# The effective field theory approach of teleparallel gravity, $f(T)$ gravity and beyond

Chunlong Li*, Yi-Fu Cai† and Ling-Qin Xue§

*Department of Astronomy, University of Science and Technology of China, Hefei, Anhui 230026, China*
*CAS Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei, Anhui 230026, China*
**E-mail: chunlong@mail.ustc.edu.cn*
*†E-mail: yifucai@ustc.edu.cn*
*§xuelq@mail.ustc.edu.cn*
*www.ustc.edu.cn*

Yong Cai

*School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*
*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*
*E-mail: caiyong13@mails.ucas.ac.cn*

Emmanuel N. Saridakis

*Department of Physics, National Technical University of Athens, Zografou Campus GR 157 73, Athens, Greece*
*CASPER, Physics Department, Baylor University, Waco, TX 76798-7310, USA*
*Chongqing University of Posts & Telecommunications, Chongqing, 400065, China*
*E-mail: Emmanuel_Saridakis@baylor.edu*

We develop the effective field theory approach to torsional modified gravities, a formalism that allows for the systematic investigation of the background and perturbation levels separately. Starting from the usual effective field theory approach to curvature-based gravity, we suitably generalize it at the background level by including terms of the contracted torsion tensor, and at the perturbation level by including pure torsion perturbative terms and mixed perturbative terms of torsion and curvature. Having constructed the effective field theory action of general torsional modified gravity, amongst others we focus on $f(T)$ gravity and we perform a cosmological application. We investigate the scalar perturbations up to second order, and we derive the expressions of the Newtonian constant and the post Newtonian parameter $\gamma$. Finally, we apply this procedure to two specific and viable $f(T)$ models, namely the power-law and the exponential ones, introducing a new parameter that quantifies the deviation from general relativity and depends on the model parameters. Since this parameter can be expressed in terms of the scalar perturbation mode, a precise measurement of its evolution could be used as an alternative way to impose constraints on $f(T)$ gravity and break possible degeneracies between different $f(T)$ models.

*Keywords*: Teleparallel Gravity, $f(T)$ Gravity, Cosmological Perturbations, Effective Field Theory

## 1. The Effective Field Theory Approach to Torsional and $f(T)$ Gravity

For more details about this work we refer readers to our recent work[1,2].

The action of the effective field theory (EFT) after combining modified teleparallel gravity is

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R - \Lambda(t) - b(t)g^{00} + \frac{M_P^2}{2}d(t)T^0\Big] + S^{(2)} \ , \qquad (1)$$

where $T^0 = g^{0\mu}T^{\nu}{}_{\mu\nu}$ and $T^{\mu}{}_{\nu\rho}$ is the torsion tensor. In the above action we have included the part $S^{(2)}$, which includes all terms that explicitly start quadratic in perturbations, and thus it does not affect the background dynamics. In addition to the terms presented in the ordinary EFT action, the part $S^{(2)}$ is expected to include also:
i) Pure torsion terms such as $\delta T^2$, $\delta T^0\delta T^0$ and $\delta T^{\rho\mu\nu}\delta T_{\rho\mu\nu}$ ($T$ is the torsion scalar.);
ii) Terms that mix curvature and torsion, such as $\delta T\delta R$, $\delta g^{00}\delta T$, $\delta g^{00}\delta T^0$ and $\delta K\delta T^0$ ($K$ is the trace of the extrinsic curvature.).

From this action we could derive the equivalent energy density and pressure of the dark energy:

$$\rho_{DE}^{\rm eff} = b + \Lambda - 3M_P^2\Big[H\dot{\Psi} + \frac{dH}{2} + H^2(\Psi - 1)\Big] \ , \qquad (2)$$

$$p_{DE}^{\rm eff} = b - \Lambda + M_P^2\Big[\ddot{\Psi} + 2H\dot{\Psi} + \frac{\dot{d}}{2} + (H^2 + 2\dot{H})(\Psi - 1)\Big] \ . \qquad (3)$$

## 2. Application to $f(T)$ Theory and Its Cosmology

For $f(T)$ gravity we have the below correspondence to the parameters of the EFT action.

$$\begin{aligned}
\Psi(t) &= -f_T(T^{(0)}) \ , \\
\Lambda(t) &= \frac{M_P^2}{2}\left[T^{(0)}f_T(T^{(0)}) - f(T^{(0)})\right] \ , \\
d(t) &= 2\dot{f}_T(T^{(0)}) \ , \\
b(t) &= 0 \ ,
\end{aligned} \qquad (4)$$

where we have used symbol $f_T$ to label the derivate of $f(T)$ with respect to $T$ and $T^0$ represents the background value. So the EFT action of $f(T)$ gravity is

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[-f_T(T^{(0)})R + 2\dot{f}_T(T^{(0)})T^0 - T^{(0)}f_T(T^{(0)}) + f(T^{(0)})\Big] \ . \qquad (5)$$

### 2.1. *Scalar perturbations*

The scalar perturbations of tetrad[9] under the Newtonian gauge up to second order is

$$e^0_\mu = \delta^0_\mu \big(1+\phi+\frac{1}{2}\phi^2+\frac{1}{2}\partial_i\chi\partial_i\chi\big) + a\delta^i_\mu\Big[\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big]\ , \tag{6}$$

$$e^a_\mu = a\delta^i_\mu\delta^a_i\big(1-\psi+\frac{1}{2}\psi^2\big)+\frac{a}{2}\delta^i_\mu\delta^a_j\partial_i\chi\partial_j\chi+\delta^0_\mu\delta^a_i\Big[\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big]\ , \tag{7}$$

$$e^\mu_0 = \delta^\mu_0\big(1-\phi+\frac{1}{2}\phi^2+\frac{1}{2}\partial_i\chi\partial_i\chi\big)+\frac{1}{a}\delta^\mu_i\Big[-\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big]\ , \tag{8}$$

$$e^\mu_a = \frac{1}{a}\delta^\mu_i\delta^i_a\big(1+\psi+\frac{1}{2}\psi^2\big)+\frac{1}{2a}\delta^\mu_i\delta^j_a\partial_i\chi\partial_j\chi+\delta^\mu_0\delta^i_a\Big[-\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big]\ , \tag{9}$$

where $\chi$ is the additional scalar that could be present in the $f(T)$ theory. This perturbed form of tetrad will give rise to the below perturbed metric

$$g_{00} = -(1+2\phi+2\phi^2)\ , \tag{10}$$

$$g_{ij} = a^2\delta_{ij}(1-2\psi+2\psi^2)\ , \tag{11}$$

$$g^{00} = -(1-2\phi+2\phi^2)\ , \tag{12}$$

$$g^{ij} = a^{-2}\delta_{ij}(1+2\psi+2\psi^2)\ , \tag{13}$$

Substituting these expressions into the EFT action and varying it with respect to the scalars, we could derive the Newtonian constant after choosing the post-Newtonian approximation

$$k^2\phi = \frac{a^2}{2M_P^2 f_T}\left(1-\frac{a^2M^2}{M_P^2 f_T H^2 k^2}\right)\delta\rho_m \approx \frac{a^2}{2M_P^2 f_T}\delta\rho_m\ . \tag{14}$$

And the post-Newtonian parameter is

$$\gamma = \frac{\psi}{\phi} = 1+\frac{a^2M^2}{f_T H^2 k^2 M_P^2 - a^2M^2} \approx 1\ . \tag{15}$$

### 2.2. *Tensor perturbations*

The tensor part of the perturbation of tetrads is

$$\bar{e}^0_\mu = \delta^0_\mu\ , \tag{16}$$

$$\bar{e}^a_\mu = a\delta^a_\mu + \frac{a}{2}\delta^i_\mu\delta^{aj}h_{ij}+\frac{a}{8}\delta^i_\mu\delta^{ja}h_{ik}h_{kj}\ , \tag{17}$$

$$\bar{e}^\mu_0 = \delta^\mu_0\ , \tag{18}$$

$$\bar{e}^\mu_a = \frac{1}{a}\delta^\mu_a - \frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij}+\frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik}h_{kj}\ , \tag{19}$$

which gives rise to the below metric

$$g_{00} = -1\ ,\quad g_{0i} = 0\ ,$$

$$g_{ij} = a^2\big(\delta_{ij}+h_{ij}+\frac{1}{2}h_{ik}h_{kj}\big)\ . \tag{20}$$

From the action (5), we know the only difference of $f(T)$ EFT action from that of $f(R)$ theory[3] is that the term $2\dot{f}_T(T^{(0)})T^0$. However because of $T^0 = 3H$, this term wouldn't contribute to the equation of motion, so we obtain the below equation of gravitational wave

$$\ddot{h}_{ij} + 3H\big(1 - \beta_T\big)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0, \tag{21}$$

where we have defined

$$\beta_T \equiv -\frac{\dot{f}_T}{3Hf_T} = \frac{d\ln f_T}{d\ln T}(1 + w_{tot}) \ . \tag{22}$$

So we could see that the speed of gravitational wave for $f(T)$ theory at the astrophysical scale is exactly the speed of light. However $\beta_T$ would modify the dispersion relation in the below way

$$\left|\frac{d\omega}{dk}\right| = \frac{1}{a}\Big[1 - \frac{9a^2}{4k^2}H^2(1 - \beta_T)^2\Big]^{-\frac{1}{2}} \ . \tag{23}$$

The above is just a rough analysis. More details can be found at the work[1,2].

## References


1. C. Li, Y. Cai, Y. F. Cai and E. N. Saridakis, arXiv:1803.09818 [gr-qc].
2. Y. F. Cai, C. Li, E. N. Saridakis and L. Xue, arXiv:1801.05827 [gr-qc].
3. G. Gubitosi, F. Piazza and F. Vernizzi, "The Effective Field Theory of Dark Energy," JCAP **1302**, 032 (2013) [arXiv:1210.0201 [hep-th]].
4. R. Aldrovandi, J.G. Pereira, "Teleparallel Gravity: An Introduction," Springer, Dordrecht, 2013.
5. Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, "Quintom Cosmology: Theoretical implications and observations," Phys. Rept. **493**, 1 (2010) [arXiv:0909.2776 [hep-th]].
6. G. R. Bengochea and R. Ferraro, "Dark torsion as the cosmic speed-up," Phys. Rev. D **79**, 124019 (2009) [arXiv:0812.1205 [astro-ph]].
7. C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, "The Effective Field Theory of Inflation," JHEP **0803**, 014 (2008) [arXiv:0709.0293 [hep-th]].
8. J. K. Bloomfield, E. E. Flanagan, M. Park and S. Watson, "Dark energy or modified gravity? An effective field theory approach," JCAP **1308**, 010 (2013) [arXiv:1211.7054 [astro-ph.CO]].
9. K. Izumi and Y. C. Ong, "Cosmological Perturbation in f(T) Gravity Revisited," JCAP **1306**, 029 (2013) [arXiv:1212.5774 [gr-qc]].
10. R. Zheng and Q. G. Huang, "Growth factor in $f(T)$ gravity," JCAP **1103**, 002 (2011) [arXiv:1010.3512 [gr-qc]].