Let's play with partitions! The distribution of the eigenvalues of the area operator

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1 Abstract

The area operator, that appears in the context of Loop Quantum Gravity, has as eigenvalues all possible numbers of the form

$$\sum_{i=1}^{n} \sqrt{n_i(n_i + 2)}$$

for some natural numbers n_i. Several approximate methods have been proposed along the years to study the distribution of these eigenvalues. They rely on approximations to get rid of the square root and known results about integer partitions, in particular the classic asymptotic estimates due to Hardy, Ramanujan and Rademacher. The main problem with these approaches is that different approximations lead to different results and, hence, are not conclusive. In this talk I will present a method that we developed, based on Laplace transforms, that provides a very accurate solution to this problem. The representation that we get is valid for any area and can be used, in particular, to obtain its asymptotics in the large area limit.

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