# CONFORMAL SYMMETRY IN THE SKY

14th Marcel Grossman Conference

Rome

July 2015

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Investigations in string theory over the last few decades have uncovered surprising & computationally powerful mathematical relations among classical & quantum systems. While the possibilities for the direct experimental verification of string theory as THE theory of nature remain remote, these relations have already had significant applications in pure mathematics, CM physics, nuclear physics & GR.

Astronomy has largely been left out of the fun. One of the most famous such stringy relations - the "AdS/CFT correspondence" - has a much-less-understood cousin - the "Kerr/CFT correspondence" - inspired by the Bardeen-Horowitz discovery of extreme Kerr conformal symmetry. In this talk I will describe recent explorations of observational signals of this correspondence.

# OUTLINE

1. Scaling limits & critical behavior in astrophysics
2. Enhanced conformal symmetry near the horizon of a rapidly spinning Kerr black hole
3. Applications to gravity wave emission
4. Applications to force-free electrodynamics & black hole energy extraction & predictions for the Event Horizon Telescope.
5. Concluding comments

Condensed matter physics, like astrophysics studies systems with many components and complicated nonlinear interactions. Much progress has been made by finding extreme or special situations in which the matter exhibits critical behavior that can be (approximately) solved. This behavior may appear at high or low temperatures or energies, magnetic fields, chemical potentials or near phase transitions. e.g carbon liquid-gas exhibits critical opalscence (1869). It is often accompanied by enhanced symmetry.

A famous example is the Ising model realized experimentally in $CoNb_2O_6$ (Colbalt Niobate) (Coldea et. al.)


COUPLED SPINS

At low energies and distances $\gg a$, a space time scaling symmetry appears

$$x \to \lambda x, \quad t \to \lambda t$$

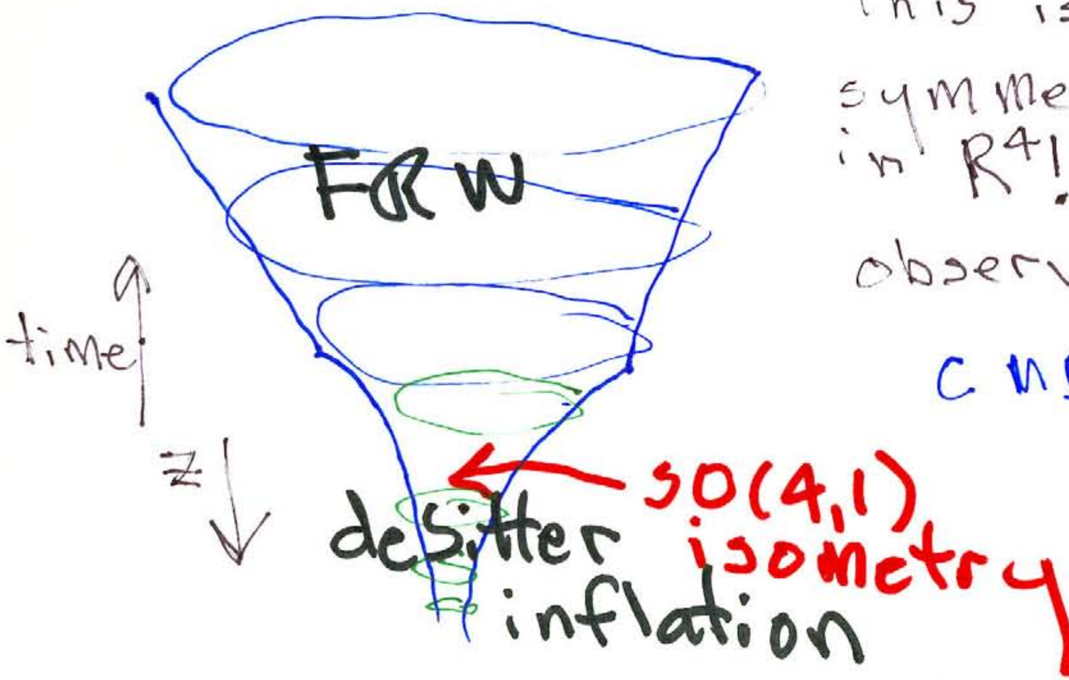
More surprisingly, there is an $\infty$-dimensional conformal symmetry

$$(x+t) \to f^{+}(x+t), \quad (x-t) \to f(x-t)$$

General arguments imply this is generic. (Zamalodchikov, Polchinski) Also quantum hall effect.

Such **2D quantum conformal field theories** are ubiquitous in physics and have been extensively studied.

Such **critical** behavior is also emerging in astrophysics. For example at extreme redshift $z \to \infty$ inflation proposes there is an enhanced de Sitter symmetry of space time.




This is the conformal symmetry of a field theory in $R^4$! It has important observational consequences:

CMB $\left(\frac{\delta\rho}{\rho}\right)^2 = \text{constant } k^{-3}$

Perhaps there will be many more examples of **critical** behavior in the **sky**.

This talk considers another region of infinite redshift: the near-horizon of an extreme Kerr BH. Critical behavior and $\infty$-dimensional enhanced symmetries – the same as in the critical Ising model – appear. This could be relevant to the observations of high spin black holes.

Warm-up:

## Near-Horizon Extreme Reissner-Nordstrom

- Metric for charge $Q = GM$ black hole is

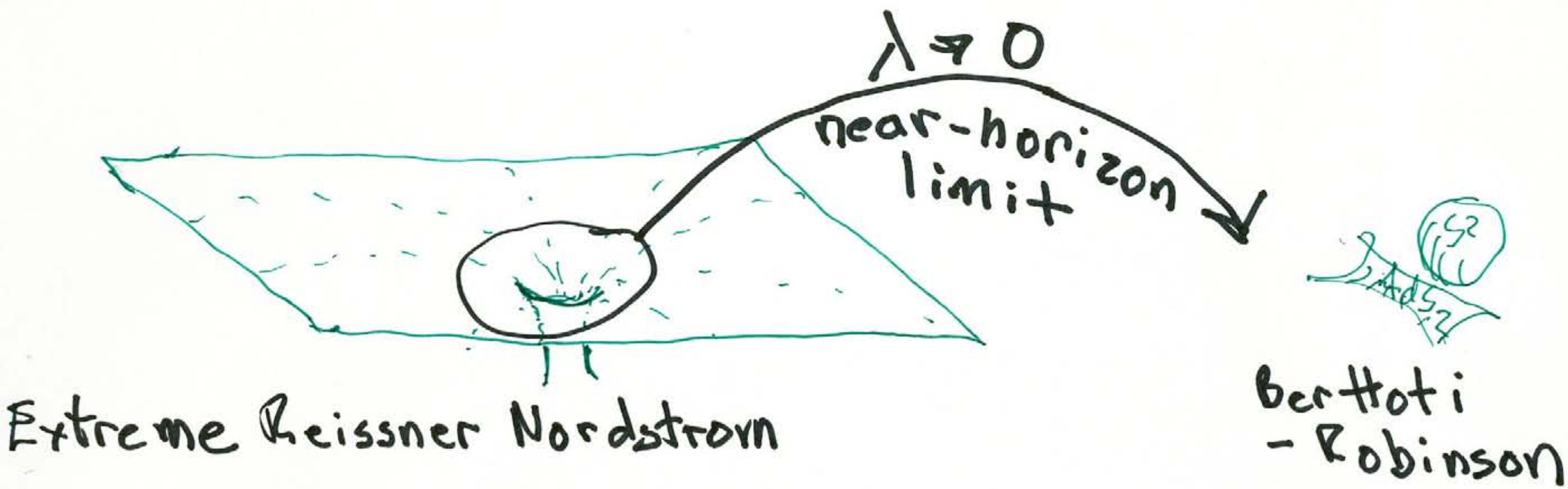
$$ds^2 = -\left(1-\frac{Q}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1-\frac{Q}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

in new coordinates $x = \frac{r-Q}{\lambda}$, $\tau = \frac{\lambda t}{Q^2}$,

$$= \underbrace{Q^2\left(-x^2 d\tau^2 + \frac{dx^2}{x^2}\right)}_{AdS_2} + \underbrace{Q^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{S^2}$$


$$+ \mathcal{O}(\lambda)$$

$\lambda \to 0 \Rightarrow$ Robinson-Berttoti Universe 1959




## Enhanced symmetries

$SO(3) \times U(1) \longrightarrow SO(3) \times SO(2,1)$

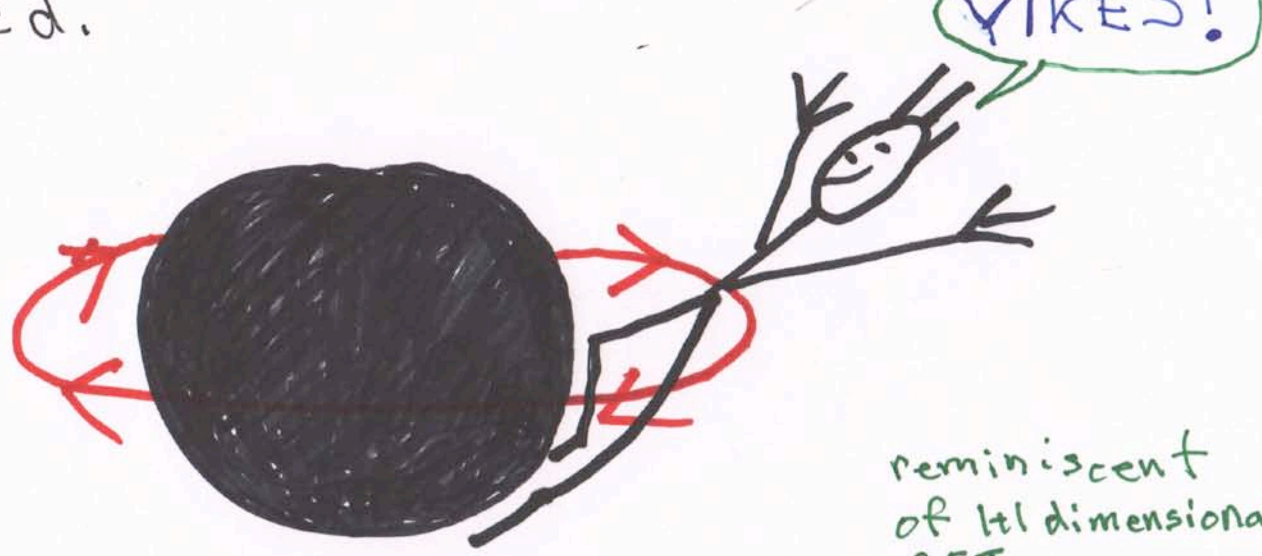
- SO(3): rotations
- U(1): time translations
- SO(3): rotations
- SO(2,1): global conformal transformations

includes scaling: $t \to \alpha t$, $x \to \dfrac{x}{\alpha}$

# EXTREME KERR



Black holes have a maximal spin $J = GM^2$, as the horizon can't exceed light speed.



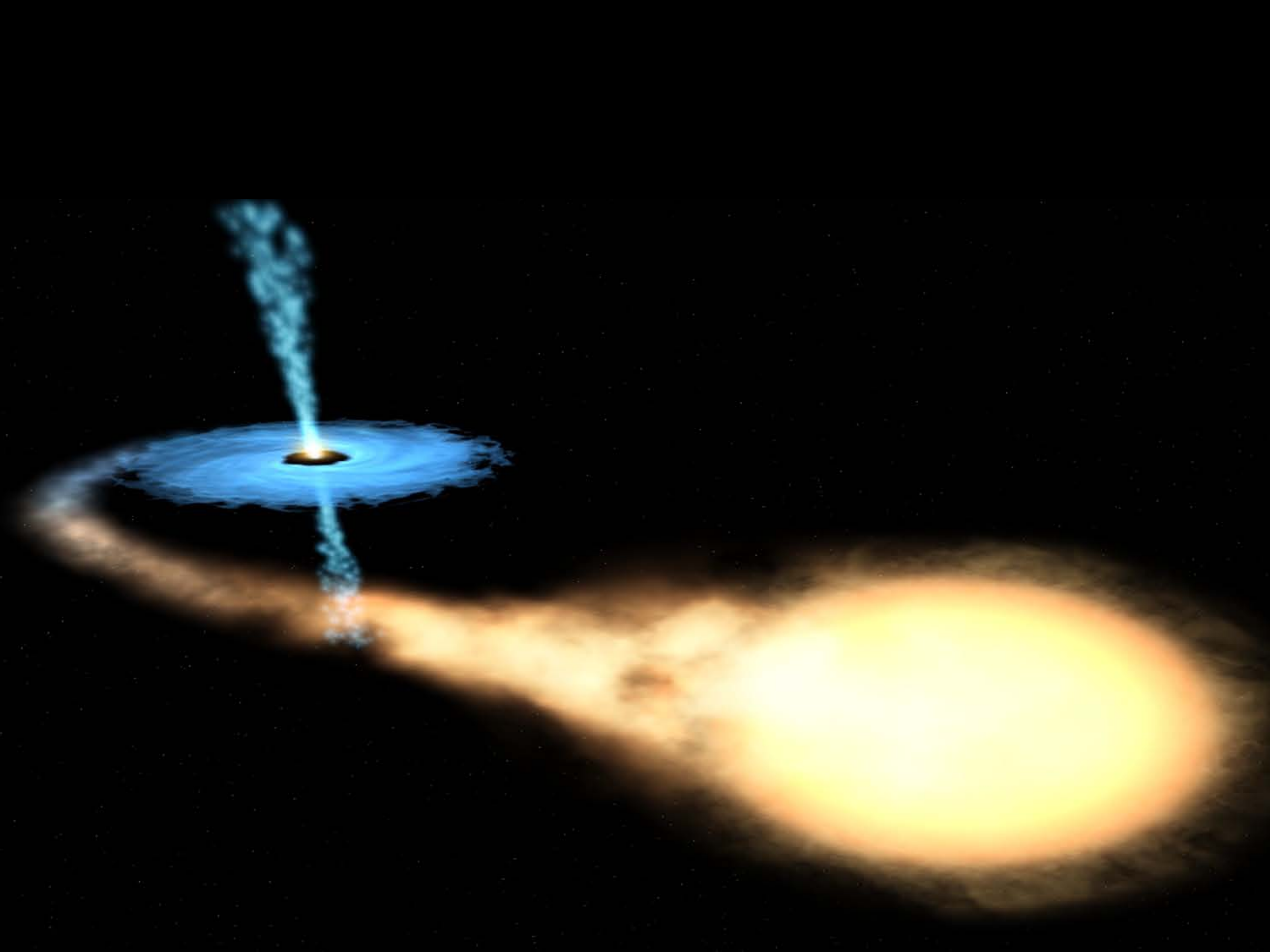

reminiscent of 1+1 dimensional CFT

The extreme ones are simple because everything is constrained to move at the speed of light when near the horizon.

GRS 1915+105 $\lesssim$ 2% below lightspeed McClintock Shafee Narayan Remillard Davis

MC-6-30-15 $\lesssim$ 1% below lightspeed Brenneman & Reynolds

Cygnus X1 $\lesssim$ 2% below lightspeed Gou McClintock Remillard Steiner Reid Orosz Narayan Hanke Garcia



# Near-Horizon Extreme Kerr

The extreme Kerr metric is

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\sin^2\theta}{\rho^2}\left((r^2+a^2)d\phi - a\,dt\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$a = \frac{J}{M},\quad J = GM^2,\quad \Delta = (r-a)^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

the near-horizon limit is

$$\tau = \frac{\lambda t}{2GM},\quad x = \frac{r-GM}{\lambda GM},\quad \psi = \phi - \frac{t}{2GM} \quad \text{with } \lambda \to 0$$

giving the **NHEK** geometry

$$ds^2 = \frac{GJ}{1+\cos^2\theta}\left[-x^2 d\tau^2 + \frac{dx^2}{x^2} + \frac{4\sin^2\theta}{(1+\cos^2\theta)^2}(d\psi + x\,d\tau)^2 + d\theta^2\right]$$

with an enhanced $SO(2,1)\times U(1)$ isometry!

Bardeen & Horowitz (99)

For $J \to GM^2$ $(a_* \to 1)$, $R_{ISCO} \to a$.

So for GRS 1915+105, the inner edge of the accretion disk is deep within the NHEK region. The geometry is the NHEK metric up to < 2% corrections, and the enhanced $SO(2,1)$ symmetry governs the dynamics.

# MORE!!

A careful analysis shows the physical symmetry group is $\infty$-dimensional, involving

$$\delta\psi = f(\psi), \quad \delta r = \frac{\partial f(\psi)}{\partial \psi} r$$

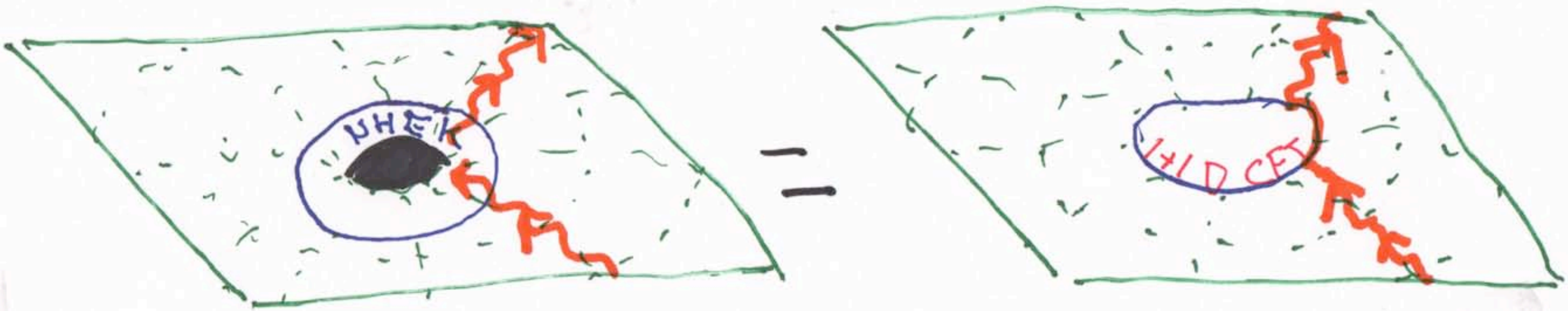
i) These are a special subgroup of diffeomorphisms which act "non-trivially" like asymptotic Poincare transformations in Minkowski space

ii) The analysis uses the methods employed by Bondi, van der Burg, Metzner & Sachs to show that the BMS group is the physical symmetry of future null infinity

iii) This group is the same as that in critical Ising model and is a very nice and familiar one from 2D conformal field theory!

Guica, Hartman, Song & AS 2008

# Computationally this implies




The extreme BH can be replaced by a 1+1 conformal field theory at the boundary of the NHEK region. This "weak" Kerr/CFT correspondence is NOT a new law of physics or a conjecture. It is a rewriting of the Einstein equation in a manner that makes the conformaly symmetry manifest.

# ASIDE

For **quantum** gravity on NHEK, it is natural to presume that the conformal symmetry acts on states. This, together with the analogy to AdS/CFT, motivates the **strong Kerr/CFT conjecture**: quantum gravity on NHEK is dual to a (warped) 2D CFT. If correct, strong Kerr/CFT potentially accounts for extreme Kerr entropy $S = 2\pi J/\hbar$. However that is the subject of a different talk.


What are the observational signals of extreme Kerr conformal symmetry?

The existence of the conformal symmetry ~~This~~ means powerful methods from 2D CFT can be used to study extreme Kerr. For example consider wave scattering

"Mathematical Theory of Black Holes"
S. Chandrasekhar


In a suitable parameter regime
Near region = NHEK

22c

S. A. TEUKOLSKY & W. H. PRESS
APJ 193 (1974)





is the fractional gain (or loss) of energy in a scattered wave, Starobinsky and Churilov find (in our notation)

$$Z = -\frac{k}{\epsilon}\,[\omega(r_+ - r_-)]^{2l+1}\left[\frac{(l-s)!\,(l+s)!}{(2l)!\,(2l+1)!!}\right]^2 \prod_{n=1}^{l}\left(1 + \frac{k^2}{4\epsilon^2 n^2}\right), \tag{6.2}$$

which shows the amplification ($Z$ positive) in the superradiant regime ($k\omega$ negative).

For $\omega = m\omega_+$, the radial equations can be solved in terms of confluent hypergeometric functions in the special case $a = M$ (maximally rotating hole). Starobinsky and Churilov perturbed away from this solution to find solutions for $Z$ in a neighborhood $\omega \approx m\omega_+$, but with $a = M$ exactly. Here their calculation is generalized to a neighborhood $a \approx M$.

When $a = M$, there are two distinct cases for the behavior of $Z$, depending on the value of

$$\delta^2 \equiv a^2m^2/M^2 - \lambda - (s + \tfrac{1}{2})^2 . \tag{6.3}$$

(Note that our definition of $\lambda$ differs from that used by Starobinsky and Churilov by $-2am\omega$.) When $\delta^2 < 0$, $Z$ passes through smoothly through zero as $\omega$ passes through $m\omega_+$. When $\delta^2 > 0$, however (which occurs for all modes with $l = m$ and for other modes with $l$ close enough to $m$), $Z$ oscillates an infinite number of times between two positive values when $\omega \to m\omega_+$ from below, while $Z$ oscillates an infinite number of times between a negative value and $-1$ when $\omega \to m\omega_+$ from above. The magnitude of the oscillations is generally too small to be seen in numerical work, with one exception: for $s = l = m = 1$, $Z_{\max}/Z_{\min} = 1.44$. (For comparison, the value for $s = l = m = 2$ is 1.00002.) However, the numerical work of the next section shows no hint of an oscillation, even for $s = l = m = 1$ and $a = 0.99999M$. The reason for this is seen in studying the behavior of $Z$ in the double limit $a \to M$, $\omega \to m\omega_+$.

Appendix A extends the calculation of $Z$ to the case where

$$\alpha \equiv 1 - \omega/(m\omega_+) \ll 1\,, \qquad \gamma \equiv (1 - a^2/M^2)^{1/2} \ll 1\,. \tag{6.4}$$

The result is

$$\begin{aligned} Z = \sinh^2 2\pi\delta \sinh \pi\kappa \exp(2\pi\hat\omega)/[&e^{-\pi\delta}\cosh^2 \pi(2\hat\omega - \delta)\cosh \pi(2\hat\omega + \delta + \kappa) \\ &+ e^{\pi\delta}\cosh^2 \pi(2\hat\omega + \delta)\cosh \pi(2\hat\omega - \delta + \kappa) \\ &- 2\cos\psi \cosh \pi(2\hat\omega + \delta)\cosh \pi(2\hat\omega - \delta)\cosh^{1/2}\pi(2\hat\omega + \delta + \kappa)\cosh^{1/2}\pi(2\hat\omega - \delta + \kappa)]\,, \end{aligned} \tag{6.5}$$

where $\kappa = ma\alpha/(\gamma M)$, $\hat\omega = \omega r_+$, and $\psi$ is defined in equation (A16). The result of Starobinsky and Churilov is recovered by letting $\gamma \to 0$ ($\kappa \to \infty$). For $\alpha$ positive, this gives

$$Z = \sinh^2 2\pi\delta/[\cosh^2 \pi(2\hat\omega - \delta) + \cosh^2 \pi(2\hat\omega + \delta) - 2\cos\psi \cosh \pi(2\hat\omega - \delta)\cosh \pi(2\hat\omega + \delta)]\,, \tag{6.6}$$

where $\psi = \text{constant} - 2\delta \ln(2m^2\alpha)$. As $\alpha \to 0$, this exhibits an infinite number of oscillations between

$$Z_{\max} = \cosh^2 \pi\delta/\sinh^2 2\pi\hat\omega \tag{6.7}$$

and

$$Z_{\min} = \sinh^2 \pi\delta/\cosh^2 2\pi\hat\omega\,. \tag{6.8}$$

However, if $\alpha \to 0$ for fixed (small) $\gamma$ in equation (6.5), we find $Z \to 0$ linearly with $\alpha$. Thus for fixed small $\gamma$ there will be a finite number of oscillations before $Z \to 0$. In the favorable case of $s = l = m = 1$, $\delta^2$ is not positive (i.e., eq. [6.5] showing oscillations is not valid) until $a > 0.995M$. Even then, numerical evaluation of equation (6.5) shows that one must have $a$ within $10^{-7}$ of $M$ before an oscillation becomes visible. Thus for all practical purposes the oscillations can be ignored as a mathematical curiosity of the nonuniform double limit $a \to M$, $\omega \to m\omega_+$.

This is exactly a 2D CFT correlator at finite temperature in disguise!

## VII. NUMERICAL RESULTS: WAVE ABSORPTION AND SUPERRADIANT SCATTERING

Our primary task is to calculate $Z$ (eq. [6.1]), the fractional gain or loss of energy in a scattered wave. $Z$ is a function of the nondimensional frequency of the wave $\omega M$, the hole's rotation $a/M$, the mode numbers $l$ and $m$, and of course $|s|$ which tells what perturbing field we are looking at. Using the results of the preceding sections, the calculation of $Z$ is a straightforward: choose a form of the radial perturbation equation (§ II) which makes the inward boundary condition easy to impose and (importantly!) numerically stable against contamination with the other solution when integrated outward. At some moderate radius, say $r \sim 3M$, use the relations between the solutions (§ III) to switch to a different form of the radial equation, one which makes the ingoing wave solution stable to numerical integration out to radial infinity, and the coefficient of this wave easy to determine there. Completing the integration, use the results of § IV to read off the ingoing energy fluxes at the horizon and at

The equality of wave scattering amplitudes in Kerr computed with standard GR and 2D CFT techniques have by now been checked in great detail in many examples. The "Kerr/CFT" dictionary is dictated by the conformal symmetry.

Hartman Song Bredberg AS
Becker Schatgin Murata
Cremonini Chen Long
Chu Cvetic Larsen

But this is not observationally relevant to my knowledge. We are now using conformal symmetry to compute the gravity wave signal from a star plunging into MC-6-30-15.


Rate estimates: 1~5000/year observable at eLisa

This was computed for the Schwarzschild case by Hadar & Kol and has been discussed as a possible eLISA signal.

First, the gravitational waveform emitted by a star on the extreme Kerr ISCO (in the extreme-mass-ratio limit) was computed by mapping to a 2D CFT and using Fermi's golden rule


$$= 2\pi \langle \mathcal{O}^\dagger_{STAR}\ \mathcal{O}_{STAR} \rangle_{2D\ CFT}$$

Fermis golden rule in 2D CFT

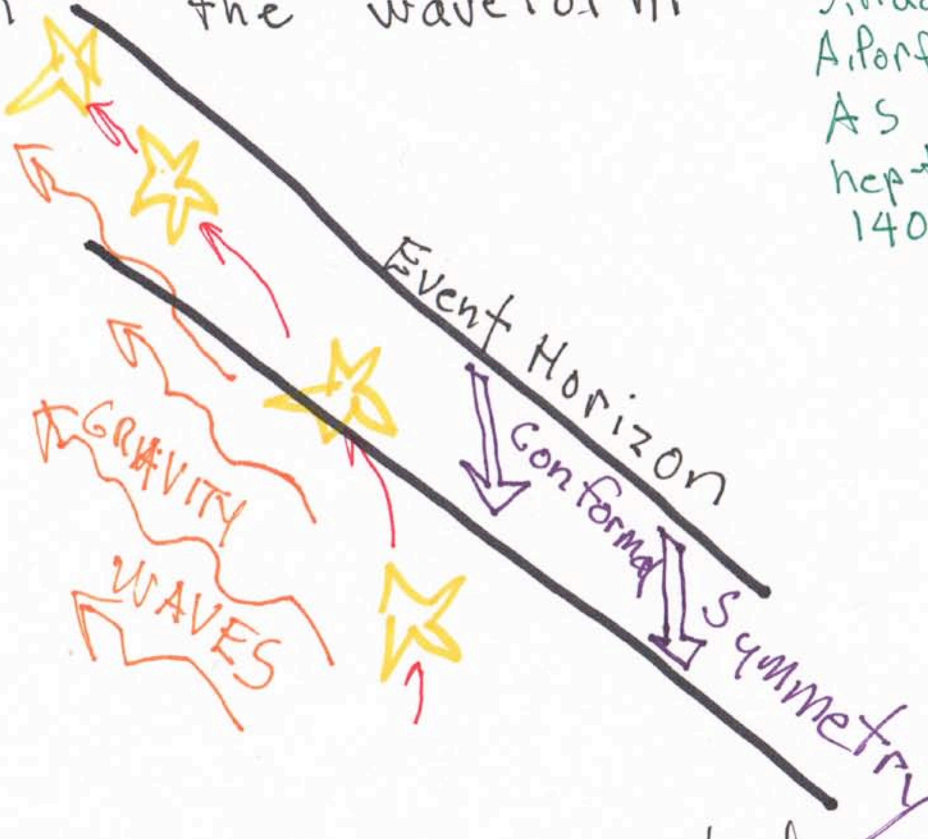
↑ operator dual to star

This might have been (but wasn't) solved by conventional methods.

Porfyriadis & AS hep-th/1401.3746

Second we used conformal symmetry ~~to map~~ (which moves the event horizon) to map this to the plunge problem and obtain the waveform

S. Hadar
A. Porfyriadis
AS
hep-th
1403.2797




Its unlikely this could be computed without exploiting the symmetry.

Most stars arrive at the ISCO with nonzero eccentricity, and plunge more rapidly into the black hole. In a 3rd paper, recently submitted, we have solved analytically for the outgoing gravitational waveform with yet another conformal mapping — to a quantum quench of the dual CFT.

"Fast plunges into a Kerr black hole"
S. Hadar, A. Porfyriadis & AS hep-th/1504.07650

# ANOTHER DIRECTION



Conformal symmetry has also been used to solve the equations of Force-Free Electrodynamics (FFE):

$$\nabla^\mu F_{\mu\nu} = J_\nu$$

$$F_{\mu\nu} J^\nu = 0$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

which govern the EM field in a dilute plasma, in the NHEK region of extreme Kerr. These are very beautiful, physically relevant but poorly understood equations.

Blandford & Znajek argued these describe energy extraction from a black hole with spin


powering many astrophysical signals.
Until recently, solutions almost exclusively numerical.

Energy extraction requires spin.
The more the better.

Occurs near the horizon.

⇒ NHEK a good place to study it!

Infinite families of exact solutions
found in last several months.
(S. Gralla & T. Jacobson)

A. Lupsasca, M. Rodriguez, AS
hep-th/1406!4133
F. Zhang, H. Yang, L. Lehner
astro-ph.HE 1409.0345
A. Lupsasca & M. Rodriguez, to appear

**HOPE**: Improved conceptual understanding of astrophysical energy extraction from BHs, and explanation of observational data.

WATCH THE MOVIE!

Brenneman & Reynolds 2006
MCG-6-30-15
$a/M > .989$!

Fe lines



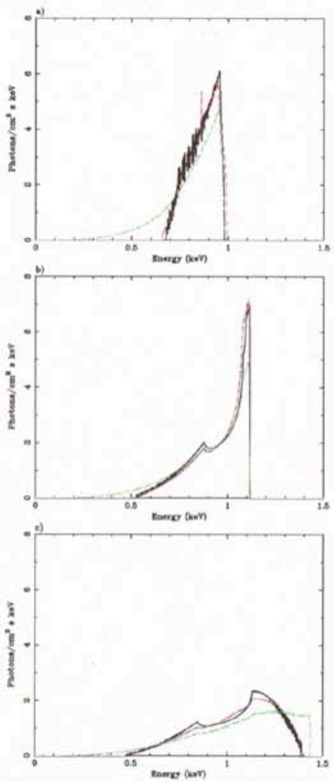


Fig. 2.— Various iron line models for a Schwarzschild black hole. The inclination angles represented are 5° for (a), 45° for (b), and 80° for (c). Here $\alpha_1 = \alpha_2 = 3.0$, and $r_{\rm min}$ and $r_{\rm max}$ are held constant at $6\,r_{\rm g}$ and $50\,r_{\rm g}$, respectively. The `diskline` profile is in solid black, the `kerrdisk` profile is in dashed red, the `kyrline` profile including emission from within the ISCO is in dash-dotted green and the `kyrline` profile not including ISCO radiation is in dotted blue.



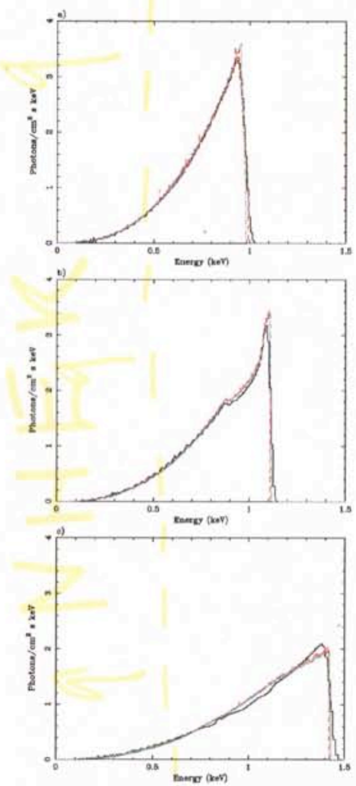
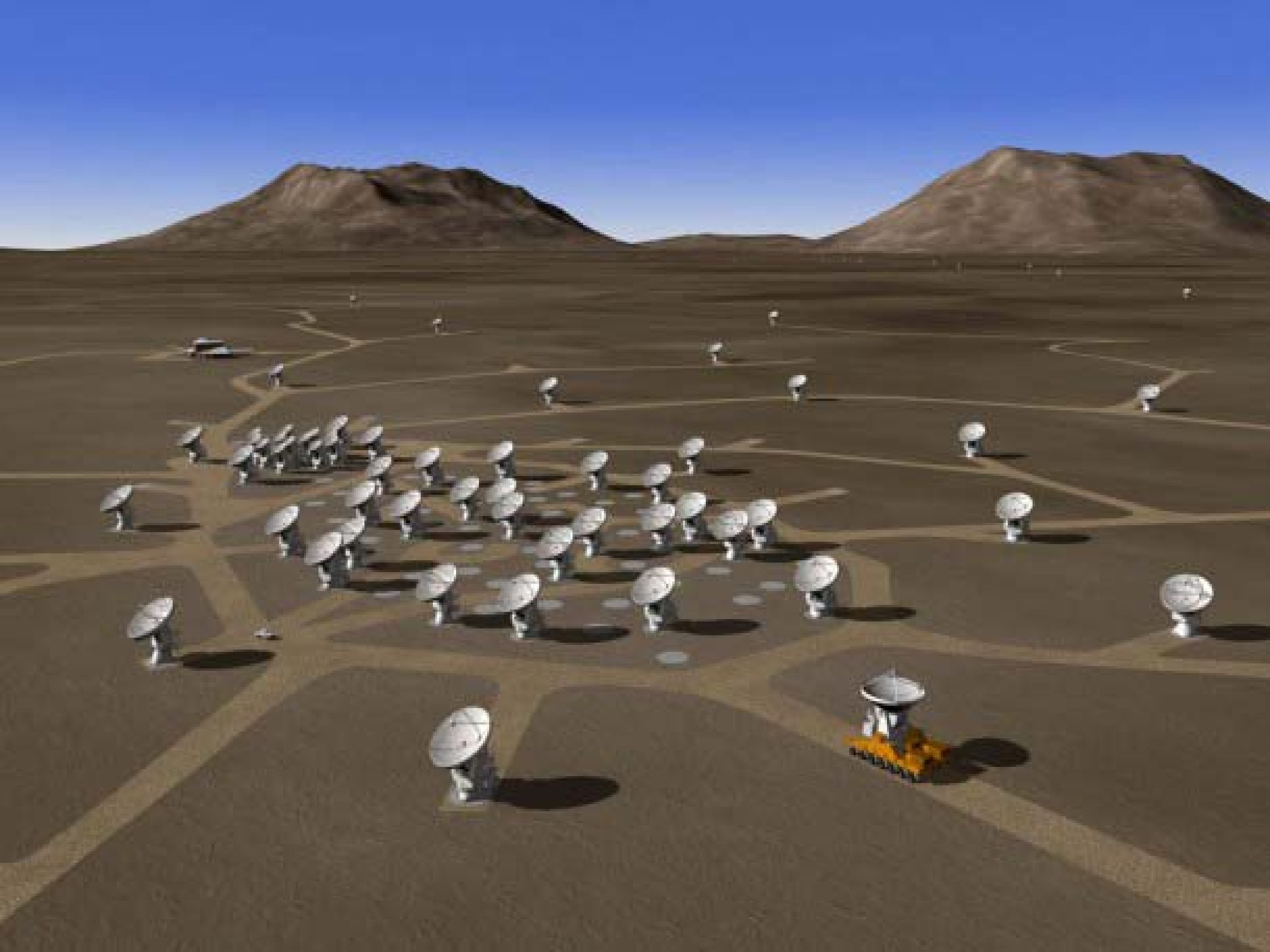


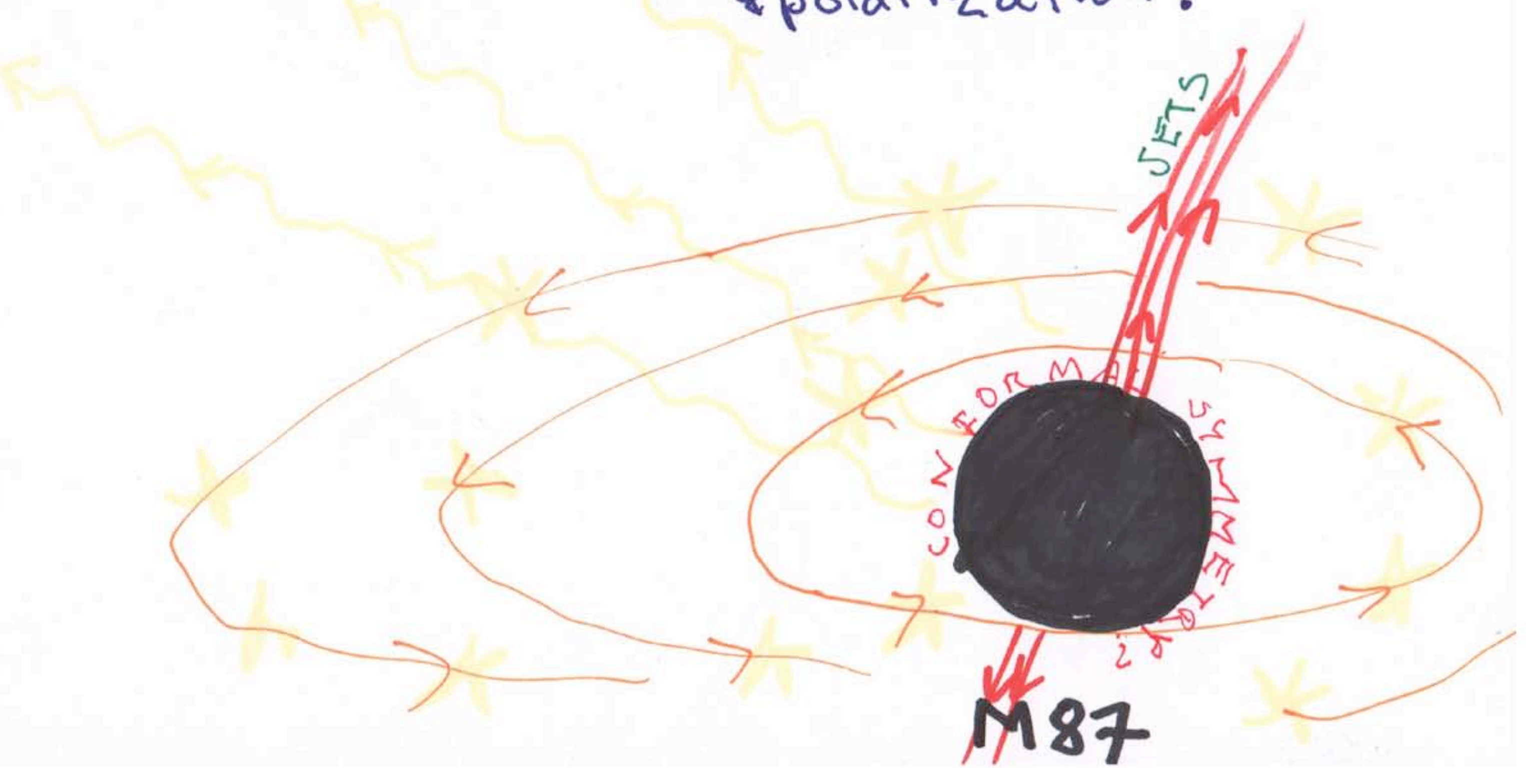
Fig. 3.— Various iron line models for a maximally spinning Kerr black hole. The inclination angles vary as above for the Schwarzschild case in (a)-(c). Other parameters are the same as those used in the Schwarzschild case, but now $r_{\rm min} = 1.235\,r_{\rm g}$, corresponding to the radius of marginal stability for a maximal Kerr black hole, rather than the $6\,r_{\rm g}$ Schwarzschild $r_{\rm ms}$. The `laor` profile is in solid black and the rest of the color scheme is the same as that used in Fig. 2 above.

IN PROGRESS: Are these curves determined by critical exponents in NHEK?



# IN PROGRESS


The Event Horizon Telescope proposes to resolve the BH at the center of M87 galaxy, observing submillimeter radiation. Highly collimated jets suggest a large spin. What are ~~the~~ the predictions of conformal symmetry for the profile & polarization?



# CLOSING REMARKS

1. conformal symmetry likely appears in the sky
2. It is computationally powerful


3. We have not found a "smoking gun" signal
4. We need help in looking from astrophysicists!

THANK YOU!