Emerging Hawking-like Radiation In Gravitational Scattering Beyond The Planck Scale

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We generalize the semiclassical treatment of graviton radiation to gravitational scattering at very large energies $E \gg M_P$ and finite scattering angles θ_s, so as to approach the collapse regime. Our basic tool is the extension of the recently proposed, unified form of radiation to the Amati-Ciafaloni-Veneziano reduced-action model. By resumming eikonal scattering diagrams, we are able to derive the corresponding (unitary) coherent-state operator. The resulting graviton spectrum, tuned on the gravitational radius R, fully agrees with previous calculations for small angles $\theta_s \ll 1$ but, for sizeable angles $\theta_s \sim 1$ acquires an exponential cutoff of the large frequency region ($\omega > 1/R$), due to energy conservation, so as to emit a finite fraction of the total energy. In the approach-to-collapse regime we find a radiation enhancement due to large tidal forces, so that the whole energy is radiated off, with a large multiplicity $\langle N \rangle \sim Gs \gg 1$ and a well-defined frequency cutoff of order $1/R$. The latter corresponds to the Hawking temperature for a black hole of mass somewhat smaller than E.

1. Introduction

One of the main unsolved problems in physics is to reconcile quantum mechanics (QM) with general relativity (GR). Its resolution is a formidable task, but important informations can be obtained by investigating some processes lying at the interface between QM and GR with the theories and tools at our disposal.

This work aims at studying gravitational scattering of particles at very high energies $E \gg M_p$ beyond the Planck scale, in order to investigate graviton bremmsstrahlung and possibly macroscopic black hole (BH) formation and evaporation at (semiclassical) quantum level, and therefore to shed light on the so-called "information paradox". What we would like to understand is whether a semiclassical picture of collapse emerges from the quantum theory, or if something different happens.

2. The Eikonal Model

Following the pioneering works of Amati-Ciafaloni-Veneziano (ACV)[1] and others, we consider two massless particles (which could be strings) colliding at center of mass energy much larger than the Planck scale $2E = \sqrt{s} \gg M_P \iff Gs \gg 1 \quad (\hbar = 1)$ with some impact parameter b. Such a simple initial state allows us to use the formalism of QM, while the condition on the energy should imply the formation of a macroscopic BH of radius $R = 4GE$, according to GR.

2.1. *Elastic scattering*

When $b \gg R$, scattering is essentially elastic. Let us consider the expansion of the elastic amplitude in terms of string diagrams. In the high-energy limit $s \gg |t|$ and in the semiclassical regime $R, b \gg l_s$, the N-loop amplitude is factorized in the convolution of N single-graviton-exchange amplitudes between the colliding particles, represented by ladder-like effective diagrams (Fig. 1(a)).

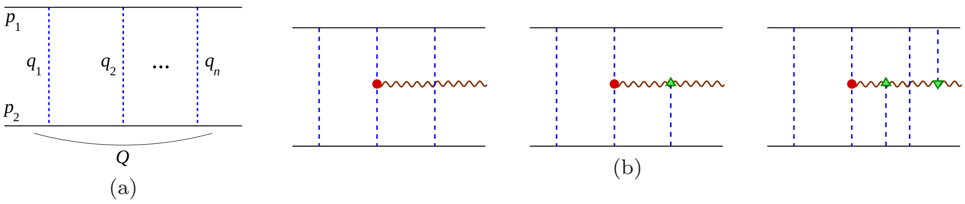


Fig. 1. Leading eikonal diagrams: (a) elastic scattering; (b) graviton emission and rescattering. The red disk represents the Lipatov vertex.

Such convolution in transverse momentum can be diagonalized by a Fourier transform in impact parameter space, and the ladder diagrams can be resummed in exponential series, yielding the elastic S-matrix in eikonal form, $S = \mathrm{e}^{\mathrm{i}2\delta(b,s)}$. The quantity $\delta(b,s) \equiv Gs\Delta(b) = Gs\log(L/b)$ can be interpreted as the (very large) phase shift in potential scattering. Here L is an IR cutoff needed to regularize the Coulomb-like infinite phase shift which occurs in long-range interactions like gravity.

2.2. *Graviton emission*

Graviton emission at high-energy (Regge kinematics) is described by the so-called effective Lipatov vertex[2] (Fig. 1(b)), which is accurate in the region where the graviton emission angle θ is much larger than the deflection angle θ_s of the incoming particles. However such vertex is inaccurate when $\theta \sim \theta_s$. In this case, the emission amplitude for soft gravitons whose energy $\omega \ll E$ is well described by the soft-graviton emission theorem in terms of the Weinberg current[3]. It turns out that, for spin-2 particles, a very simple and elegant unifying amplitude can be derived which interpolates both angular regimes for all the relevant graviton energies, even $\omega \gg M_P$, provided $\omega \ll E$.

In addition we have to take into account the so-called rescattering diagrams[1], which describe the gravitational interaction of the radiated gravitons with the original sources, as shown in the last two diagrams of Fig. 1(b).

By neglecting correlations such as emission from the same rung, we were able to resum all diagrams of Fig. 1(b) and we obtained a remarkably simple result[4]:

$$\mathcal{M}_{2\to 2+N} = \mathrm{e}^{2\mathrm{i}\delta(\boldsymbol{b},s)} \prod_{j=1}^{N} \mathfrak{M}_{\lambda_j}(\boldsymbol{b},\omega_j,\boldsymbol{q}_j) \times \left[1+\mathcal{O}\left(\omega_j^2/E^2\right)\right] \tag{1}$$

$$\mathfrak{M}_\lambda(\boldsymbol{b},\omega,\boldsymbol{q}) = \sqrt{Gs}\frac{R}{\pi}\mathrm{e}^{\mathrm{i}\lambda\phi_{\boldsymbol{q}}} \int \frac{\mathrm{d}^2\boldsymbol{x}}{2\pi|\boldsymbol{x}|^2\mathrm{e}^{\mathrm{i}\lambda\phi_{\boldsymbol{x}}}}\ \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}} \frac{\mathrm{e}^{2\mathrm{i}\omega R\Phi_B(\boldsymbol{x})}-\mathrm{e}^{2\mathrm{i}\omega R\Phi_A(\boldsymbol{x})}}{2\mathrm{i}\omega R} \tag{2}$$

$$\Phi_A(\boldsymbol{x}) \equiv \frac{E}{\omega}\left[\Delta(\boldsymbol{b}-\tfrac{\omega}{E}\boldsymbol{x})-\Delta(\boldsymbol{b})\right]\ , \qquad \Phi_B(\boldsymbol{x}) \equiv \left[\Delta(\boldsymbol{b}-\boldsymbol{x})-\Delta(\boldsymbol{b})\right] \tag{3}$$

In words, the total emission amplitude is given by the elastic amplitude times an emission factor $\mathfrak{M}$ for each emitted gravition. $\mathfrak{M}$ represents the basic object for the description of graviton radiation.

Soft infrared divergencies coming from phase-space integrations of emitted soft gluons are cancelled by virtual correction, that we incorporate by means of the Weinberg method[3], which amounts to write the final state of graviton in Fock space as a coherent state operator S acting on the vacuum with creation (real emission) and destruction (virtual corrections) operators:

$$S = \mathrm{e}^{2\mathrm{i}\delta} \exp\left\{\int \frac{\mathrm{d}^3 q}{\sqrt{2\omega}}\ 2\mathrm{i}\sum_\lambda \left[\mathfrak{M}_\lambda a_\lambda^\dagger(q) + \mathfrak{M}_\lambda^* a_\lambda(q)\right]\right\} \tag{4}$$

We are therefore able to construct an explicitly unitary S matrix for particle scattering and associated bremmsstrahlung. From eq. (4) we can compute the energy spectrum of graviton radiation $\mathrm{d}E^{\mathrm{GW}}/\mathrm{d}\omega$ which is given by $2\omega|\mathfrak{M}|^2$, integrated over all angular directions. The result is depicted in fig. Fig. 2(a). We notice that the spectrum at low frequencies tends to a constant, in agreement with the zero-frequency-limit[5]. At values of $\omega \sim 1/R$ the spectrum starts decreasing as $1/\omega$, almost independently of the impact parameter b, i.e., of the deflection angle θ_s of the hard particles. Note that R increases with energy E. This means that if we increase the energy of the process, the typical frequencies of the radiation decrease, in a way reminiscent of how Hawking radiation depends upon the BH mass.

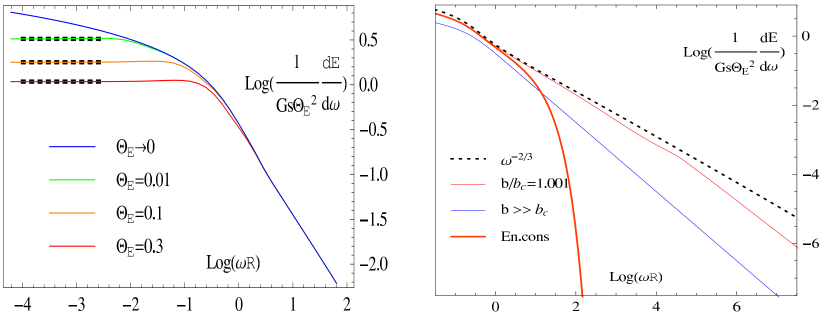
3. Strong Gravity Regime

By decreasing the impact parameter b towards the gravitational radius R, we enter the strong gravity regime, the regime we are really interested in. In this regime we have to consider subleading corrections.

3.1. *Elastic scattering*

ACV were able to identify and resum also the class of subleading diagrams, the so-called H diagrams[6]. These are diagrams with a trilinear interaction (the Lipatov vertex) which is responsible for graviton emissions from exchanged gravitons.

As a consequence, a very interesting feature appears: the phase shift $\delta(b,s)$ acquires an imaginary part for impact parameters $b < b_c \simeq 1.6R$ so that the elastic



(a) (b)

Fig. 2. Rescaled frequency spectrum of graviton radiation. (a) Spectrum in the leading eikonal approximation ($b \gg R$) for various values of θ_s; the black dashes on the left represent the zero-frequency limit. (b) Resummed spectrum with subleading contribution at $b \sim b_c \sim R$, showing the intermediate enhanced region and the exponential fall-off when including energy conservation.

S-matrix is suppressed. The question then arises: is this unitarity deficit in the elastic channel compensated by inelastic graviton production? Or is this critical value a signal of gravitational collapse? Let us note that the unitarity deficit has a fractional critical exponent $= 3/2$, resembling a sort of phase transition.

3.2. *Graviton emission*

By including graviton production from the multi-H diagrams we argue that the resummed emission amplitude maintains the form of eq. (1) by replacing the leading phase shift $\delta(b, s)$ with the resummed one. For $b \to b_c^+$ the non-analytic behaviour of $\delta(b)$ severely affects the energy spectrum. In particular, for $\omega > 1/R$ there is an intermediate region where the frequency spectrum decreases less rapidly than $1/\omega$, actually $\omega^{-2/3}$ (Fig. 2(b)), implying a much higher radiation due to strong tidal forces[7].

3.3. *Energy Conservation*

Because of the logarithmic behaviour of $\mathrm{d}E^{GW} \sim Gs\theta_s^2 \mathrm{d}\omega/\omega$, and also because of the enhanced emission of radiation at large angles, it is mandatory to take into account energy conservation, i.e., to require that the energy carried by emitted gravitions be less than the total CM energy.

Following the proposal of[8], we impose energy conservation event by event, by requiring $\sum_j \omega_j < E$ in each hemisphere, and extending this bound to virtual corrections on the basis of AGK cutting rules[9]. Without entering here into technical details, such procedure amounts to multiply each amplitude by $\Theta(E - \sum_j \omega_j)$ and maintaining phase coherence them. By a saddle point evaluation of the usual integral representation of the Θ-function one can compute the radiated energy distribution[7]. We observe that, while b approaches the critical value $b_c = 1.6R$, the energy distribution shows an intermediate regime where it is enhanced by tidal forces and decreases only as $\sim \omega^{-2/3}$, before the eventual exponential decrease $\sim \mathrm{e}^{-\omega/T}$ at large ω's. Such exponential behaviour is typical of a thermal radiation. But here the radiation is coherent and the S matrix is unitary. The "quasi-temperature" parameter T is of order of the Hawking temperature $T_H = 1/(4\pi R)$ for $b \to b_c$: $T \simeq 0.8/R$. In particular it decreases with increasing energy of the collision.

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