
\begin{aligned}
&\Gamma(\theta) = r_0^2 + a^2\cos^2\theta, \qquad \gamma(\theta) = \frac{(r_0^2 + a^2)^2\sin^2\theta}{r_0^2 + a^2\cos^2\theta}, \\
&b = \frac{2ar_0}{r_0^2 + a^2}, \qquad r_0 = \sqrt{Q^2 + P^2 + a^2}. && (3)
\end{aligned}
$$

Here a is the angular momentum parameter and Q, P are the electric and magnetic charges of the original dyonic KN black holes, and B labels a deviation from the extreme limit and acts as the new horizon radius in this geometry.

The physical quantities associated to the black hole thermodynamics, Hawking temperature, entropy, angular velocity and chemical potentials, are ($\bar{\Phi}_{\mathrm{H}}$ is given from the Hodge dual of Maxwell field $dA_{[1]}$)

$$
\begin{aligned}
&T_{\mathrm{H}} = \frac{B}{2\pi}, \qquad S_{\mathrm{BH}} = \pi(r_0^2 + a^2 + 2Br_0), \qquad \Omega_{\mathrm{H}} = -\frac{2ar_0 B}{r_0^2 + a^2}, \\
&\Phi_{\mathrm{H}} = \frac{Q(Q^2 + P^2)B}{r_0^2 + a^2}, \qquad \bar{\Phi}_{\mathrm{H}} = \frac{P(Q^2 + P^2)B}{r_0^2 + a^2}. && (4)
\end{aligned}
$$

The KG equation for the scalar dyons can be solved exactly. There is an effective potential due to the electromagnetic and gravitational interactions and the pair production becomes a tunneling process. By imposing a suitable boundary condition, one can obtain the production rate from the ratio of incoming and outgoing fluxes on the boundaries (at the horizon and asymptotic) [3]. For example, one can impose no incoming flux at the asymptotic outer boundary and then the outgoing (transmitted) flux at the asymptotic region represents the spontaneous produced particle, the outgoing (incident) flux at the horizon represents the total particles created by vacuum fluctuations, and the incoming (reflected) flux represents the re-annihilated pairs.

For bosonic particles, the flux conservation [12]

$$|D_{\text{incident}}| = |D_{\text{reflected}}| + |D_{\text{transmitted}}|, \tag{5}$$

is related to the Bogoliubov relation

$$|\mathcal{A}|^2 - |\mathcal{B}|^2 = 1, \tag{6}$$

where the vacuum persistence amplitude $|\mathcal{A}|^2$ and the mean number of produced pairs $|\mathcal{B}|^2$ are given by the ratios of the flux components

$$|\mathcal{A}|^2 \equiv \frac{D_{\text{incident}}}{D_{\text{reflected}}}, \qquad |\mathcal{B}|^2 \equiv \frac{D_{\text{transmitted}}}{D_{\text{reflected}}}. \tag{7}$$

Moreover, from the viewpoint of scattering of an incident flux from the asymptotic boundary, we can define the absorption cross section ratio as

$$\sigma_{\text{abs}} \equiv \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{|\mathcal{B}|^2}{|\mathcal{A}|^2}. \tag{8}$$

Using the following ansatz (hereafter parameters m, q, p are mass, electric and magnetic charges of scalar field)

$$\Phi(\tau, \rho, \theta, \varphi) = \mathrm{e}^{-i\omega\tau + i[n \mp (qP - pQ)]\varphi} R(\rho) S(\theta), \tag{9}$$

a straightforward calculation leads to the Bogoliubov coefficients and the absorption cross section ratio as (the detail derivation can be found in [6])

$$|\mathcal{A}|^2 = \frac{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}, \tag{10}$$

$$|\mathcal{B}|^2 = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}, \tag{11}$$

$$\sigma_{\text{abs}} = \frac{|\mathcal{B}|^2}{|\mathcal{A}|^2} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}, \tag{12}$$

where three essential parameters are (λ_l is the separation constant)

$$\tilde{\kappa} = \frac{\omega}{B}, \qquad \kappa = \frac{(qQ + pP)(Q^2 + P^2) - 2nar_0}{r_0^2 + a^2}, \qquad \mu = \sqrt{\kappa^2 - m^2(r_0^2 + a^2) - \lambda_l - \frac{1}{4}}, \tag{13}$$

in which μ^2 is positive due to the BF bound violation in the AdS_2 spacetime [6].

Following our previous studies [3, 5], the mean number of produced pairs (11) can be reexpressed as

$$\mathcal{N} = |\mathcal{B}|^2 = \left(\frac{\mathrm{e}^{-2\pi\kappa + 2\pi\mu} - \mathrm{e}^{-2\pi\kappa - 2\pi\mu}}{1 + \mathrm{e}^{-2\pi\kappa - 2\pi\mu}}\right)\left(\frac{1 - \mathrm{e}^{-2\pi\tilde{\kappa} + 2\pi\kappa}}{1 + \mathrm{e}^{-2\pi\tilde{\kappa} + 2\pi\mu}}\right). \tag{14}$$

Note that the mean number (14) has a similar form except for different quantum numbers as that of charged scalars in a near-extremal RN [3] and KN [5] since the near-horizon geometry is an $AdS_2 \times S^2$ for the near-extremal RN black hole while it is a warped AdS_3 for the near-extremal KN black hole. Following Refs. [7, 8], we introduce an effective temperature and its associated counterpart

$$T_{\text{KN}} = \frac{\bar{m}}{2\pi\kappa - 2\pi\mu} = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \qquad \bar{T}_{\text{KN}} = \frac{\bar{m}}{2\pi\kappa + 2\pi\mu} = T_U - \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \tag{15}$$

where the effective mass $\bar{m}$ is

$$\bar{m} = \sqrt{m^2 - \frac{\lambda + 1/4}{2}\mathcal{R}}, \tag{16}$$

and the corresponding Unruh temperature T_U and AdS curvature $\mathcal{R}$ are

$$T_U = \frac{\kappa}{2\pi\bar{m}(r_0^2 + a^2)} = \frac{(qQ + pP)(Q^2 + P^2) - 2nar_0}{2\pi\bar{m}(r_0^2 + a^2)^2}, \qquad \mathcal{R} = -\frac{2}{r_0^2 + a^2}. \tag{17}$$

Therefore the mean number (14) can be expressed as

$$\mathcal{N} = \mathrm{e}^{\frac{\bar{m}}{T_{\rm KN}}} \times \left(\frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\rm KN}}} - \mathrm{e}^{-\frac{\bar{m}}{T_{\rm KN}}}}{1+\mathrm{e}^{-\frac{\bar{m}}{T_{\rm KN}}}} \right) \times \left\{ \frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\rm KN}}} \left(1 - \mathrm{e}^{-\frac{\omega - q\Phi_{\rm H} - p\bar{\Phi}_{\rm H} - n\Omega_{\rm H}}{T_{\rm H}}} \right)}{1 + \mathrm{e}^{-\frac{\omega - q\Phi_{\rm H} - p\bar{\Phi}_{\rm H} - n\Omega_{\rm H}}{T_{\rm H}}} \mathrm{e}^{-\frac{\bar{m}}{T_{\rm KN}}}} \right\}. \tag{18}$$

Now the physical meaning of each term in Eq. (18) becomes clear: we interpret the first parenthesis as the Schwinger effect with the effective temperature $T_{\rm KN}$ in AdS_2 [13] and the second parenthesis as the Schwinger effect in the Rindler space [14], in which the Unruh temperature is given by the Hawking temperature and the charges have the chemical potentials of $\Phi_{\rm H}, \bar{\Phi}_{\rm H}$, and $\Omega_{\rm H}$, while the effective temperature for the Schwinger effect due to the electric field on the horizon is still determined by $T_{\rm KN}$.

According to the KN/CFTs duality [11, 15], the absorption cross section ratio of scalar field in Eq. (12) corresponds to that of its dual operator in the dual two-dimensional CFT with left- and right-hand sectors

$$\sigma_{\rm abs} \sim T_{\rm L}^{2h_{\rm L}-1} T_{\rm R}^{2h_{\rm R}-1} \sinh\left(\frac{\tilde{\omega}_{\rm L}}{2T_{\rm L}} + \frac{\tilde{\omega}_{\rm R}}{2T_{\rm R}} \right) \left| \Gamma\left(h_{\rm L} + i\frac{\tilde{\omega}_{\rm L}}{2\pi T_{\rm L}} \right) \right|^2 \left| \Gamma\left(h_{\rm R} + i\frac{\tilde{\omega}_{\rm R}}{2\pi T_{\rm R}} \right) \right|^2, \tag{19}$$

where $T_{\rm L}, T_{\rm R}$ are the temperatures, $h_{\rm L}, h_{\rm R}$ are the conformal dimensions of the dual operator, $\tilde{\omega}_{\rm L} = \omega_{\rm L} - q_{\rm L}\Phi_{\rm L}$ and $\tilde{\omega}_{\rm R} = \omega_{\rm R} - q_{\rm R}\Phi_{\rm R}$ are the total excited energy in which $(q_{\rm L}, q_{\rm R})$ and $(\Phi_{\rm L}, \Phi_{\rm R})$ are respectively the charges and chemical potentials (both including the electric and the magnetic contributions for the dyonic KN black hole case) of the dual left and right-hand operators. The complex conformal dimensions $(h_{\rm L}, h_{\rm R})$ of the dual operator be read out from the asymptotic expansion of the bulk dyonic charged scalar field at the AdS boundary [6]

$$h_{\rm L} = h_{\rm R} = \frac{1}{2} \pm i\mu. \tag{20}$$

For the dyonic KN black holes, there are in general three different pictures, namely J-, Q- and the P-picture, in the dual CFTs descriptions. Here we only show the result of J-picture (refer to [6] for the other two).

In the J-picture, the left- and right-hand central charges of the dual CFT are determined by the angular momentum [11, 15]

$$c_{\rm L}^J = c_{\rm R}^J = 12J, \tag{21}$$

and the associated left- and right-hand temperatures for the near extremal dyonic KN black hole are

$$T_{\rm L}^J = \frac{r_0^2 + a^2}{4\pi a r_0}, \qquad T_{\rm R}^J = \frac{B}{2\pi a}. \tag{22}$$

The CFT microscopic entropy is calculated from the Cardy formula

$$S_{\rm CFT} = \frac{\pi^2}{3}(c_{\rm L}^J T_{\rm L}^J + c_{\rm R}^J T_{\rm R}^J) = \pi(r_0^2 + a^2 + 2r_0 B), \tag{23}$$

which agrees with the macroscopic entropy (4) of the near extremal KN black hole.

Besides, by matching the first law of black hole thermodynamics with that of the dual CFT, i.e., $\delta S_{\rm BH} = \delta S_{\rm CFT}$, the following relation holds

$$\frac{\delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - \bar{\Phi}_{\rm H}\delta P}{T_{\rm H}} = \frac{\tilde{\omega}_{\rm L}}{T_{\rm L}} + \frac{\tilde{\omega}_{\rm R}}{T_{\rm R}}, \tag{24}$$

where the angular velocity and chemical potentials at $\rho = B$ are given in Eq. (4). To probe the rotation we need to turn off the charges of the probe scalar field and set $T_{\rm L} = T_{\rm L}^J$ and $T_{\rm R} = T_{\rm R}^J$, then for the dyonic KN black hole $\delta M = \omega,\ \delta J = n,\ \delta Q = 0,\ \delta P = 0$. Thus, we have

$$\tilde{\omega}_{\rm L}^J = n \quad \text{and} \quad \tilde{\omega}_{\rm R}^J = \frac{\omega}{a} \quad \Rightarrow \quad \frac{\tilde{\omega}_{\rm L}^J}{2T_{\rm L}^J} = -\pi\kappa \quad \text{and} \quad \frac{\tilde{\omega}_{\rm R}^J}{2T_{\rm R}^J} = \pi\tilde{\kappa}, \tag{25}$$

where q, p are set to zero. Consequently, the agreement between the absorption cross section ratio (12) of the scalar field (with $q = p = 0$) in the near extremal dyonic KN black hole and that of its dual scalar operator in Eq. (19) is found in the J-picture.

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