Scalar-Gauss-Bonnet Theories: Evasion of No-Hair Theorems and Novel Black-Hole Solutions

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We consider a general Einstein-scalar-GB theory with a coupling function $f(\phi)$. We demonstrate that black-hole solutions appear as a generic feature of this theory since a regular horizon and an asymptotically-flat solution may be easily constructed under mild assumptions for $f(\phi)$. We show that the no-hair theorems are easily evaded, and a large number of regular, black-hole solutions with scalar hair are then presented for a plethora of coupling functions $f(\phi)$.

Keywords: Modified theories; Gauss-Bonnet term; No-Hair theorems; novel black holes

The existence or not of black holes associated with a non-trivial scalar field in the exterior region has attracted the attention of researchers over a period of many decades. The *no-hair theorem*[1], that excluded static black holes with a scalar field, was outdated by the discovery of black holes with Yang-Mills[2], Skyrme fields[3] or conformally-coupled scalar fields[4]. The novel no-hair theorem[5] (for more recent analyses, see[6–8]) was also shown to be evaded in the context of the Einstein-Dilaton-Gauss-Bonnet theory[9] and in shift-symmetric Galileon theories[10,11].

Here, we consider a wide class of gravitational theories where the scalar field has a general coupling $f(\phi)$ to the Gauss-Bonnet (GB) term $R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$. In[12], we demonstrated that the above theory evades the no-hair theorems and that black-hole solutions, with a regular horizon and an asymptotically-flat limit, may be constructed under mild only constraints on the coupling function $f(\phi)$. We then determined[12] the characteristics of those black-hole solutions such as the horizon area, scalar charge and entropy. The proposed presentation is based on these two works.

We thus consider the following generalised gravitational theory

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + f(\phi)R^2_{GB}\right]. \tag{1}$$

The gravitational field equations and the equation for the scalar field have the covariant form:

$$G_{\mu\nu} = T_{\mu\nu}\,, \qquad \nabla^2\phi + \dot{f}(\phi)R^2_{GB} = 0\,, \tag{2}$$

where a dot denotes the derivative with respect to the scalar field. The energy-momentum tensor has the form

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}{}_{\alpha\beta}\nabla_\gamma\partial_\kappa f, \tag{3}$$

with $\tilde{R}^{\rho\gamma}{}_{\alpha\beta} = \eta^{\rho\gamma\sigma\tau}R_{\sigma\tau\alpha\beta} = \epsilon^{\rho\gamma\sigma\tau}R_{\sigma\tau\alpha\beta}/\sqrt{-g}$. In the context of the above theory, we seek spherically-symmetric solutions, with a line-element

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{4}$$

that describe regular, static, asymptotically-flat black holes. By employing the line-element (4), the Einstein's equations take the explicit form

$$\begin{aligned} 4e^B(e^B + rB' - 1) = \phi'^2\big[r^2 e^B + 16\ddot{f}(e^B - 1)\big] \\ -8\dot{f}\left[B'\phi'(e^B - 3) - 2\phi''(e^B - 1)\right], \end{aligned} \tag{5}$$

$$4e^B(e^B - rA' - 1) = -\phi'^2 r^2 e^B + 8\left(e^B - 3\right)\dot{f}A'\phi', \tag{6}$$

$$\begin{aligned} e^B\big[rA'^2 - 2B' + A'(2 - rB') + 2rA''\big] = -\phi'^2 re^B \\ +8\phi'^2\ddot{f}A' + 4\dot{f}[\phi'(A'^2 + 2A'') + A'(2\phi'' - 3B'\phi')], \end{aligned} \tag{7}$$

while the scalar equation reads

$$2r\phi'' + (4 + rA' - rB')\,\phi' + \frac{4\dot{f}e^{-B}}{r}\big[(e^B - 3)A'B' - (e^B - 1)(2A'' + A'^2)\big] = 0. \tag{8}$$

In the above, the prime denotes differentiation with respect to r – throughout this work, we assume that the scalar field shares the symmetries of the spacetime.

Equation (6) may be algebraically solved to determine the function e^B. Then, the remaining field equations reduce to a system of two independent, ordinary differential equations of second order for the functions A and ϕ:

$$A'' = \frac{P}{S}\,, \qquad \phi'' = \frac{Q}{S}\,, \tag{9}$$

where the functions P, Q and S are lengthy expressions of $(r, \phi', A', \dot{f}, \ddot{f})$.

For a spherically-symmetric spacetime, the presence of a regular horizon is realised for $e^A \to 0$, while ϕ, ϕ' and ϕ'' remain finite, as $r \to r_h$. Demanding the above, the 2nd of Eqs. (9) yields the constraint

$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \tag{10}$$

where the additional bound $\dot{f}_h^2 < r_h^4/96$ should hold. Then, employing the above, the 1st of Eqs. (9) determines the form of A', leading to the near-horizon solution

$$\begin{aligned} e^A = a_1(r - r_h) + \dots\,, \qquad e^{-B} = b_1(r - r_h) + \dots\,, \\ \phi = \phi_h + \phi'_h(r - r_h) + \phi''_h(r - r_h)^2 + \dots\,. \end{aligned} \tag{11}$$

At asymptotic infinity, on the other hand, assuming power-law expressions for the metric functions and scalar field, and substituting in the field equations, we obtain

$$\begin{aligned} e^A = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \dots\,, \quad e^B = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \dots\,, \\ \phi = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \frac{32M^2D - D^3}{24r^3} + \dots\,. \end{aligned} \tag{12}$$

in terms of the ADM mass M and scalar charge D. Therefore, a general coupling function $f(\phi)$ for the scalar field does not interfere with the existence of either a regular horizon or an asymptotically-flat limit for the spacetime (4).

Can the above two asymptotic solutions be smoothly matched for a complete black-hole solution to emerge? The no-hair theorem[5] forbids the existence of such a solution in the context of a wide class of scalar-tensor theories. Its applicability is based on the following assumptions: first, at asymptotic infinity, the $T^r{}_r$ component of the energy-momentum tensor, that has the form

$$T^r{}_r = \frac{e^{-B}\phi'}{4}\Big[\phi' - \frac{8e^{-B}\left(e^B-3\right)\dot{f}A'}{r^2}\Big], \tag{13}$$

is positive and decreasing: indeed, using the asymptotic expansions (12), we find that $T^r{}_r \simeq \phi'^2/4 \sim \mathcal{O}(1/r^4)$. Second, in the near-horizon regime, $T^r{}_r$ should be negative and increasing[5]. However, employing the asymptotic solution (11), we find that in our case

$$T^r{}_r = -\frac{2e^{-B}}{r^2}A'\phi'\dot{f} + \mathcal{O}(r-r_h)\,. \tag{14}$$

The above expression is positive-definite since, close to the horizon, $A' > 0$, and $\dot{f}\,\phi' < 0$ according to Eq. (10) for a regular horizon. We also find that, in our case, $T^r{}_r$ is in fact always decreasing close to r_h for every solution we have found, therefore, the novel no-hair theorem is non-applicable in our theory.

In order to demonstrate the validity of the aforementioned arguments, we have numerically solved the system of equations (9), and produced a large number of black-hole solutions with scalar hair. The scalar field and profile of $T^r{}_r$ are depicted in Fig. 1, for a variety of forms of the coupling function $f(\phi)$: exponential, odd and even power-law, odd and even inverse-power-law. In all cases, for a given value of ϕ_h, Eq. (10) uniquely determines the quantity ϕ'_h. The integration of the system (9) with initial conditions (ϕ_h, ϕ'_h) then leads to the presented solutions.

We have also studied[12] in detail the characteristics of the black-hole solutions, and in Fig. 2 we present the indicative case of $f(\phi) = \alpha/\phi$. The scalar charge has a

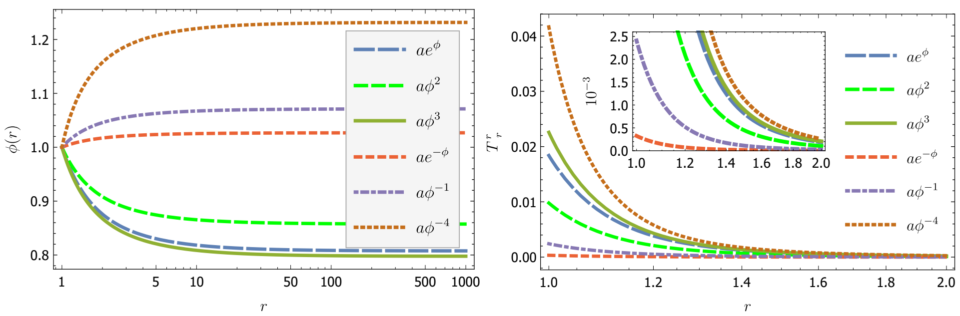

Fig. 1. The scalar field ϕ (left plot) and the $T^r{}_r$ component (right plot) for different coupling functions $f(\phi)$, for $a = 0.01$ and $\phi_h = 1$.

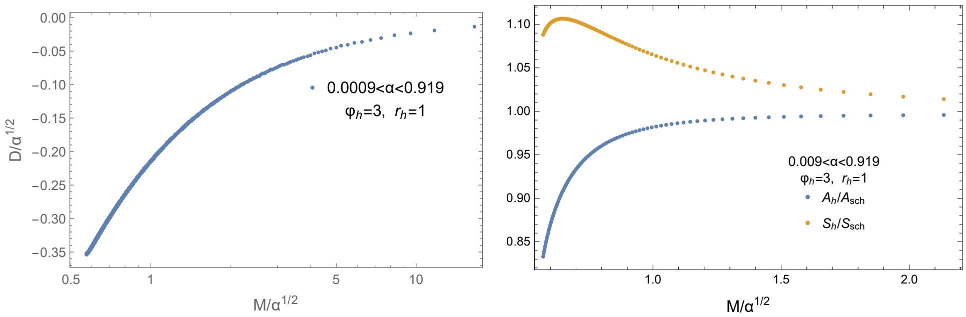

Fig. 2. The scalar charge D (left plot), and the ratios A_h/A_{Sch} and S_h/S_{Sch} (right plot, lower and upper curve respectively) in terms of the mass M, for $f(\phi) = \alpha/\phi$.

monotonic dependence on the mass M while its horizon area is always smaller than the one of the Schwarzschild solution exhibiting also a lower value beyond which the black hole ceases to exist. Its entropy is larger than that of the Schwarzschild case and thus thermodynamically more stable. Other classes of solutions exhibit a variety of characteristics. In all cases, however, our analysis clearly demonstrates that the presence of the GB term in a scalar-tensor theory leads to the emergence of novel families of black holes with scalar hair.

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