Black Holes and Soft Hair

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Black Hole Entropy:
Why do we believe that black holes have entropy?

- First law of Black Hole Mechanics (Bardeen, Carter and Hawking):
  An infinitesimal change in black hole equilibrium states is described by

$$dM = \frac{\kappa dA}{8\pi} + \Phi dQ + \Omega dJ$$

A = Surface area of the event horizon.
κ = Surface gravity.
Φ = Electrostatic potential of the black hole.
Q = Electric charge of the black hole.
Ω = Angular velocity of the black hole.
J = Angular Momentum of the black hole.

- Second Law of Black Hole Mechanics (Hawking):
  Any change in the equilibirum state of a black hole, subject to the weak energy condition for matter, the area of the event horizon increases.

$$dA \geq 0$$

.

- Hawking Temperature

$$T_H = \frac{\kappa}{2\pi}$$

.

Identification of the first law of black hole mechanics with the first law of thermodynamics leads to the Bekenstein-Hawking entropy.

$$S = A/4.$$

- Euclidean derivation of Black Hole Entropy (Gibbons and Hawking).
  Construct the path integral for gravity and evaluate it at tree-level in Euclidean space. The horizon becomes an irreducible S^2 but is otherwise no special place in Euclidean geometry. The interior of the black hole is not part of the Euclidean geometry and so only questions about the exterior of the black hole can be answered. The states of the black hole are implicitly averaged over in this approach.
- All of these approaches are thermodynamic in nature.

Boltzmann interpretation of entropy:

$$S = \ln W$$

W is density of states for black holes of fixed M, J and Q.

Central Question:

WHAT ARE THE QUANTUM STATES OF A BLACK HOLE?

- Covariant phase space formalism. Start with the action

$$I = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\;\; R + \text{possible boundary terms}$$

- Variation of the action under $g_{ab} \to g_{ab} + h_{ab}$ gives the equations of motion and a boundary term:

$$\delta_h I = \frac{1}{16\pi}\int_{\mathcal{M}} (\text{Einstein Tensor})^{ab} h_{ab} + \int_{\partial\mathcal{M}} \theta$$

$\theta(g; h) =$ presymplectic potential three-form.

Think of θ as being $\sum p_i \delta q_i$ with p, q being the generalised coordinates and momenta.

Explicitly

$$(*\theta)_a = \frac{1}{16\pi}(\nabla_b h_a^b - \nabla_a h)$$

.

The presymplectic current is defined by

- $\omega(g, h, h') = \delta_h \theta(g, h') - \delta_{h'} \theta(g, h)$

  ω is the presymplectic current, a three-form in spacetime. The integral of ω on a spacelike surface Σ in $\mathcal{M}$ defines the symplectic form $\Omega(g, h, h')$. Think of this as being $\sum dp_i \wedge dq_i$.

  $\Omega = \int_\Sigma \omega$

- The phase space of the theory is the classical solutions of the Einstein equations g_{ab}, together with the tangent vectors in the phase space h_{ab}, the solutions of the linearised Einstein equations.

  Ω is a two-form in the infinite-dimensional phase space of general relativity.

- The Iyer-Lee-Wald charges are defined by

$$\Delta Q_\zeta(g;h) = \int_\Sigma \omega(g,h,L_\zeta g) = \int_S F(g,h,L_\zeta g)$$

where $dF = \omega$ and S is a closed two-surface in Σ.
- ΔQ is the change in the charge conjugate to the vector field ζ between the spacetime g_{ab} and $g_{ab} + h_{ab}$.
- For example, if ζ were a time translation then Q would be the quasilocal mass enclosed by the surface S.
- S might be a boundary of spacetime.
  Then, another example would be S being a section of null infinity. If the spacetime were stationary and ζ the time translation Killing vector, Q would be the Komar mass.
- If S were null infinity and ζ a BMS supertranslation or super-rotation, then Q would be the corresponding supertranslation or super-rotation charges.

An explicit formula for Q is

$$Q_\zeta(g,h) - Q_\zeta(g,0) = \frac{1}{16\pi}\int_S dS_{ab}\ \ [-4\zeta^a\nabla^b h$$
$$+2\zeta^a\nabla_c h^{cb} - 2\zeta_c\nabla^b h^{ac} - h\nabla^a\zeta^b + 2h^{ac}\nabla_c\zeta^b].$$

We have been a bit cavalier with our definition of Q.
Q must be an exact two-form in phase space in order for Q to be a function of state and not depend on the path taken from the metric g_{ab} to $g_{ab} + h_{ab}$. In the derivation of Q from the action, there are various ambiguities which allow us to add extra terms to Q. These were cataloged by Wald and Zoupas. One may have to add a counterterm Q_{ct} to Q to make it exact.

Diffeomorphisms ζ should form an algebra. So that

$$L_\zeta L_{\zeta'} - L_{\zeta'} L_\zeta = L_{[\zeta,\zeta']}$$

.

The same should be true of the charges:

$$[Q_\zeta, Q_{\zeta'}] = Q_{[\zeta,\zeta']}$$

as long as diffeomorphism invariance is a symmetry of nature.

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The same should be true of the charges:
But explicit calculations reveal the possibility that

$$[Q_\zeta, Q'_\zeta] = Q_{[\zeta,\zeta']} + K_{\zeta,\zeta'}$$

and if $K \neq 0$ then diffeomorphism symmetry would be violated.

Suppose now that we deal with observers external to a black hole. Their observations are made regarding the event horizon as the boundary of space.

Thus Q are the charges of the black hole when we choose S to be the event horizon.

We can find vector fields that represent superrotations on the event horizon. Suppose in the Kerr metric the inner and outer horizons are at $r = r_\pm$.

Let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R}, \quad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L - t/2M},$$
$$y = \sqrt{\frac{r_+-r_-}{r-r_-}}e^{\pi(T_L+T_R)\phi - t/4M}$$

where

$$T_L = (r_+ + r_-)/4\pi a, \, T_R = (r_+ - r_-)/4\pi a$$

M is the mass of the Kerr black hole and a the rotation parameter.

- The vector field

$$\zeta^+ = \epsilon_n(w^+), \quad \zeta^y = \frac{1}{2} y \epsilon_n', \quad \zeta^\theta = \zeta^- = 0, \ n \in \mathbf{Z}.$$

with $\epsilon_n = 2\pi T_R e^{(1+\frac{in}{2\pi T_R})w^+}$ has a Lie bracket that reproduces the Virasoro algebra.

- Replacing $+ \leftrightarrow -$, $T_R \leftrightarrow T_L$ produces a second vector field that obeys a second Virasoro algebra that commutes with the first.
- To define the exact charges associated to these vector fields, one needs to add a counterterm

$$Q_{ct} = \frac{1}{8\pi} \int dS^{cd} \nabla_d (\zeta^a h_c^b) N_{ab}.$$

where N_{ab} is the volume form on the normal bundle to the horizon.

One then finds that, for both families of vector fields, their charges obey algebras with a non-vanishing central term

$$[Q_n, Q_m] = i(n-m)Q_{n+m} + in^3 J\delta_{n+m,0}$$

This central term corresponds to a conventional central charge $12J$ with J being the angular momentum of the black hole.

Observers exterior to the black hole will observe a violation of diffeomorphism invariance unless they can find a way to cancel the central term.

Black hole scattering for Kerr black holes:

- For quanta of energy δE and angular momentum δJ the absorption probability P contains a factor of

$$P \sim |\Gamma(1 + \frac{i\omega_L}{2\pi T_L})|^2 |\Gamma(1 + \frac{i\omega_R}{2\pi T_R})|^2$$

where

$$\omega_L = \frac{2M^3}{J}\delta E, \quad \omega_R = \frac{2M^3}{J}\delta E - \delta J.$$

This formula is well known from the ancient history of black hole scattering.

It is also part of the probability for absorption in a two-dimensional conformal field theory whose left movers are at a temperature T_L and right movers at a temperature T_R for quanta of energies ω_L and ω_R.

We hypothesize that on a section of the horizon, an observer outside the black hole would see such a conformal field theory with central charges c_L and c_R at temperatures T_L and T_R.

The statistical entropy of such a conformal theory is given by a formula due to Cardy.

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R).$$

Substituting in the results from the Kerr metric results in

$$S = \frac{1}{4}A.$$

We believe that the soft hair therefore accounts for the black hole entropy and the quantum states of the black hole are described by the states of such a theory.

- A similar calculation works in Kerr-Newman.
- The final results hold in Schwarzschild although there is something a bit degenerate since as $a \to 0$, $T_{L,R} \to \infty$ and $c_{L,R} \to 0$.
- Inherently holographic.
- Not a solution to the information paradox as it leaves unclear how to deal with the species problem or how the information in collapse gets encoded into this field theory.
- Freely falling observers do not see this conformal field theory as the horizon is not a boundary for their description of spacetime. An explanation of black hole complementarity?

- Related work:
  - Strominger and Vafa + ...: A CFT from string theory for states that are supersymmetric. Can be regarded as a special case of our treatment. Similarly, counting of supersymmetric brane configurations that have the same quantum numbers as black holes.
  - Kerr-CFT: A special case.

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