# Are dark matter and dark energy opposite effects of the quantum vacuum?

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## ABSTRACT

Properties as curvature, viscous fluid, dragging frame and gravity action are attributed mistakenly to the spacetime by the materialist substantivalism, the most credible philosophical interpretation of the General Relativity, caused by its absence of physical definition of spacetime and static gravitational field as immaterial, but which violates, the conception of gravity as an effect of coordinates of the generalization of the inertial motion to the accelerated motion and, in particular, the description of the metric tensor of gravity as a geometric field. These properties are really of the quantum vacuum, the main existence form of the matter. In this paper we propose that the quantum vacuum is the source of dark matter and dark energy, therefore, the components of the quantum vacuum are of them. Both are opposite effects of the quantum vacuum that when gravitationally interacts with the cosmic structures, the vacuum it curves and when such interaction tends to cease by declination of the formation of these structures.

## 1. INTRODUCCION

Quantum and Relativity dispute the physical conception on universe. Quantum theory based in quantum phase coherence, dynamical vacuum and complex scalar field existent in the vacuum, in the phase dynamics describes the universe as a superfluid [1]. We have proposed a model on celestial mechanic based in a quantum force of gravity that can be descript by Newton-Poisson equations and the parameters PPN α that would measure the curvature of quantum vacuum produced by masses [2] and PPN λ, which would measure the nonlinearity in the overlap of gravity called parametrized post-Newtonians equations [3]. Of course, these effects of curvature are not gravity [4] but external effects of the curvature of the space geometry of the vacuum, since according to author spacetime is the structural property of the dynamic matter [5]. Therefore, we abandon General Relativity describing gravity as the interaction of particles with spacetime geometry, and we assume they interact with the vacuum fluid [6] adopting Quantum theory from Newton. However, we recognize the great heuristic power of the Grossmann-Einstein equations that we consider they unify the effects of quantum gravity force and curvature of quantum vacuum integrating mistakenly both as only the curvature of the spacetime [7], that according to the author based in the measurement of the deflection of photons traveling closed to the sun gravity and vacuum curvature each one contributes half. Since, in Opticks, Newton formulated that photons should be affected by gravity in the same way as is ordinary matter and Cavendish and Von Soldner calculated deviation of light in the vicinity of the sun by an angle of 0.875 arcseconds, the half of 1.75 arcseconds according Einstein. Therefore, if it removes the effect of Newton gravity then it obtains the effect of curvature of the vacuum, since according the most recent observations of gamma rays in the GLAST project, the gravitational interaction between the photons and the vacuum is extremely weak, being quantitatively negligible. Too, we retain of Special Relativity the equivalence between mass and energy [8] fundament of that energy gravitates.

Of other hand, we consider any material existence compound of fermions as substance, bosons as field: real bosons as radiation and virtual bosons and virtual gluons as vacuum. So, universe is matter, radiation and quantum vacuum. They are material forms of existence and together they are the Matter. The quantum vacuum is filled with virtual particles, which are in a continuous state of fluctuation; these virtual particles are created in quantum vacuum fluctuations, which are the temporary change in the amount of energy in a point in space, as explained by Heisenberg uncertainty: $\Delta E.\Delta T \geq \hbar/4\pi$. According to the author, quantum vacuum satisfies the functions that were attributed to the aether without the inconvenient of its mechanical fermion structure, because the quantum vacuum is a bosonic medium, elastic, gifted of inertia, subject to the superposition principle of Bose-Einstein, and viscous fluid nature that in the Gravity Probe-B, NASA [9], [10], [11] attributes mistakenly to the spacetime.

In 1981, Mukhanov and Chibisov, authors of Quantum Origin of the Universe Structure, published that in the present the structure of the Universe in the scale $\leq 10^{-27}$ cm are quantum fluctuations that produced originally the spectrum of inhomogeneities, as the galaxies and their clusters, in the early Universe [12]. The numerous experiments, during the era of the high precision cosmology, characterized by the use of the satellites COBE, in 1992, WMAP, in 2003, and completed by mission Planck, in 2013, in which there were measured the temperature fluctuations of the Cosmic Microwave Background Radiation, CMB, are in highly agree with their predictions confirmed definitely that assures us that everything in our Universe was originated from quantum fluctuations [13].

Inside of previous context the dark energy and dark matter are necessarily aspects of the quantum vacuum. Although, we consider the universe as a superfluid, we defer of the dark energy as energy density of the superfluid, and dark matter as fluctuations of the superfluid of Kerson Huang [1] because we believe they are opposite effects of the dynamical vacuum as a fluid under the action of their two opposite gravity and pressure forces. When pressure is negligible the effects of dark matter occur and when gravity is negligible the effects of dark energy are produced. This is the thesis that it presents in this work. With such objective, as Matter cosmologically is considered a fluid, we describe it through of the relativistic equation of fluid perfect, i.e, when there is no dissipative current in it, building a very simple cosmological model through equation (1) using variables (ρ, p, u), representing respectively: energy/mass rest density, pressure and velocity vector field of a matter flow, the constant c of the electromagnetic wave speed in vacuum, the metric tensor $g_{\mu\nu}$ that describe the geometry of spacetime structural form of the Matter, according the author and the Matter tensor $T_{\mu\nu}$ according equation (2). Too, we use the cosmological equation of state (3).

$$T^{Matter}_{(\mu\nu)} = (\rho c^2 + p) u_\mu u_\nu + p g_{(\mu\nu)} \quad \text{(Eq. 1)}$$

$$T^{Matter}_{(\mu\nu)} = T^{vacuum}_{(\mu\nu)} + T^{(bosonic\ radiation)}_{(\mu\nu)} + T^{(fermionic\ matter)}_{(\mu\nu)} \quad \text{(Eq. 2)}$$

$w = p/\rho$ (Eq. 3)

In the current bibliography we find that some our previous considerations coincide in general with the alternative of Vacuum dark fluid proposing that dark matter and dark energy are two different manifestations of the vacuum dark fluid. On galactic scales, the dark fluid behaves like dark matter, and at larger scales it behaves like the dark energy [14], [15], [16], [17], [18], [19]. Our differences are: that vacuum fluid is feedbacked by the boson disintegration, of the fermion matter, therefore energy of vacuum is not constant, although , according to the observations, its increase tends to 0; its gravitational interaction with itself is low, tending to 0, and causing that at larger scales, away of the galactic structures its geometry, it becomes flat Minkowskian, while when it is trapped its gravitational interaction with the galaxies is strong, causing, in the terms of its geometry, spheroidal curves.

## 2. QUANTUM VACUUM

Quantum vacuum is a medium that permeates totally the Universe: the called outer space believed by antiques as absolutely empty generating the mistaken theory of the space existing itself of the philosophical substantivalism [20]; that is, the space apparently emptiness between stars, where the density is $10^{-24}$ g/cm$^3$, in the Universe $10^{-30}$ g/cm$^3$, and in apparent empty space regions with density of $10^{-33}$ g/cm$^3$ [21] and in the atoms where more than 99,999% is vacuum. So, Matter is mainly quantum vacuum.

In 1916, from the Quantum theory and Planck's law for the radiation of a black body, Nernst proposed that the vacuum is a medium filled with radiation. Near 1925, from Quantum electrodynamics, the electromagnetic field was treated as a collection of quantized harmonic oscillators with non-vanishing zero-point energy [22]. In 1948, in the Philips laboratories, the energy of the vacuum was stablished through of the Casimir effect [23]. According, Quantum physics and Quantum field theory, the vacuum is filled with a collection of quantum fields, in special with low-energy electromagnetic waves, random in phase and amplitude and propagating in all directions. Everywhere space exhibits zero-point fluctuations, even in regions devoid of matter and radiation [24], [25], [26]. Virtual particle-antiparticle pair are created from vacuum and annihilated back to vacuum during the Heisenberg uncertainty lapse. The Standard Model includes an additional coupling of its constituent fields to Higgs fields permeating totally the vacuum and generating the masses of the massive particles. In 2012, the Higgs boson was experimentally discovered by the CERN [27], [28].

In the decade of 1980, from Starobinsky model of eternal inflation of Universe, Mukhanov elaborated the theory of inflationary perturbations of metric, valid for a broad class of inflationary models. Now, the standard method of investigation of inflationary perturbations. Mukhanov started with the simple Newtonian approach to the theory of density perturbations in an expanding universe that extended to the General Relativity, and finished with the full quantum theory of production and subsequent evolution of inflationary perturbations of metric [29]. In 2008 Mukhanov said: "Our results were obtained for the first particular working model of inflation based on $R^2$−gravity, which is conformally equivalent to a model with a scalar field" [30] i.e, gravitons.

In this work, we describe dynamic vacuum through of the relativistic equation of fluid perfect through equation (4) using variables (p, p, u), representing respectively: energy density of vacuum, pressure vacuum and velocity vector field of energy flow of vacuum, the constant c of the electromagnetic wave speed in vacuum and the metric tensor $g_{\mu\nu}$ that describe the geometry of spacetime of vacuum and the vacuum tensor $T^{vacuum}_{\mu\nu}$. In such case:

$$T^{vacuum}_{\mu\nu} = (\rho^{vacuum} c^2 + p^{vacuum}) u^{vacuum}_\mu u^{vacuum}_\nu + p^{vacuum} g^{vacuum}_{\mu\nu} \quad \text{(Eq. 4)}$$

## 3. DARK ENERGY

Independent, the groups: High-Z Supernova Search Team (1998) and the Supernova Cosmology Project (1999) stablished accelerating expansion of the universe. To explain it has been proposed the dark energy that would be highly homogeneous, lowly dense and interacting only with the gravity force [31]. Dark energy is formulated in two main forms: As constant energy density filling vacuum homogeneously i.e cosmological constant and as scalar field such as quintessence whose energy density would change extremely very slowly in time and space, filling the vacuum with minimum low inhomogeneity, very difficult of distinguish of cosmological constant. In both cases such energy may be identified with energy of vacuum. Although, other alternatives exist we do not consider because they would violate the identity of dark energy with energy of vacuum.

Accelerating expansion of the universe it produces since the last 5000 million years; after of the eras: radiation and baryon matter domains. Current era is of dark energy domain, i.e. each era corresponds to domain of a form of the material existence. Of course, the current era is of vacuum domain. As there is no compelling explanation for cosmic acceleration; we believe that with final of the domain of the formation of the great cosmic structures of the era of matter, these structures could are contributing in a very low magnitude to vacuum, that is, the Matter could be ruled by the contrary two tendencies: quantum fluctuations of the vacuum (boson fields) produce fermion matter and the disintegration of the fermion matter in virtual boson fields like real boson particles (radiation) and virtual boson particles (vacuum), increasing softly energy of vacuum. Thus, we prefer dark matter as quintessence.

Being vacuum dark fluid the most credible source of dark energy because “the vacuum has a pressure equal to minus its energy density, $p_{VACUUM} = -\rho_{VACUUM}$. This also means that the energy of vacuum is mathematically equivalent to a cosmological constant” [31] , [32], with w =−1.

If we consider vacuum very distant of the gravitational influence of the great cosmic structures then we find vacuum as dark energy from equations (5, 6 and 7), using variables (ρ, p, u), representing respectively: energy rest density of vacuum, pressure vacuum and velocity vector field of energy vacuum flow, cosmological constant Λ, the metric tensor of Minkowski $\eta_{\alpha\beta}$ that describe the geometry of flat spacetime of vacuum since we supposed 0 gravity and the vacuum tensor $T_{\mu\nu}$ is deduced of the equations:

Energy of dynamic complex scalar vacuum =

$$\rho_{\Lambda} + \rho_{\text{bosonic disintegration matter}} - \rho_{\text{lost by quantum fluctuations that produces matter}} \quad \text{(Eq. 5)}$$

Energy of dynamic complex scalar vacuum according observations $\approx \Lambda \rightarrow \rho \approx 0$ (Eq. 6)

$w = -1 \rightarrow -p$ (Eq. 7)

$$T_{\alpha\beta}^{vacuum} = -p^{vacuum} u_{\alpha}^{vacuum} u_{\beta}^{vacuum} - p^{vacuum} \eta_{\alpha\beta}^{vacuum} \quad \text{(Eq. 8)}$$

Thus vacuum accelerated expansion of the universe.

## 4. DARK MATTER

The method used in cosmology to determine the mass of stars and the great cosmic structures as a galaxy, or a cluster of galaxies is measuring how its gravity determines the motion of other stars or cosmic structures around it, using Newton’s form of Kepler’s third Law rewritten as: $m = v * r^2 / G$, variables (m, v, r), represent respectively: mass enclosed by orbit, orbital velocity and average orbital separation and the gravitational constant G. In 1922, Kapteyn, in 1932, Dutchman and Oort and finally, in 1933, Zwicky discovered that the motion of galaxies externs gyrate more rapidly for their gravity to maintain them in a group, in last case, of more of 1000, of the Coma cluster, being necessary 100 times more hide mass to explain such speed of those galaxies [33]. Only closed 1970, this discovery was accepted when Rubin and Ford showed, measuring the Doppler shift of clouds of hydrogen gas, in several distances, around the center of their galaxies to require far more mass than by its optical detectable matter. Depending on the gravity, the hydrogen gas clouds or galaxies closer to the center they would have a greater speed than the external, since the gravity diminishes in inverse proportion to the square of distance. However, the observations give a result different since the speeds are higher than the expected. The discrepancy between the amount of visible matter and the strength of gravity is most pronounced in the smallest galaxies in cluster, known as dwarf spheroidals [33].

Oort explained as dark matter the possible hide matter; in its searches, after excluding all the particles of the Standard Model it has explored various possible sources that insufficient or don't emit radiation electromagnetic, interact gravitationally and they are detectable by gravitational lensing as black holes, all kind of Machos, like brown dwarf. But there weren't enough to account for the amount of dark matter needed [34]. Too, neutrinos that were discarded by their high speed since they would have prevented the clumping of the universe and thus the density fluctuation would have collapsed on large scales [34].

The particles of the dark matter would have the properties: strong gravitational interaction, uncharged since they don't respond to electromagnetic force, color force does not appear, massive, stable (13,7 billion years) [35], freely-slowly motion. There are various types of particles proposed as: WIMPs, axion and very heavy Planckian interacting massive particle [36]. Of all the mass and energy in the universe, the 95.1% is considered dark matter and dark energy, therefore, its proportion with respect of baryon matter would be 19.4 to 1, being 2.5:1 ratio of dark energy to dark matter and by each gram of baryon matter there would be 5.68 grams of dark matter at least on cosmological scales [37].

In 1983, Milgrom proposed MOND, which modifies second Newton´s law for that in its formulation as third Kepler´s law adjust it to the anomalous speeds observed but without that could explain totally the dynamics of galaxies within clusters in particular Bullet cluster [33]. Other alternatives related or derived of MOND for e.g.: Bekenstein, 2004, relativistic version adapted to test of General Relativity and significant gravitational lensing [38], Brownstein and Moffat, 2007 based in modified gravity, a covariant generalization of General Relativity with auxiliary (gravitational) fields, adjusted to include Bullet cluster [37].

A new experimental discovery realized in 2006 appoint to the existence of hide matter in form of energy of vacuum since it did not interact during crash of two galaxies. The theory of Vacuum dark fluid considers that “when the dark fluid is in the presence of the matter, it slows down and coagulates around it; this then attracts more dark fluid to coagulate around it, thus amplifying the force of gravity near it. This effect is always present but only becomes noticeable in the presence of a really large mass, like a galaxy.” [39].

Of other hand, the author considers that the geometry of the vacuum dark fluid, trapped by structures of the fermion matter under its gravitational action, it curves. In the scale of our solar system the maxim curvature exists closed to Sun causing the deflection of the electromagnetic wave by an angle of 0.875 arcseconds that it can assume like the curvature of the vacuum at such position. In the scale of galaxies and cluster the curvature of vacuum, it should close spheroidally, due to the great gravitational force exerted from the gravity center of the galaxy or cluster in rotation, causing furthermore its rotation and concentration of its density

peripherally by the action of the centrifugal force, producing the observed effect that dark matter it concentrates peripherally and of course annulling its pressure, unavoidable consequence of the closed curvature since the energy of vacuum lost its pressure and only gravity remains. Due to that the geometry of the vacuum, it curves spheroidally, exactly coinciding with the geometry of the dwarf spheroidals, it produces the maximum exploitation of energy of vacuum as dark matter, effect also observed. Therefore under this condition of the vacuum completely trapped by the fermionic matter the cosmological model that it should apply is of dust matter, thus "the gravitational field is produced entirely by the mass, momentum, and stress density of a perfect fluid which has positive mass density but vanishing pressure", therefore, fluid acts like only gravity [40] being we suppose:

$$T_{(\mu\nu)}^{(galaxy\,cluster)} = T_{(\mu\nu)}^{(fermionic\,matter\,of\,galaxy\,cluster)} + T_{(\mu\nu)}^{(vacuum\,trapped\,by\,galaxy\,cluster)} \qquad \text{(Eq. 9)}$$

$T_{(\mu\nu)}^{(galaxy\,cluster)}$ 3 dimensional spheroid immerse in a 4-dimensional Euclidean spacetime of the existent vacuum beyond galaxy cluster, that is, of the vacuum no trapped, therefore, with metric:

$$ds^2 = e^{(\nu(r))}dt^2 - e^{(\lambda(r))}dr^2 - r^2 d\,\theta^2 - r^2 \sin^2\theta\, d\,\varphi^2 \qquad \text{(Eq. 10)}$$

$T_{(\mu\nu)}^{(galaxy\,cluster)}$, to Einstein clusters on pseudo spheroidal space-times, should satisfy Einstein-Florides´ field equations (13). Einstein cluster (dust matter) is a special type of anisotropic distribution of fluid for which the radial stress is zero, since it is at rest, and non-vanishing tangential stress maintaining the equilibrium, i.e. counter gravitational collapse. Florides (1974) provides a scheme for construct solutions from Einstein's field equations for Einstein clusters [41] with metric:

$$R_{\mu}^{\nu} - \frac{1}{2} R\,\delta_{\mu}^{\nu} = -8\,\Pi\,T_{\mu}^{\nu} \qquad \text{(Eq. 11)}$$

So, from p=0 according to observations, energy of vacuum it manifests as dark matter (Eq. 12)

and from equations (9) and (11) it obtains:

$$T_{(\mu\nu)}^{(dust\,matter)} = T_{(\mu\nu)}^{(galaxy\,cluster)} \qquad \text{(Eq. 13)}$$

that implicates the use of a model in extreme simplified. Of other hand, like according to ρ:

$$T_{(\mu\nu)}^{(fermionic\,matter\,of\,galaxy\,cluster)} = \rho^{(fermionic\,matter\,of\,galaxy\,cluster)} c^2 u_{\mu} u_{\nu} \qquad \text{(Eq. 14)}$$

$$T_{(\mu\nu)}^{(vacuum\,trapped\,by\,galaxy\,cluster)} = \rho^{(vacuum\,traped\,by\,galaxy\,cluster)} c^2 u_{\mu} u_{\nu} \qquad \text{(Eq. 15)}$$

Therefore:

$$T_{(\mu\nu)}^{(dust\,matter)} = \rho^{(fermionic\,matter\,of\,galaxy\,cluster)} c^2 u_{\mu} u_{\nu} + \rho^{(vacuum\,traped\,by\,galaxy\,cluster)} c^2 u_{\mu} u_{\nu} \qquad \text{(Eq. 16)}$$

Thus vacuum dark fluid acts like dark matter.

## 6. CONCLUSIONS

Dark matter and dark energy, it can explain from energy of the vacuum as a dark fluid, subject to the two opposite tendencies of the forces of gravity and pressure, present in its typical configuration of perfect fluid. Since, the dark vacuum as matter it gravitates and the dark vacuum as energy it presses. That is, as matter the vacuum attracts and as energy the vacuum repels. Thus, under external influences can arise, from dark vacuum, two extreme states: - When the dark vacuum is trapped inside great cosmic structures of dominion of the matter like galaxy cluster, it vanish the pressure and, therefore, dark vacuum it manifests only as a quantum force of gravity. - When the dark vacuum is free of the dominion of the matter due to that its gravity self it tending to 0, dark vacuum it manifest only as a force of pressure.

The existence of these two states of the vacuum are supported in the astronomic register of the anomalies of the orbital velocities of peripheral galaxies in the galaxy clusters highest according is estimated by equations of the celestial mechanic, that driving to dark matter and the accelerated expansion of the universe that driving to dark energy.

Finally, author proposes:- The two most general opposite tendencies that rule the dynamic Matter are the energy of the vacuum is feedbacked from the disintegration of the fermionic matter in bosonic matter and the fluctuations of the quantum vacuum generates the fermionic matter. -The presence or absence of fermionic matter generates the two opposite geometric states of the configuration of the vacuum. The geometry of the spacetime of the vacuum, it curves under the action of the fermionic matter, causing on spheroids, null pressure and the Euclidean spacetime of the vacuum free of fermionic matter produces null gravity.

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