Loop Quantum Cosmology and the CMB

Ivan Agullo

Louisiana State University

MG15

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Goal

Use Quantum Gravity to push the boundaries of our understanding of the early Universe

Goal

Use a proposal for quantum gravity to push the boundaries of our understanding of the early Universe

Goal

Use loop quantum cosmology to push the boundaries of our understanding of the early Universe

An example of:

(a) How, by using well-motivated assumptions, physical input, and approximations, one can build a quantum theory of the cosmos

(b) How one can use it to address some of the open questions in cosmology

(c) How one can use this theory to make predictions that can help us to test the underlying ideas

PLAN:

1. A brief introduction to LQC

2. Cosmic perturbations in LQC

3. LQC and the CMB

Here, brief overview of work done by many researchers:

Alesci, Ashtekar, Barrow, Benitez-Martinez, Bojowald, Bonga, Bolliet, Brizuela, Cailleteau, Cianfrani, Corichi, Campiglia, Dapor, Diener, Engle, Freishhack, Garay, Grain, Gupt, Hanusch, Hernandez, Joe, Karami, Martin-Benito, Martin de Blas, Mena-Marugan, Megevan, Mielczarek, Montoya, Lewandowski, Linsefors, Liegener, Nelson, Pawlowski, Payli, Putchta, Olmedo, Singh, Taveras, Thiemann, Vandersloot, Vidoto, Vijayakumar, Wilson-Ewing,…

More details, session QG3, chaired by Pullin and Singh, Thursday afternoon.

1. A brief introduction to LQC

Loop Quantum Gravity rests on Ashtekar's reformulation of GR in connexion variables:

$g_{\mu\nu} \longrightarrow A_i^I(\vec{x})$, $E_J^j(\vec{x})$ **Ashtekar variables**

$A_j^I(\vec{x})$ **is a** $SU(2)$ **connection** $I, J = 1, 2, 3$

$E_J^j(\vec{x})$ **its conjugate variable**

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Quantum theory:

The quantum representation is chosen using symmetries: (spatial) diffeomorphisms invariance

→ unique kinematical Hilbert space: $\Psi(A^i_I)$ **Quantum Geometry!**

Dynamics: $\hat{H}\Psi(A^i_I)=0$ Wheeler-De Witt-like equation

Loop Quantum Cosmology is a mini-superspace version of
Loop Quantum Gravity:
quantization of spacetimes with the symmetries of cosmology

First: the simplest, homogeneous + isotropic model: FLRW

Classical system: scalar field $\phi(t)$ + gravity $a(t)$. In connexion variables:

$$A_i^I(t) = c(t)\, e_i^I \qquad E_I^i(t) = p(t)\, e_I^i$$

orthonormal triad in space

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Relational time interpretation:
Klein-Gordon-like equation in "time" ϕ

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Relational time interpretation: Klein-Gordon-like equation in "time" ϕ

Solving this equation, one obtains the Hilbert space of physical states and physical observables in it.

This is a theory of quantum cosmology

Ashtekar, Bojowald, Corichi, Martin-Benito, Mena-Marugan, Olmedo, Pawloswki, Singh, Wilson-Ewing….

Physical consequences

Analytical results: Ashtekar, Corichi, Pawloswki, Singh

All physical observables (e.g. curvature invariants, energy density of ϕ) are bounded above. No singularity in the entire Hilbert space. For instance:

$$\rho_{\rm sup} = \frac{18\pi}{G^2\hbar\Delta_o} \approx 0.4\,\rho_{Pl} \qquad\qquad R_{\rm sup} = 48\pi G\rho_{\rm sup}$$

Minimum are eigenvalue in LQG

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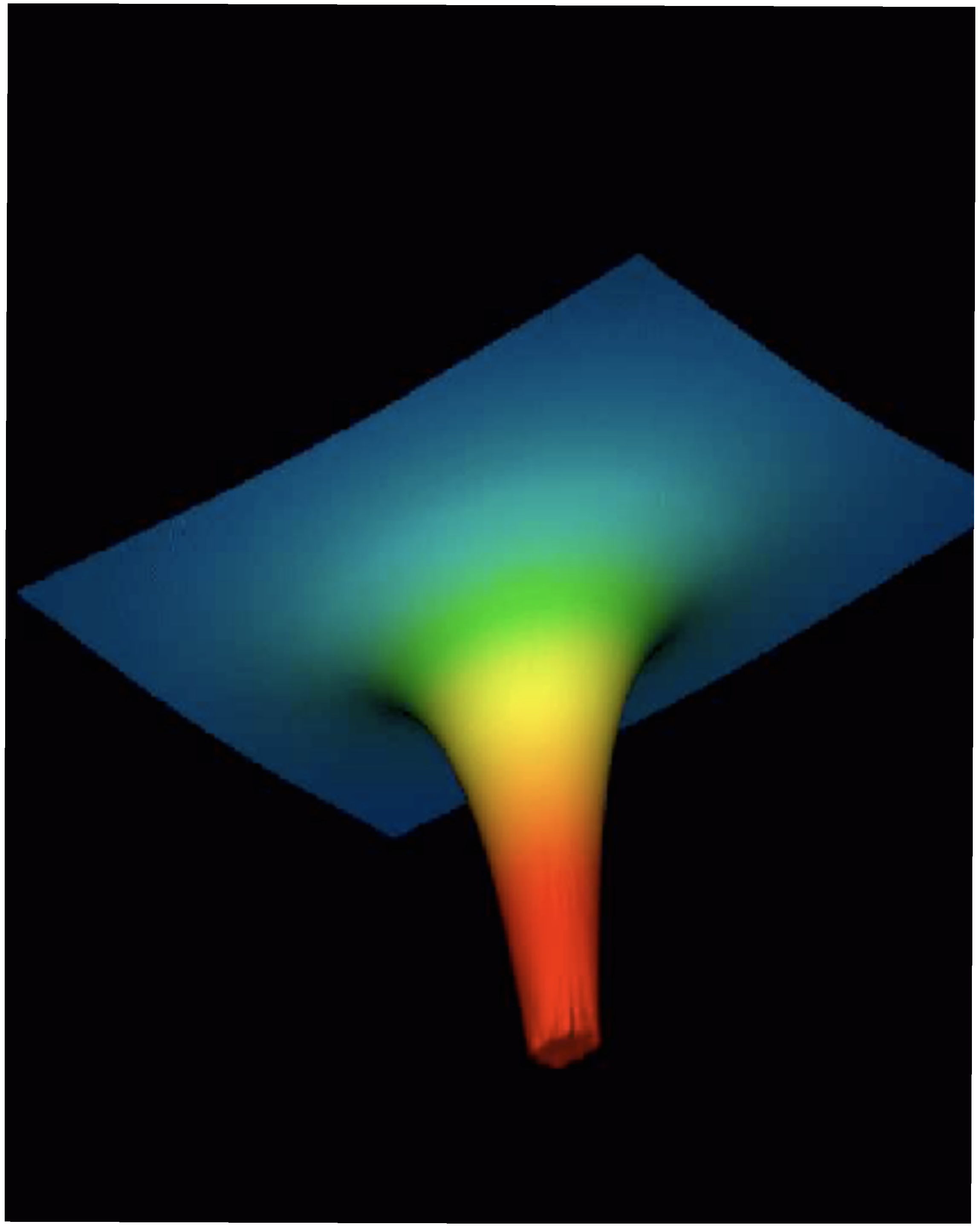
Additionally:

All states during the evolution go through an instant (in ϕ-time) of minimum volume and maximum curvature: Bounce

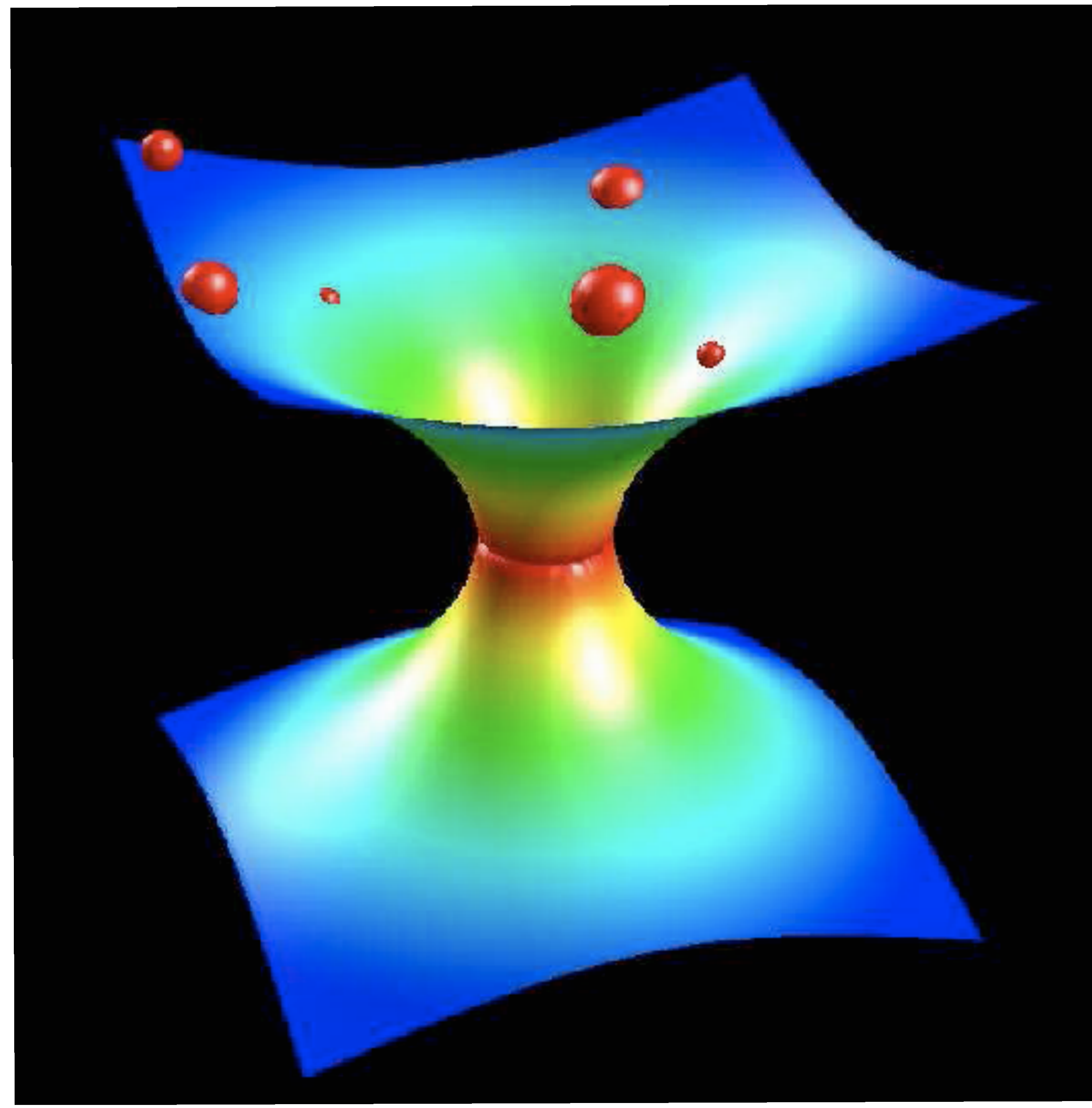
Artistic conceptions of the Big Bang and Big Bounce

Big Bang



Credits: Pablo Laguna

Big Bounce



Credits: Cliff Pikover

To gain some intuition about the spacetime geometry:

Equations that follow the evolution of $\langle \hat{a} \rangle$ for "sharply peaked" wave functions $\Psi(c, \phi)$

Effective equations

$$H^2 = \frac{8\pi G}{3}\,\rho\left(1 - \frac{\rho}{\rho_{\rm sup}}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\,\rho\left(1 - 4\frac{\rho}{\rho_{\rm sup}}\right) - 4\pi G\,P\left(1 - 2\frac{\rho}{\rho_{\rm sup}}\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$

where, as usual: $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ **and** $P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Work has been extended to more complex cosmological models:

-with spatial curvature

-with cosmological constant

-Bianchi I, IX

-Gowdy

Relation LQC and LQG

Lots of recent work on relating LQC to LQG in a more systematic way (symmetry reduction at the quantum level)

Alesci, Cianfrani, Engle, Brunnemann, Freishhack

Goal: Apply this framework to the early universe

I will use LQC to complete inflation, rather than to replace it

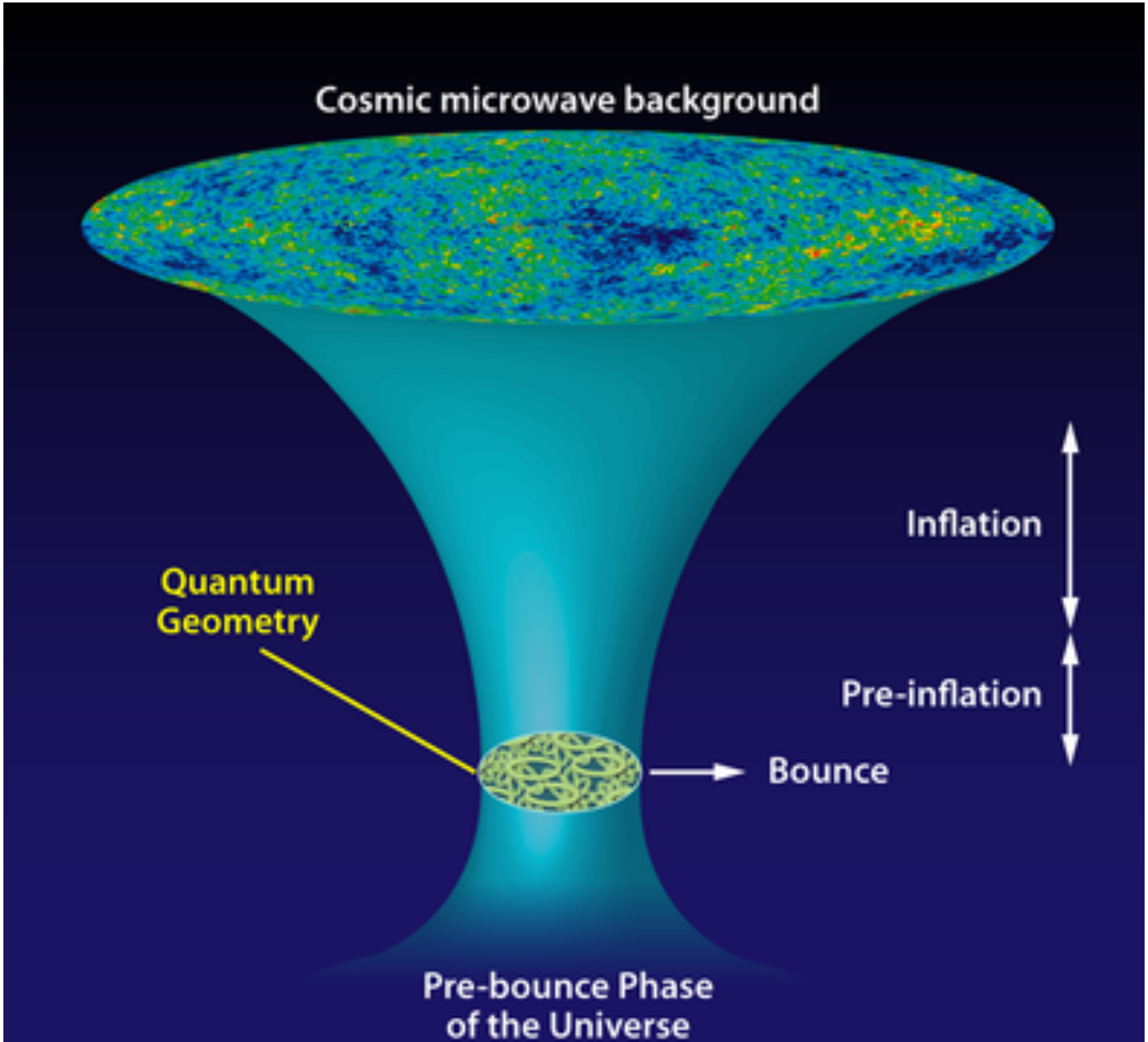


Fig. Credits:
P. Singh, Physics 5, 142 (2012)

2. Scalar and tensor perturbations in LQC

Here, review of one approach

Other approaches exists: see e.g. Mena-Marugan, Martin-Benito, Martin de Blas, Castello-Gomar, Olmedo

Similar results

(See Mena-Marugan's talk on Thursday, session QG3)

Ashtekar, Kaminski, Lewandowski 2010

I.A., Ashtekar, Nelson 2013

Brief summary of the strategy:

Starting point: $\Psi(a, \phi, \delta\phi, \delta g_{\mu\nu})$

Perturbation theory $\Psi(a, \phi, \delta\phi, \delta g_{\mu\nu}) = \Psi_{\rm FRW}(a, \phi) \otimes \psi_{\rm pert}(a, \phi, \delta\phi, \delta g_{\mu\nu})$

Equations of motion:

$$\hat{H}\,\Psi(a, \phi, \delta\phi, \delta g_{\mu\nu}) = 0 \xrightarrow[\text{take expectation value in } \Psi_{\rm FRW}]{} \partial_t^2 \psi_{\rm pert} + f(\langle \hat{a}^n \rangle, \langle \hat{\phi}^m \rangle)\,\psi_{\rm pert} = 0$$

One obtains a QFT in a quantum spacetime

Result:

The resulting equations are formally equivalent to the equations normally used in cosmology:

$$(\tilde{\Box} + \tilde{\mathcal{U}})\mathcal{Q}(x) = 0 \qquad \tilde{\Box}\,\mathcal{T}^{(+,\times)}(x) = 0$$

scalar pert **tensor perts (two polarizations)**

with the exception that the classical FRW metric has been replaced by:

$$d\tilde{s}^2 = \tilde{a}^2\left(-d\tilde{\eta}^2 + d\vec{x}^2\right)$$

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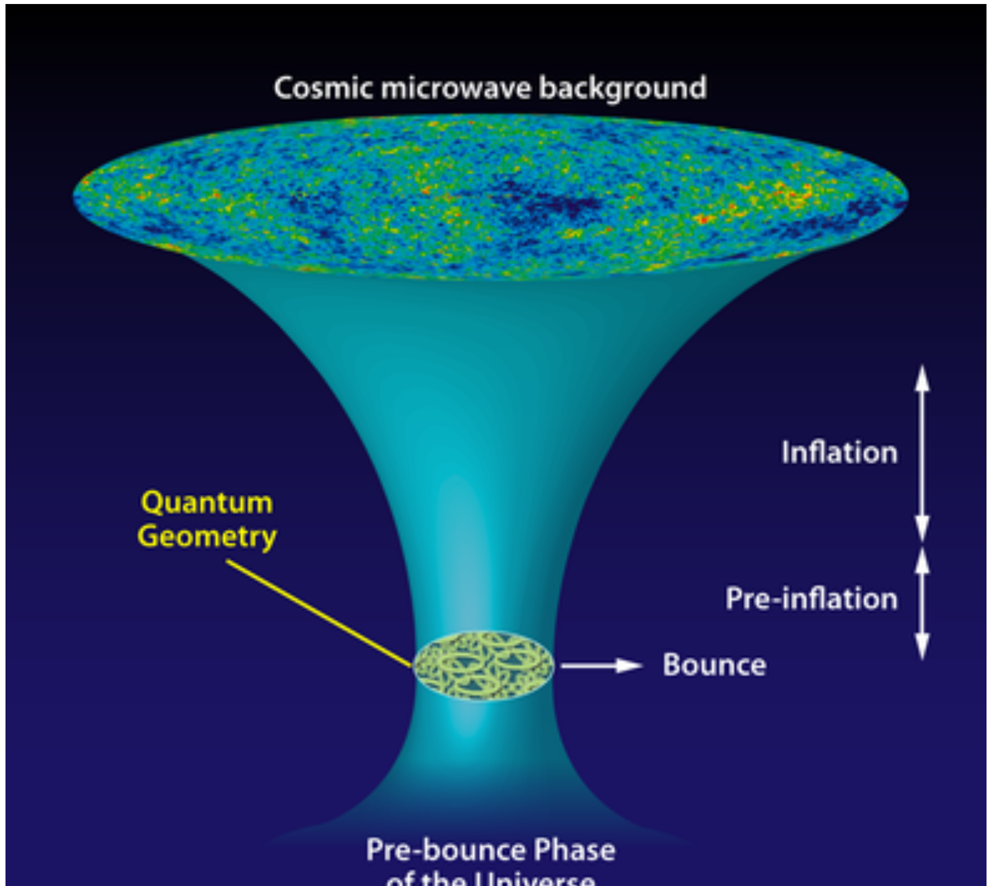
$$d\tilde{s}^2 = \tilde{a}^2\,(-d\tilde{\eta}^2 + d\vec{x}^2)$$ Dressed, effective metric

where

$$\left.\begin{aligned} \tilde{a}^4(\phi) &:= \frac{\langle \hat{\Theta}^{-1/4}\,\hat{a}^4(\phi)\,\hat{\Theta}^{-1/4}\rangle_{\Psi_{\rm FRW}}}{\langle\, \hat{\Theta}^{-1/2}\rangle_{\Psi_{\rm FRW}}} \\ d\tilde{\eta} &:= \tilde{a}^2(\phi)\,\langle \hat{\Theta}^{-1/2}\rangle_{\Psi_{\rm FRW}}\,d\phi \end{aligned}\right\} \longrightarrow \tilde{a}(\tilde{\eta})$$

Perturbations only sensitive to a couple of “moments” of $\Psi_{\rm FRW}$ (simple result, although the specific moments are non-trivial)

3. Phenomenology of LQC



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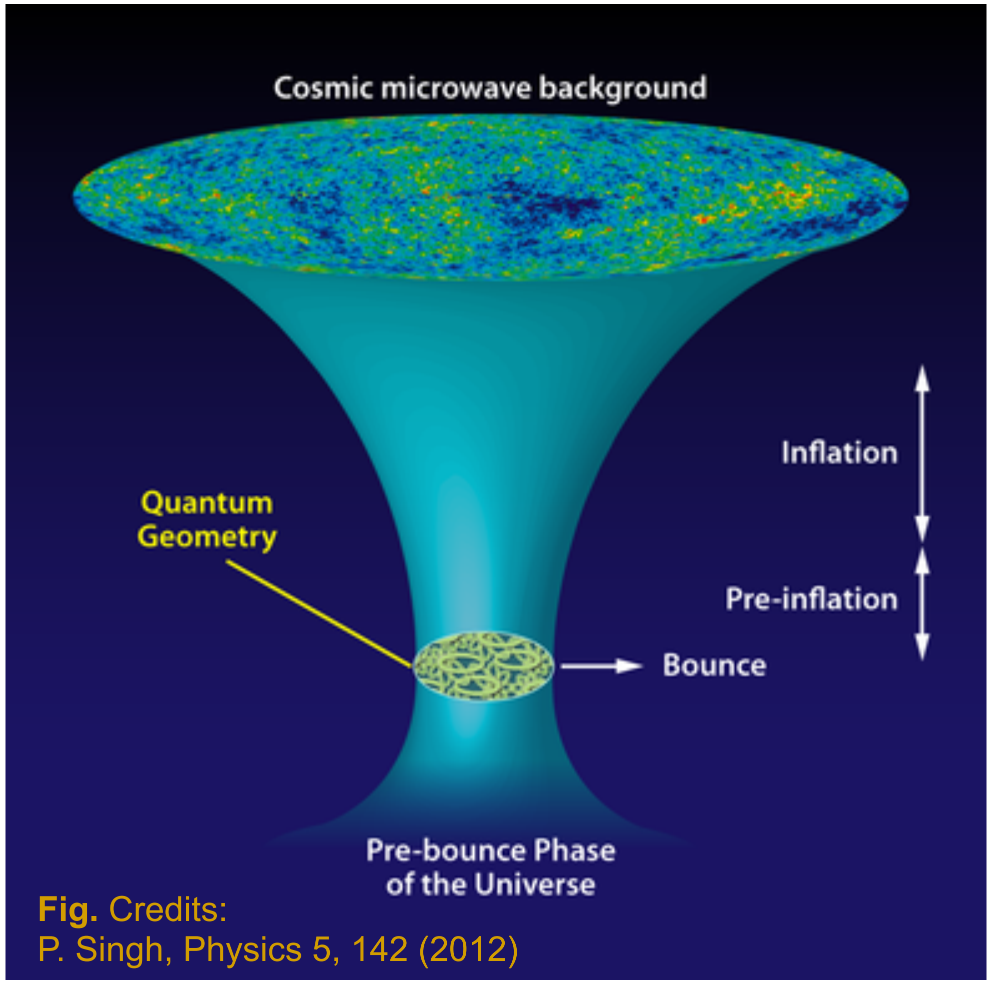
Strategy:



Strategy:

1) Perturbations **start in the vacuum** at early times

2) Evolution **across the bounce amplifies** curvature perturbations

3) Then standard slow-roll inflation begins, but **perturbations** reach the **onset of inflation** in an **excited state**, rather than the vacuum

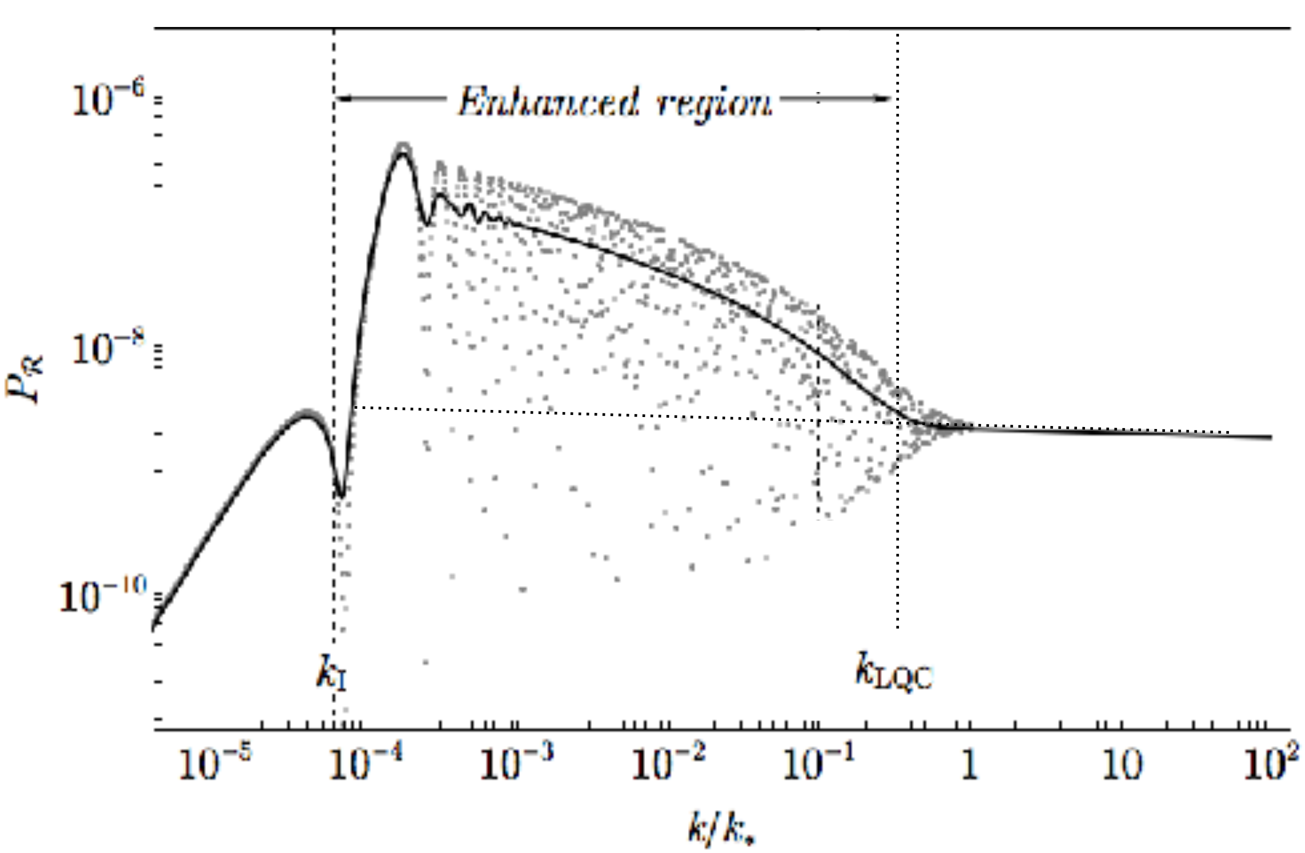
4) These **excitations impact observables** quantities

Two-point function: The power spectrum

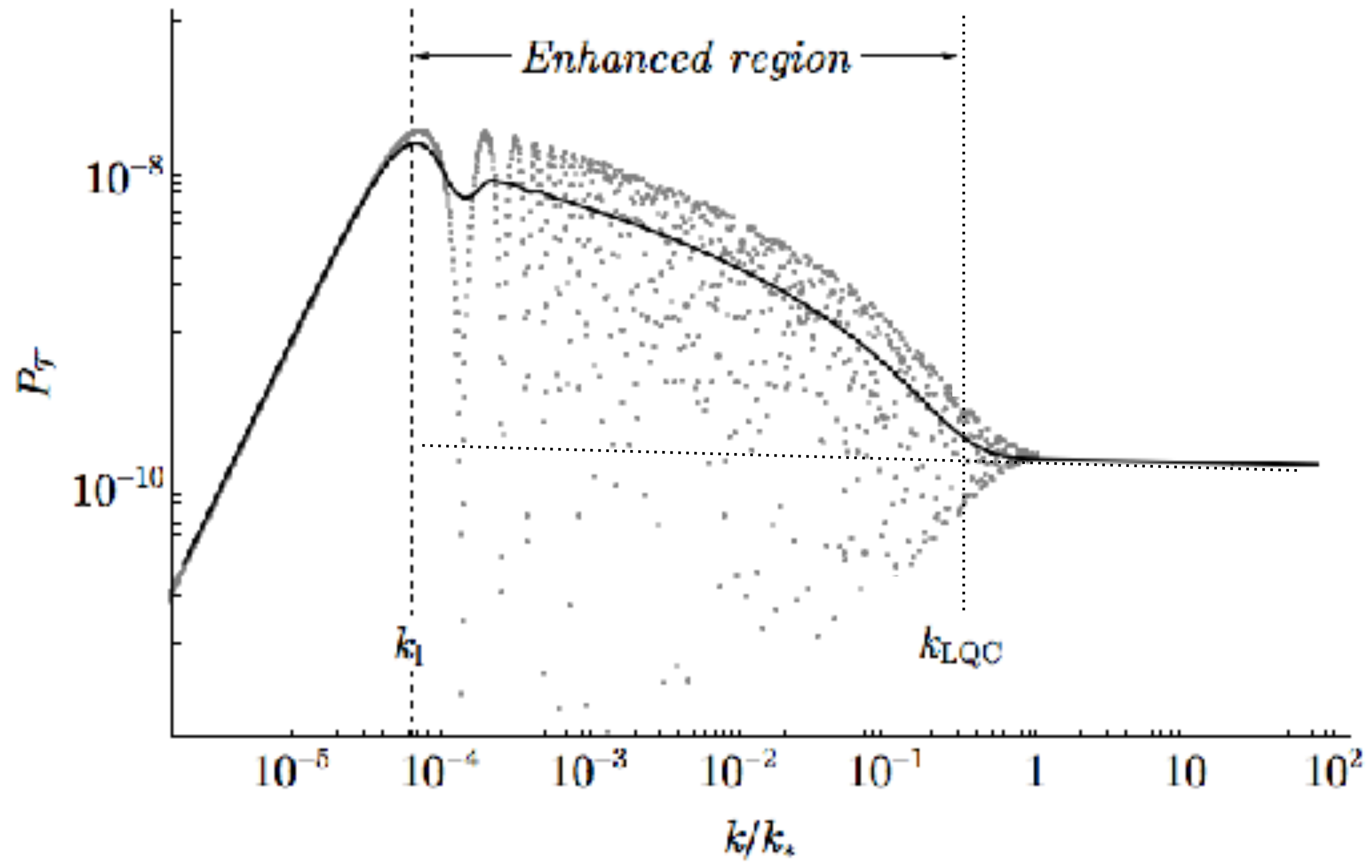
Results of numerical evolution

(I.A.-Ashtekar-Nelson 2012-13, I.A.-Morris 2015)

Scalar Power Spectrum



Tensor Power Spectrum



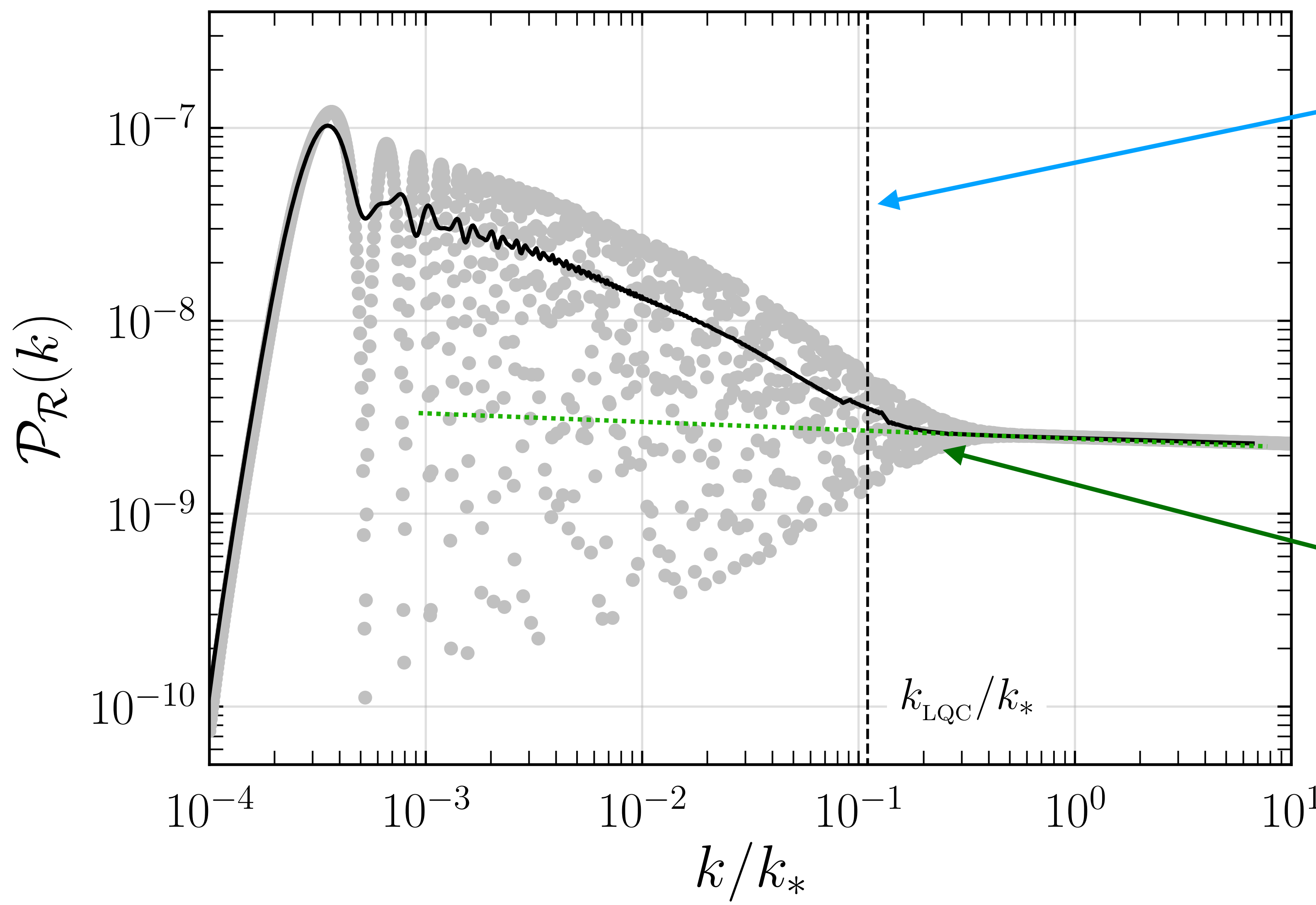
$\phi_B = 1.22 \quad m = 1.1 \times 10^{-6}$ **and vacuum initial condition in the past**

Grey point: numerical result for individual k **'s**

Black line: average of grey points

$k_\star/a_0 = 0.002\,\mathrm{Mpc}^{-1}$

Scalar Power Spectrum



LQC introduces a new scale in the problem kLQC. It is defined by the Ricci curvature at the bounce

Inflation without LQC

The pre-inflationary evolution modifies the power for low k-values (long wavelengths)

Free parameter: amount of expansion before slow-roll inflation

- For large post-bounce expansion, predictions are indistinguishable from standard inflation

  → QG extension of the inflationary scenario

- For smaller expansion, QG corrections at large angles in CMB.

  Most important:

  - modification of power for low k
  - effects on spectral indices and runnings
  - reduction of tensor-to-scalar ratio (slightly alleviates constrains on quadratic potential)
  - modification of consistency relation $r < -8\, n_t$

(see Mena-Marugan, Elizaga de Navascués, Bedic, Martineau's talks on Thursday session QG3 for many more details)

Three-point functions: Non-Gaussianity

Work in collaboration with B. Bolliet and V. Sreenath, 2017

(See talk by V. Sreenath for more details, Thursday afternoon)

Why to study Non-Gaussianity?

(a) If it is too large, the perturbative expansion used to compute the power spectrum would break down.

…a real possibility, because the bounce takes place at the Planck

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(a) If it is too large, the perturbative expansion used to compute the power spectrum would break down.

…a real possibility, because the bounce takes place at the Planck

(b) Even if perturbation theory turns out to be OK, there are strong observational upper bounds

Goal: Compute three-point correlation function

$$\langle 0|\hat{\mathcal{R}}_{\vec{k}_1}\hat{\mathcal{R}}_{\vec{k}_2}\hat{\mathcal{R}}_{\vec{k}_3}|0\rangle =: (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, B_{\mathcal{R}}(k_1,k_2,k_3)$$

We need:

To go beyond linear perturbation theory: expand Einstein action at third order

Hard calculation. Done for the first time by Maldacena in 2003.

Even harder in pre-inflationary regime: absence of slow-roll approx.

- We have developed a numerical code to compute non-Gaussianity in generic FRLW spacetime
- Embedded in the numerical infrastructure CLASS
- We have made it publicly available: https://github.com/borisbolliet/class_lqc_public
- This code will be useful beyond LQC

Non-Gaussianity parameterized by the function $f_{NL}(k_1, k_2, k_3)$ defined as:

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where $B_{\mathcal{R}}(k_1,k_2,k_3) \equiv -\frac{6}{5} f_{\rm NL}(k_1,k_2,k_3) \times (\Delta_{k_1}\Delta_{k_2} + \Delta_{k_1}\Delta_{k_3} + \Delta_{k_2}\Delta_{k_3})$

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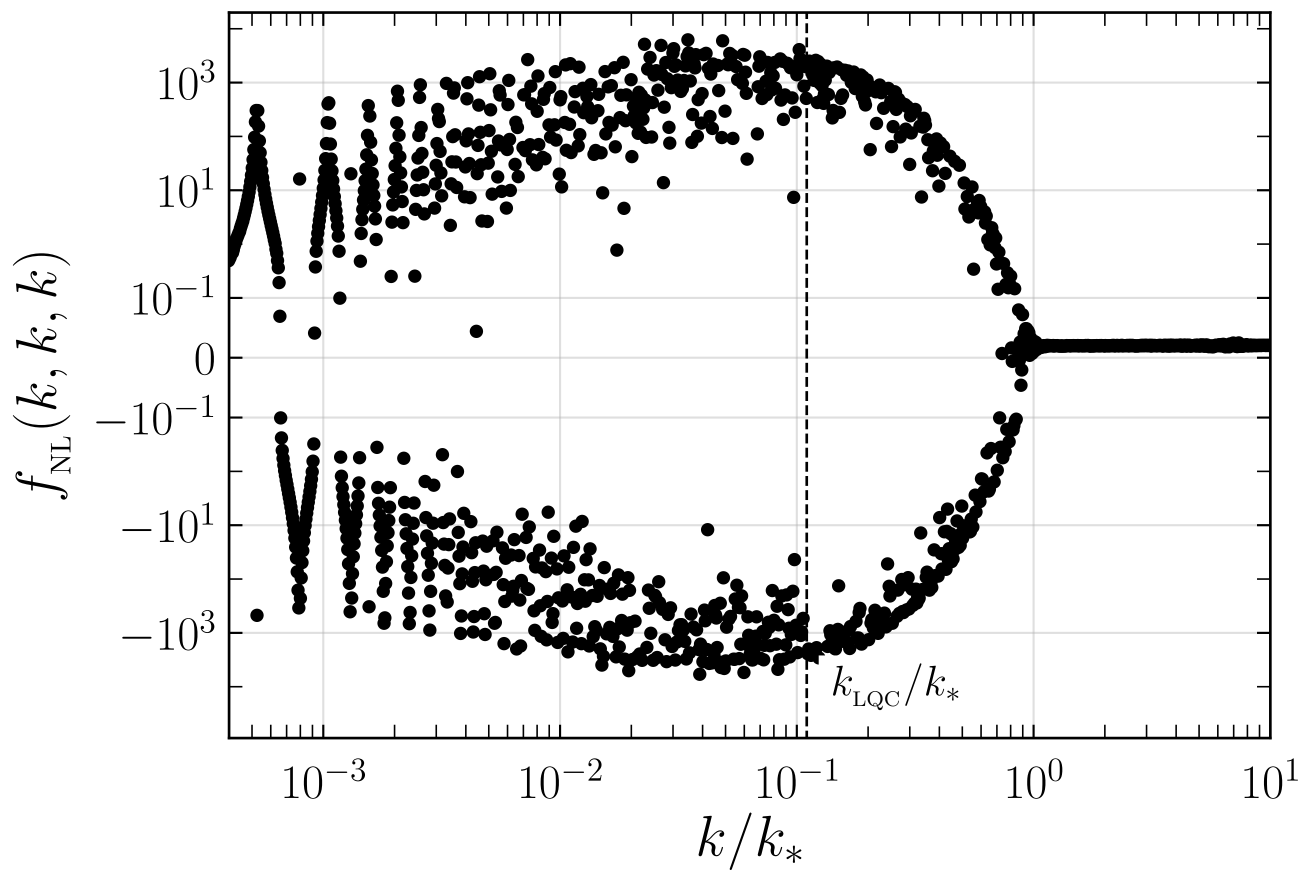
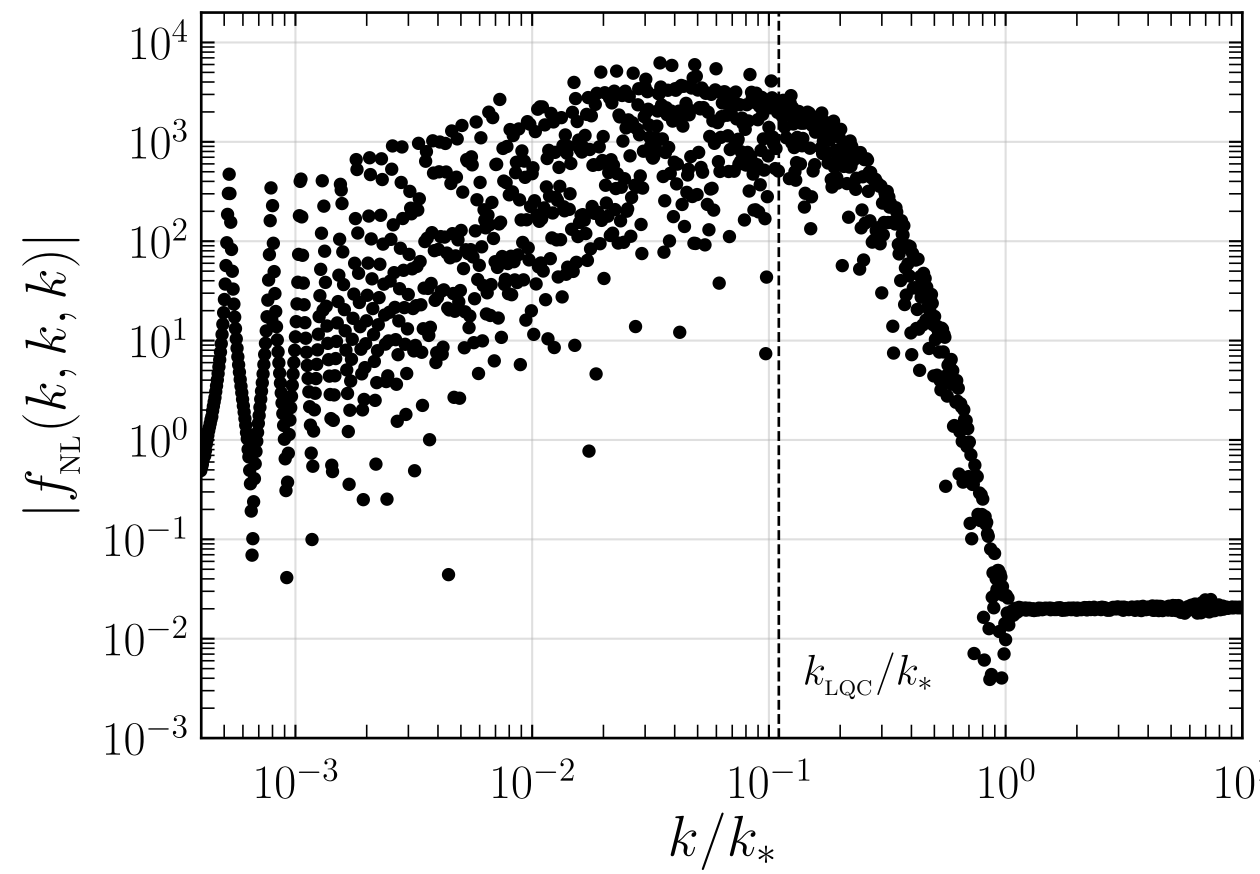
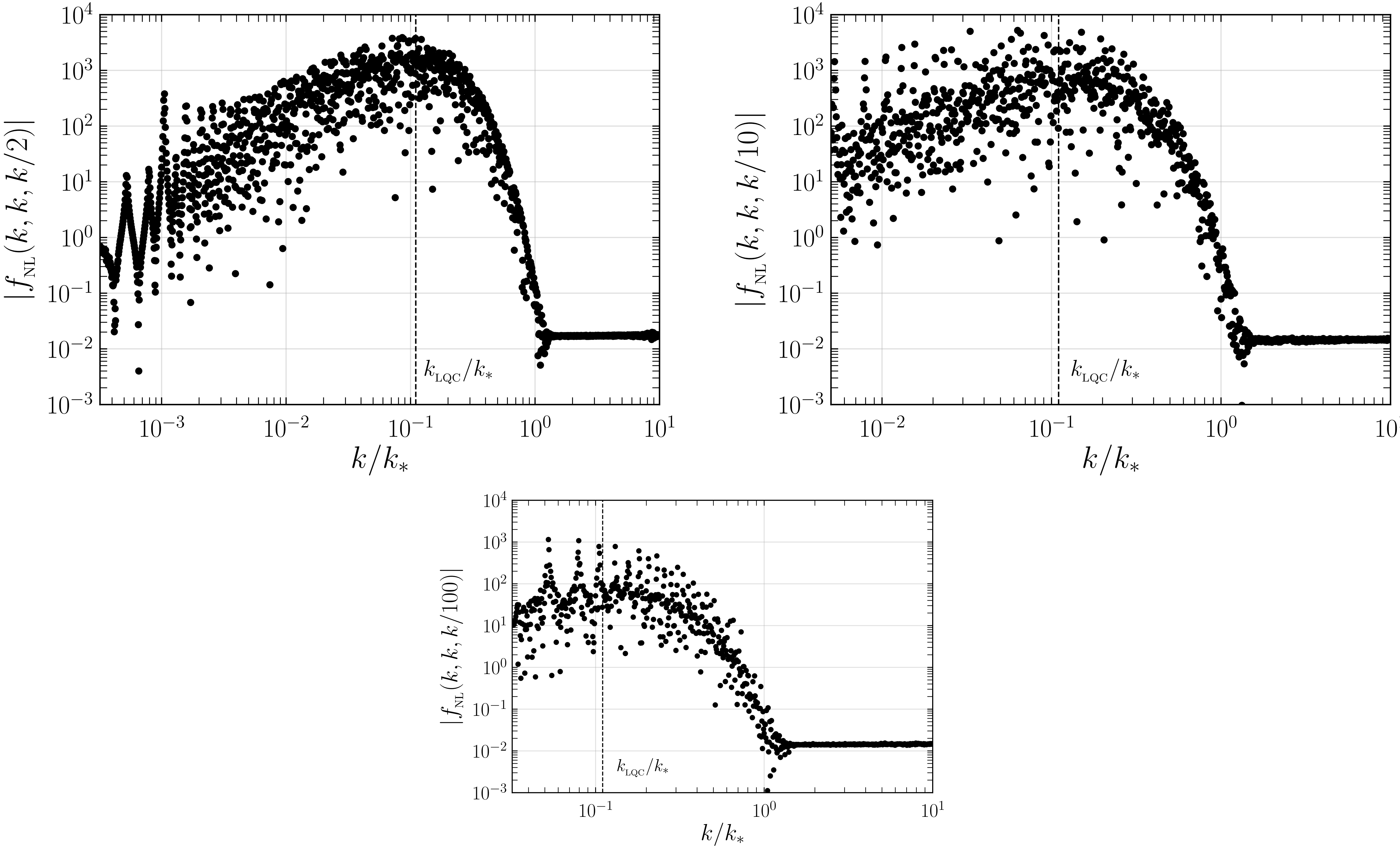
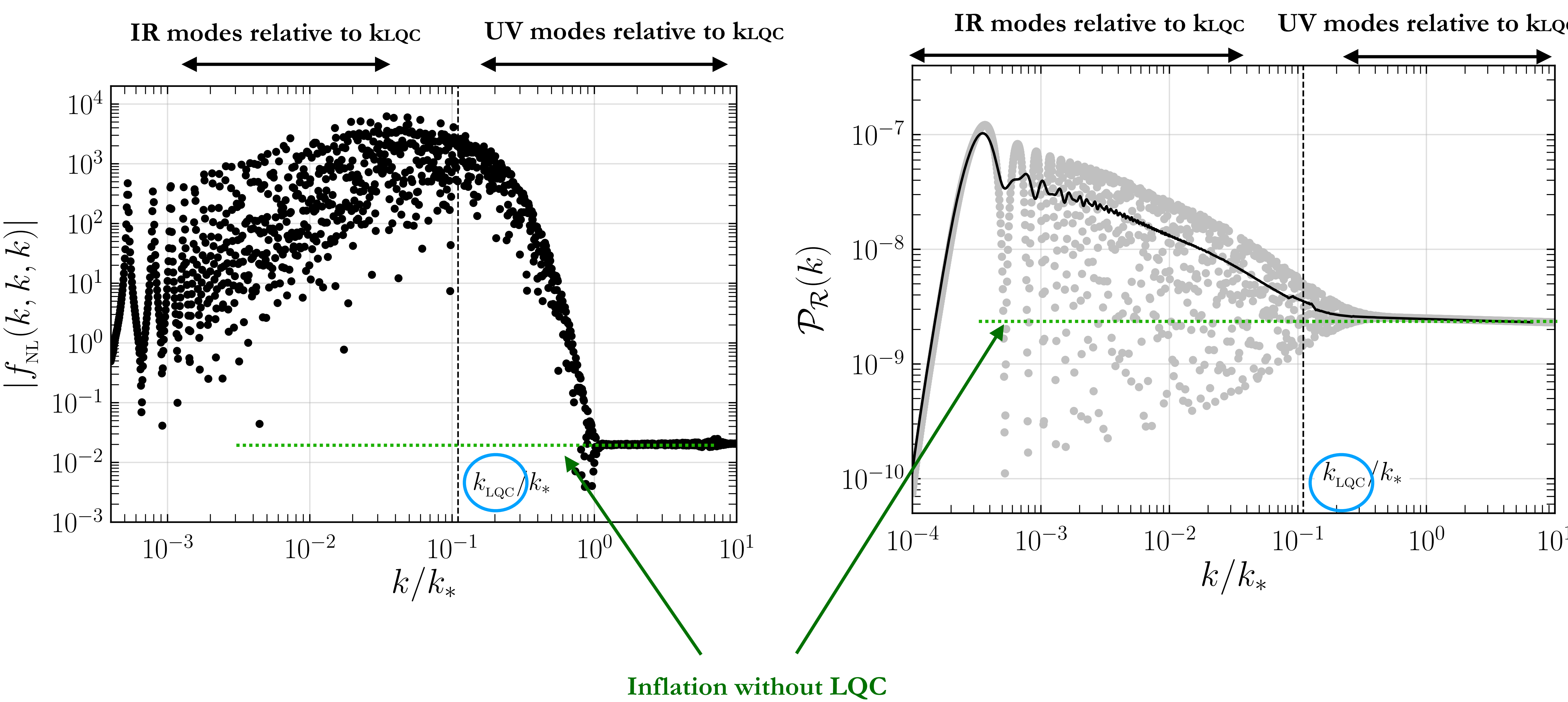
First, we show equilateral configurations, i.e. k1 = k2 = k3



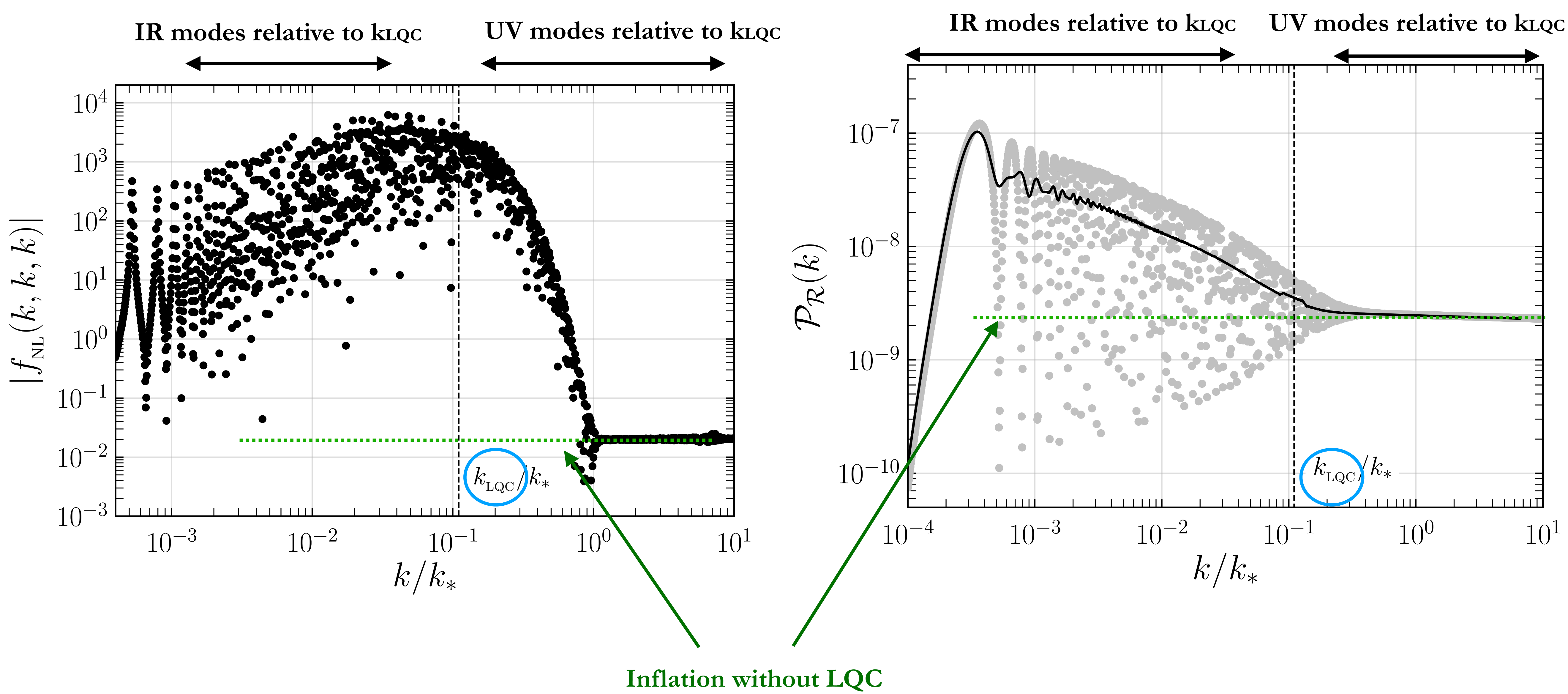


Similar results for other configurations





Qualitative understanding: similar to the power spectrum



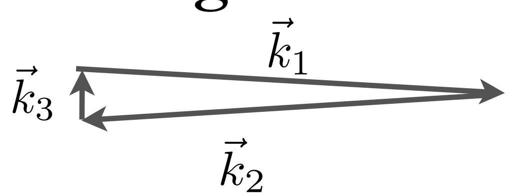
Qualitative understanding: similar to the power spectrum

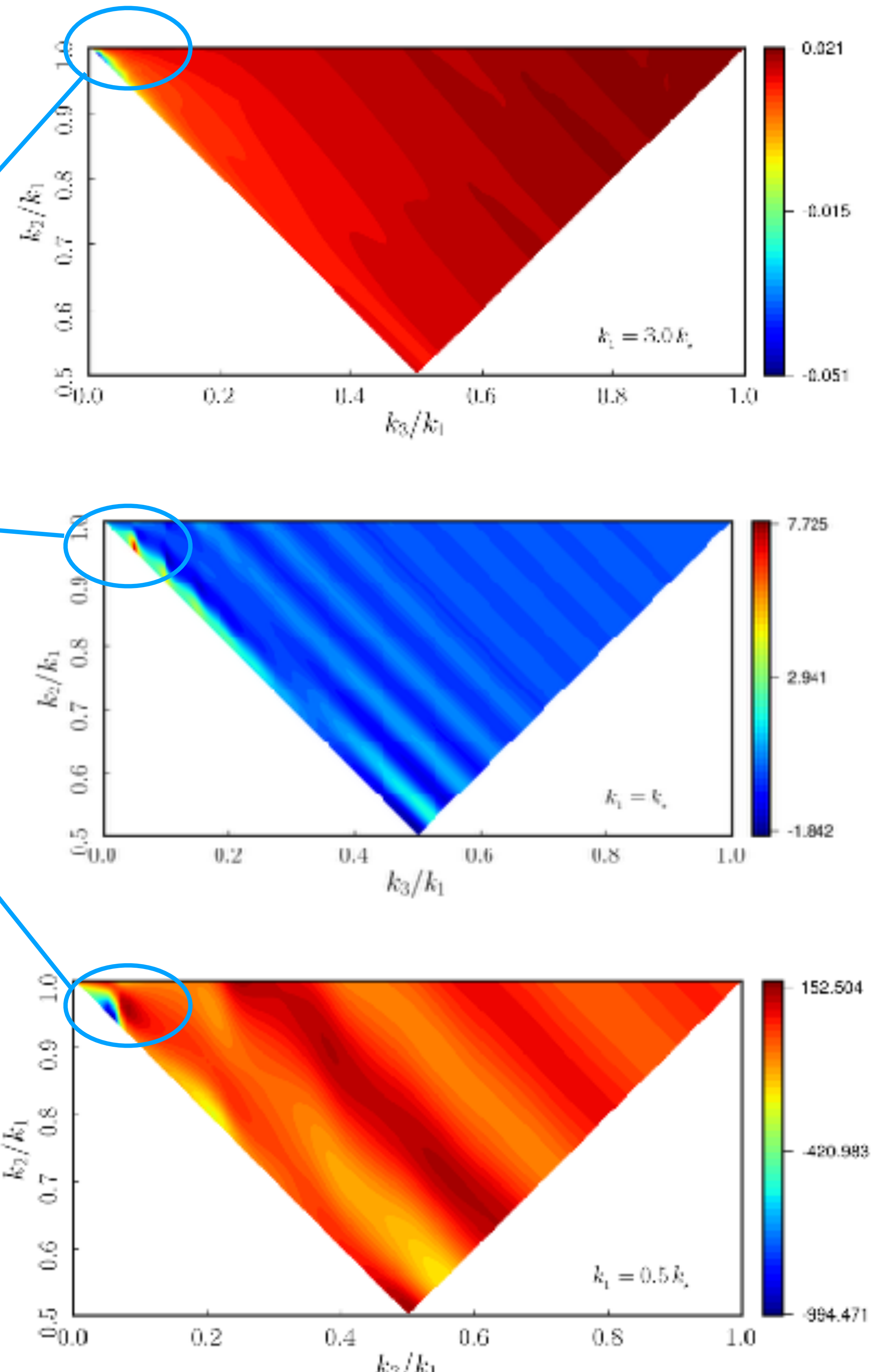
The bounce amplifies non-Gaussianity significantly, for modes that are of the same order or more infrared than the curvature radius at the bounce

Non-Gaussianty in LQC are strongly scale dependent, in contrast to a majority of models in the market

Two dimensional plots: fNL vs k2 and k3, for fixed k1

The amplitude of fNL is quite uniform, although larger in squeezed configurations





$$k_1 = 3\,k_*$$

$$k_1 = k_*$$

$$k_1 = 0.5\,k_*$$

Summary of the main results:

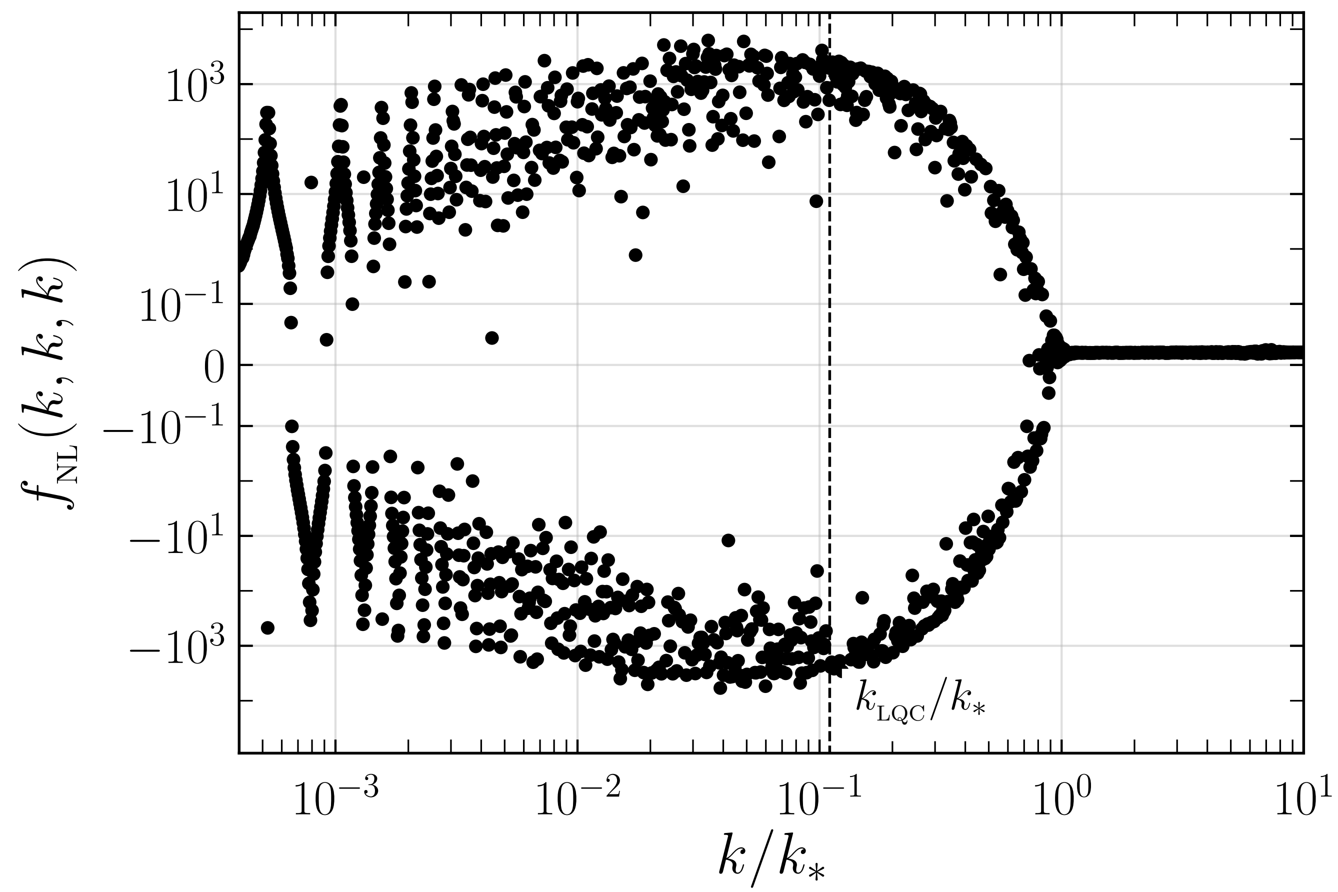
(1) The results of standard inflation exactly recovered for UV modes (nice check)

(2) Non-Gaussianity is very oscillatory

(3) The amplitude largely enhanced by the bounce for IR modes

(4) We have checked that, despite the large enhancement, perturbation theory is under control

(5) Comparison with observations:



The non-Gaussianity generated by the bounce in LQC has precisely the shape needed to respect observational constraints on large k's, and still to produce some observable effect at low k's

We are exploring whether this non-Gaussianity can produce effects in the CMB similar to the "large anomalies" observed by WMAP and PLANCK

4. Summary

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(5) Observable effects concerning tensor perturbations and Non-Gaussianity