Direction of Arrival Estimation for Transient GW Sources via Time-Frequency Representations

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Direction of arrival (DOA) estimation for transient gravitational wave (GW) observations is a crucial aspect of GW and multi-messenger astronomy, using data from GW interferometric detectors and several different instruments (e.g., radio and gamma-ray telescopes, and neutrino observatories). The DOA can be retrieved with high accuracy from the observed arrival-time delays between the GW signals detected by three (or more) non-colocated interferometers. Time-Frequency representations (TFR) are widely used for un-modeled transient GW data analysis. The arrival-time delay between two detectors can be estimated by aligning the related TFR. The aligning algorithm is easily parallelizable - an appealing feature for GPU implementation, and a well suited for 3rd generation detectors (ET), with high expected detection rates. In this work we compare different TFRs (including the Q-Transform and the smoothed Wigner-Ville distributions) and sub-pixel alignment techniques (phase correlation and t-norms) by using numerical simulations for different Signal-to-Noise ratios, and public domain LIGO data.

Keywords: Gravitational Waves, Time-Frequency representations, Wigner-Ville distribution, Continuous Q Transform, Triangular Norms, Phase Correlation, Time Delay Estimation.

1. Time-Frequency Representations

Among the many proposed tools for gravitational waves (GW) detection and processing, time-frequency representations (TFR) play an important role. For example, in the time-frequency (TF) domain many GW transients, such as GW chirps from inspiraling binaries[1], are well represented by one-dimensional signatures (*ridges*) that reproduce their *instantaneous-frequency* evolution[2]. In this work we will compare the performance of the following TF representations:

- *Constant Q-Transform* (CQT)[3,4], currently employed in many flavors (Q, Omega and Omicron pipelines), by the LIGO-Virgo Collaboration (LVC);
- *Wigner-Ville* (WV) distribution, featuring the best localization/resolution properties[5,6];
- *Smoothed WV*[7], that removes the intermodulation artifacts which affect the visual readability of the WV distribution.

In Fig. 1 we show the TFR of GW150914 signal. Compared to CQT, both WV-based representations provide a sharper instantaneous frequency. Therefore they are better candidates for arrival-time delay estimation.

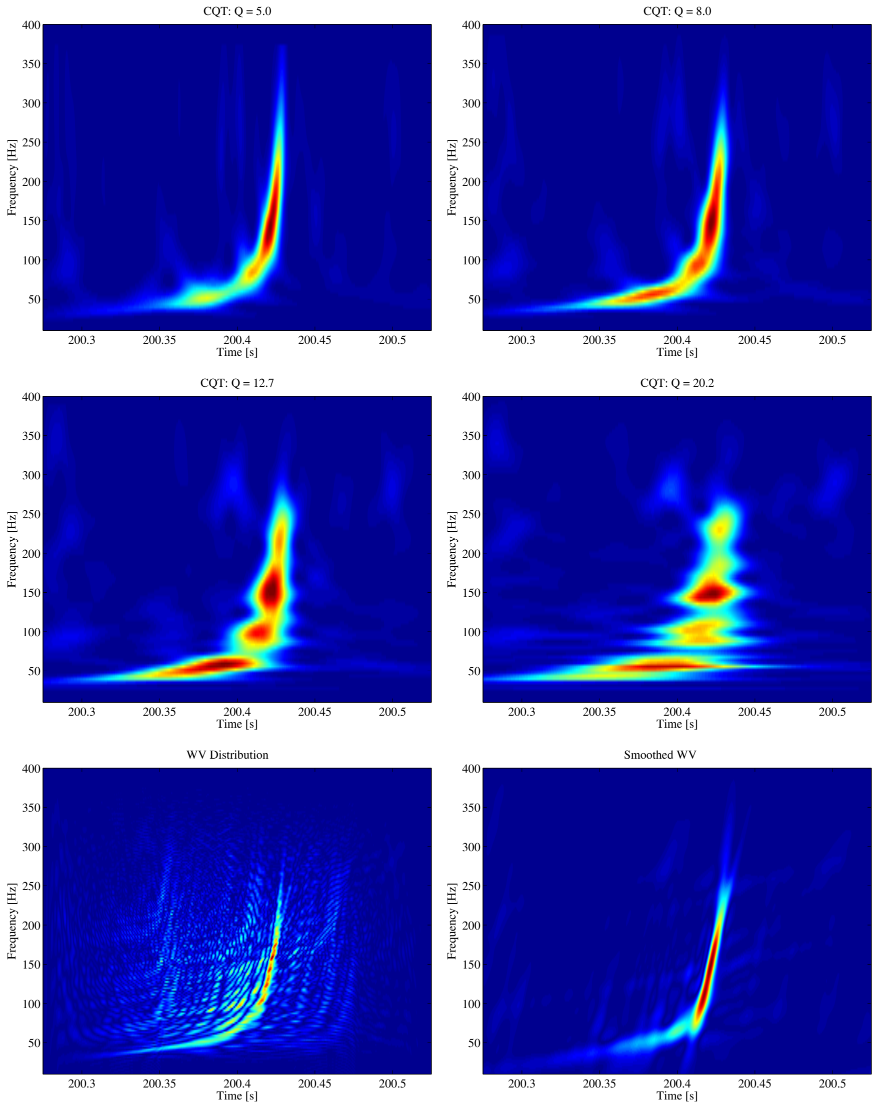

Fig. 1. TFRs for GW150914 (Hanford): the first two rows contain CQT representations (for increasing values of Q factor); the last row is devoted to WV-based TFRs.

2. Arrival-Time Delay Estimation

The main purpose of this paper is to evaluate the effectiveness of the TF representations in estimating the arrival-time delay between the signals originated by the same source and received by two (or more) non-colocated detectors. To this end, it is possible to use several aligning techniques typically used in image processing

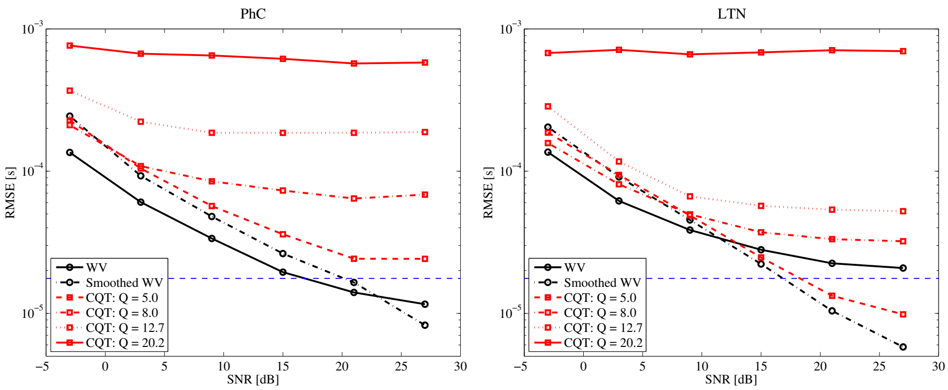

Fig. 2. RMSE vs SNR for Phase Correlation (PhC) (left plot) and *Łukasiewicz Triangular Norm* ($\widehat{d}_{LTN}$) (right plot) arrival time estimators, computed via $N_{MC} = 100$ Monte Carlo trials. The blu dashed line, in both plots, corresponds to the quantization error $= 1/(f_s\sqrt{12})$.

so as to align the TFR representations. In particular, we will test the performance of the *Phase Correlation* (PhC) method[8] and the *Łukasiewicz Triangular Norm* (LTN) method[9], both allowing sub-pixel resoluton.

3. Numerical Results

In this section we illustrate the performance of the TF-based time delay estimation procedures via numerical simulations. We generate two chirpy signals $s_x(t)$ and $s_y(t)$ using a commonly adopted model for GW sources, namely, the *IMR-PhenomPv2* model[10], whose relevant parameters are the masses M_1 and M_2 of the two merging black holes, the *effective spin* χ_{eff} and the observed time delay d between $s_x(t)$ and $s_y(t)$. We set these parameters so as to reproduce the estimated GW150914 waveform[11,12]. We set the observation interval $T = 1/8$ s and the sampling frequency $f_s = 16384$ Hz

Then, disregarding (for sake of simplicity) the different gain of the two interferometers, Gaussian white noise is added to have the same *Signal-to-Noise Ratio* (SNR) in both detectors. We compute the TFR of $x(n) = s_x(n) + w_x(n)$ and $y(n) = s_y(n) + w_y(n)$, where $w_x(n)$ and $w_y(n)$ are independent Gaussian white noises with the same variance. Finally, we estimate the arrival-time delay and we assess the estimation performance via a standard Monte Carlo method, computing the Root Mean Square Error (RMSE), defined as

$$RMSE = \sqrt{\frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} (\widehat{d}_i - d)^2}, \tag{1}$$

where N_{MC} is the number of Monte Carlo trials (we set $N_{MC} = 100$), $\widehat{d}_i$ is the time delay estimate for the i-th trial, and d is the actual time delay.

The results are shown in Fig. 2. We can see that, as expected, the sharpness of the TF representation is the key for both (PhC and LTN) estimators. Indeed, the CQT exhibits a bad performance for high values of the Q factors, since it gives good frequency resolution but poor time resolution, while the RMSE is significantly lower for low Q values. As regards to the WV-based TF representations, we can see that, for low and medium SNR values, the sharpness of the WV distribution yields a clear advantage, but for high SNR the intermodulation artifacts cause a performance degradation. This degradation is absent in the *Smoothed* WV, that outperforms the other TF representations for high SNR values when LTN arrival-delay estimation is used.

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