Gravity, Null Surfaces and Thermodynamics

Sumanta Chakraborty*

Department of Theoretical Physics, Indian Association for the Cultivation of Science,
Kolkata, 700032, India
**E-mail: sumantac.physics@gmail.com*

T. Padmanabhan

IUCAA, Post Bag 4, Ganeshkhind
Pune, 411007, India
E-mail:paddy@iucaa.in

In this talk I will explore the consequences of the gravity-thermodynamics connection for an arbitrary null surface and highlight the thermodynamic significance of various geometrical quantities. In particular, I will demonstrate that: (a) A conserved current, associated with the time development vector in a natural fashion, has direct thermodynamic interpretation when evaluate on null surfaces. (b) Three different projections of a suitably defined gravitational momentum related to an arbitrary null surface in the spacetime lead to three different equations, all of which have thermodynamic interpretation. The first one reduces to a Navier-Stokes equation for the transverse drift velocity. The second can be written as a thermodynamic identity TdS = dE + P dV. The third describes the time evolution of the null surface in terms of suitably defined surface and bulk degrees of freedom. The implications will be discussed.

Keywords: Null Surfaces; Black Hole Thermodynamics

The key conclusions that we will reach in this talk are as follows:

- There is considerable amount of evidence to suggest that gravitational field equations have the same status as, say, the equations of fluid mechanics. They describe the macroscopic, thermodynamic, limit of an underlying statistical mechanics of the microscopic degrees of freedom of the spacetime. The macro and micro descriptions are connected through the heat density Ts of the spacetime. Here, the temperature T arises from the interpretation of the null surfaces as local Rindler horizons. The entropy density is a phenomenological input, the form of which determines the theory. For a very wide class of theories, it can be defined in terms of a function $F(R^{cd}_{ab}, \delta^i_j)$ built from (2,2) curvature tensor R^{cd}_{ab} and the Kronecker deltas, as:

$$s = -\frac{1}{8}\sqrt{q}P^{ab}_{cd}\epsilon_{ab}\epsilon^{cd}; \quad P^{ab}_{cd} = \frac{\partial F}{\partial R^{cd}_{ab}}; \quad \nabla_a P^{ab}_{cd} = 0 \tag{1}$$

- Given a Vector field v^a, one can construct three currents: (a) the Noether current $J^a(v)$, (b) the gravitational momentum $P^a(v)$ and (c) the reduced gravitational momentum $\mathcal{P}^a(v)$. Interestingly enough, one can attribute *thermodynamic* meaning to these quantities which are usually considered to be *geometrical*. For example, the conserved current J^a, associated with the time-development vector ξ^a of the spacetime, leads to a conserved charge (i.e., integral of $u_a J^a(\xi)$ defined either on a spacelike surface or on a null surface) that can be related to the boundary heat density Ts, where T is the Unruh-Davies temperature and s stands for entropy density.
- One can also define the notion of gravitational momentum P^a for all the Lanczos-Lovelock models of gravity such that $\nabla_a(P^a + M^a) = 0$ (where M^a is the momentum density of matter) for all observers, leads to the field equation of the Lanczos-Lovelock model. This generalizes a previous result for general relativity.
- The field equations can also be derived from a thermodynamic variational principle, which essentially extremises the total heat density of all the null surfaces in the spacetime. This variational principle can be expressed directly in terms of the total gravitational momentum, thereby providing it with a simple physical interpretation.
- One can associate with any null surface the two null vector fields ℓ_a, k_a with $\ell_a k^a = -1$ and ℓ_a being the tangent vector to the congruence defining the null surface, as well as the 2-metric $q_{ab} = g_{ab} + \ell_a k_b + \ell_b k_a$. These structures define three natural projections of the gravitational momentum $(P^a\ell_a, P^a k_a, P^a q_{ab})$, all of which have thermodynamic significance. The first one leads to the description of time evolution of the null surface in terms of suitably defined bulk and surface degrees of freedom; the second leads to a thermodynamic identity which can be written in the form $TdS = dE + PdV$; the third leads to a Navier-Stokes equation for the transverse degrees of freedom on the null surface which can be

interpreted as a drift velocity.

These results again demonstrate that the emergent gravity paradigm enriches our understanding of the spacetime dynamics and the structure of null surfaces, by allowing a rich variety of thermodynamic backdrops for the geometrical variables.

References

1. J. D. Bekenstein, "Black holes and entropy," *Phys.Rev.* **D7** (1973) 2333–2346.
2. J. D. Bekenstein, "Generalized second law of thermodynamics in black hole physics," *Phys.Rev.* **D9** (1974) 3292–3300.
3. S. Hawking, "Particle Creation by Black Holes," *Commun.Math.Phys.* **43** (1975) 199–220.
4. S. Hawking, "Black Holes and Thermodynamics," *Phys.Rev.* **D13** (1976) 191–197.
5. P. Davies, "Scalar particle production in Schwarzschild and Rindler metrics," *J.Phys.* **A8** (1975) 609–616.
6. W. Unruh, "Notes on black hole evaporation," *Phys.Rev.* **D14** (1976) 870.
7. G. W. Gibbons and S. W. Hawking, "Cosmological Event Horizons, Thermodynamics, and Particle Creation," *Phys. Rev.* **D15** (1977) 2738–2751.
8. V. Iyer and R. M. Wald, "Some properties of Noether charge and a proposal for dynamical black hole entropy," *Phys.Rev.* **D50** (1994) 846–864, `arXiv:gr-qc/9403028 [gr-qc]`.
9. T. Padmanabhan, "Thermodynamical Aspects of Gravity: New insights," *Rept. Prog. Phys.* **73** (2010) 046901, `arXiv:0911.5004 [gr-qc]`.
10. T. Padmanabhan, "Gravity and the thermodynamics of horizons," *Phys.Rept.* **406** (2005) 49–125, `arXiv:gr-qc/0311036 [gr-qc]`.
11. T. Padmanabhan, "Emergent Gravity Paradigm: Recent Progress," *Mod. Phys. Lett.* **A30** no. 03n04, (2015) 1540007, `arXiv:1410.6285 [gr-qc]`.
12. T.Padmanabhan, *Gravitation: Foundations and Frontiers.* Cambridge University Press, Cambridge, UK, 2010.
13. T. Padmanabhan and D. Kothawala, "Lanczos-Lovelock models of gravity," *Phys.Rept.* **531** (2013) 115–171, `arXiv:1302.2151 [gr-qc]`.
14. T. Padmanabhan and A. Paranjape, "Entropy of null surfaces and

dynamics of spacetime," *Phys.Rev.* **D75** (2007) 064004, arXiv:gr-qc/0701003 [gr-qc].

15. T. Padmanabhan, "Classical and quantum thermodynamics of horizons in spherically symmetric space-times," *Class.Quant.Grav.* **19** (2002) 5387–5408, arXiv:gr-qc/0204019 [gr-qc].