

A General Relativistic Thomas-Fermi Treatment of Neutron Star Cores

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Introduction

Traditional approach of neutron star cores

Gravitational interaction	⇒	General Relativity
Weak interaction	⇒	β -equilibrium
Strong interaction	⇒	Different approaches
Electromagnetic interaction	⇒	NONE



Local charge neutrality condition

$n_e(r) = n_p(r)$ decouples Microphysics and Macrophysics



EOS embedded into Einstein equations

Introduction

Global charge neutrality

$$n_e(r) \neq n_p(r) \Rightarrow N_e = N_p$$

How to incorporate this new GLOBAL condition self-consistently ?



Thomas-Fermi model: from atoms to neutron stars !!

Toward a GR Thomas-Fermi treatment of neutron star cores

General Relativistic Equilibrium Equations

EINSTEIN-MAXWELL EQUATIONS

+

β -EQUILIBRIUM

+

GR THOMAS-FERMI EQUILIBRIUM CONDITION FOR e^- s

+

GLOBAL NEUTRALITY

+

STRONG INTERACTION ?? (see next talk)

General Relativistic Thomas-Fermi treatment: formulation of the simplest example $\rightarrow$ “non-interacting” npe-gas

Metric

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2,$$

Einstein-Maxwell Equations

$$e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) - \frac{1}{r^2} = -8\pi G T_0^0,$$

$$e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = -8\pi G T_1^1,$$

$$P' + \frac{\nu'}{2}(\mathcal{E} + P) = -(eV)'(n_p - n_e)e^{\nu/2},$$

$$(eV)'' + (eV)' \left[\frac{2}{r} - \frac{(\nu' + \lambda')}{2} \right] = -4\pi\alpha e^{\lambda}(n_p - n_e)e^{\nu/2},$$

The simplest example: “non-interacting” npe-gas

GR Thomas-Fermi equilibrium + β -equilibrium

$$E_e^F = e^{\nu/2} \mu_e - eV = \text{constant}, \quad E_n^F = E_p^F + E_e^F$$

(using the Einstein-Maxwell equations)



Minimum energy condition

$$E_i^F = e^{\nu/2} \mu_i + q_i V = \text{constant}$$

for all i -species



Generalization for the case of a fluid with charged particles
(Olson and Baylin 1975, 1978; Jorge A. Rueda et al. 2010)

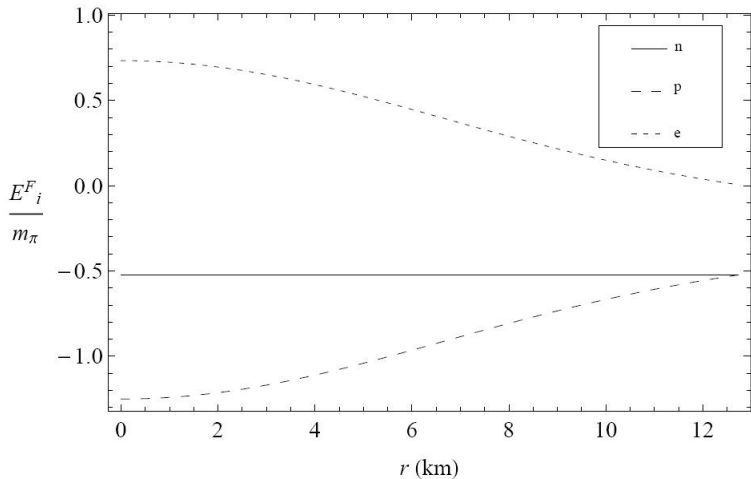


Figure: Fermi energy of neutrons, protons and electrons inside a locally charge neutral configuration.

The simplest example: “non-interacting” npe-gas

Global neutrality condition

$$eV(R_e) = 0, \quad eV'(R_e) = 0$$

Generalized particle Fermi energies

$$E_e^F = e^{\nu/2} \mu_e - eV = m_e e^{\nu(R_e)/2},$$

$$E_p^F = e^{\nu/2} \mu_p + eV = m_p e^{\nu(R_p)/2} + eV(R_p),$$

$$E_n^F = e^{\nu/2} \mu_n - eV = m_n e^{\nu(R_n)/2},$$

$$n_i(R_i) = 0 \quad \Rightarrow \quad R_n < R_p < R_e !!$$

Some analytic results ...

$$\frac{n_p(r)}{n_e(r)} = \left(\frac{m_p}{m_e}\right)^3 \left[\frac{(m_e/m_p)^2 f^2(r) \mu_e^2(r) - m_e^2}{\mu_e^2(r) - m_e^2} \right]^{3/2},$$
$$\frac{n_p(r)}{n_n(r)} = \left(\frac{m_p}{m_n}\right)^3 \left[\frac{(m_n/m_p)^2 g^2(r) \mu_n^2(r) - m_n^2}{\mu_n^2(r) - m_n^2} \right]^{3/2},$$

where the functions $f(r)$ and $g(r)$ are given by

$$f(r) = \frac{E_p^F - eV(r)}{E_e^F + eV(r)}, \quad g(r) = \frac{E_p^F - eV(r)}{E_n^F}$$

$$\frac{n_p(r)}{n_e(r)} = \left(\frac{m_p}{m_e}\right)^3 \quad \left\{ \text{For } eV(r) \equiv 0 \quad !!! \right.$$

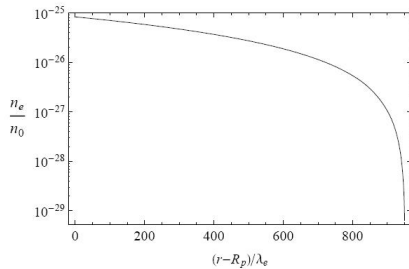
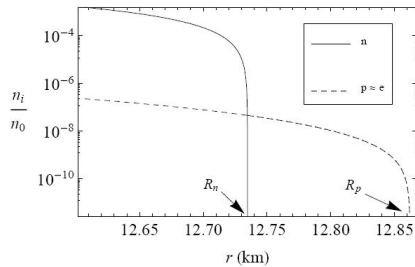
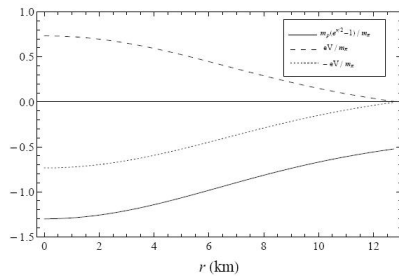
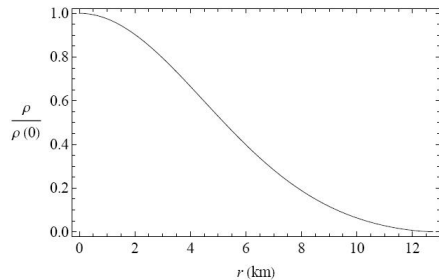
The condition $n_p = n_e$ must be satisfied at the center of the star:

Estimating the Coulomb potential at $r = 0$...

$$eV(0) = \frac{(m_p - m_e)}{2} \left[1 + \left(\frac{E_p^F - E_e^F}{m_p - m_e} \right) - \left(\frac{m_p + m_e}{E_n^F + m_n} \right) e^{\nu(0)} \right],$$
$$eV(0) \approx \frac{1}{2} \left[m_p e^{\nu(R_p)/2} - m_e e^{\nu(R_e)/2} - \frac{m_n e^{\nu(R_n)/2}}{1 + (P_n^F(0)/m_n)^2} \right]$$

For $\rho(0) > \rho_{\text{nuc}} \approx 0.5 m_\pi^3$

$$eV(0) \gtrsim 0.35 m_\pi c^2$$



Summary I

- 1 The local neutrality is indeed never reachable. The boundary conditions must take into account the presence of the Coulomb potential self-consistently: global neutrality.
- 2 Due to the gravito-electrodynamic coupling Microphysics and Macrophysics can not be decoupled ! the description is then necessary **GLOBAL**.
- 3 Despite the Coulomb potential depth is about $\sim m_\pi$, the system is stable against proton repulsion.

Open questions I

- 1 Rotation $\Rightarrow$ Magnetic Fields ! (see Boshkayev's talk, Rohollah's talk and Xue's talk)
- 2 Stability against radial oscillations (see Haney's talk)
- 3 Introducing a Crust (see next slides ...)

Introducing a crust ...

Core

npe-fluid in beta equilibrium: charged

Crust

Nuclei lattice + free-electrons (and possibly neutrons): neutral



Main Problem: Core-Crust Transition

Introducing a crust ...

Generalized particle Fermi energies

$$\begin{aligned}E_e^F &= e^{\nu(R_c)/2} \mu_e^{\text{crust}}, \\E_n^F &= e^{\nu(R_c)/2} \mu_n^{\text{crust}}, \\E_p^F &= E_n^F - E_e^F\end{aligned}$$



Boundary conditions at the core-crust interface

Global neutrality + n_e at the edge of the crust

Results for $\rho(0) = 4\rho_0$

$\frac{E_e^F}{m_\pi}$	$\frac{eV(0)}{m_\pi}$	$\frac{eV(R_c)}{m_\pi}$	$\frac{E_{max}}{E_c}$	$\frac{\rho_{crust}}{\rho_{drip}}$	$\frac{M_{crust}}{M_\odot}$	$\Delta_r^{ic}(\text{m})$	$\Delta_r^{oc}(\text{km})$
0.10	0.65	0.271	378.10	0.125	2.85×10^{-6}	0.00	0.654
0.15	0.60	0.221	373.09	0.384	1.42×10^{-5}	0.00	1.027
0.20	0.55	0.167	358.45	1.000	5.16×10^{-5}	0.00	1.482
0.30	0.45	0.070	270.14	46.19	5.46×10^{-5}	1.62	1.482
0.35	0.40	0.023	139.48	80.83	6.17×10^{-5}	2.43	1.482

Table: Results of the integration for selected values of E_e^F (Jorge A. Rueda et al. 2010).

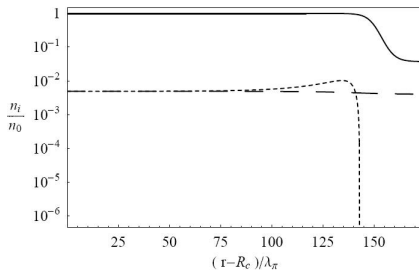
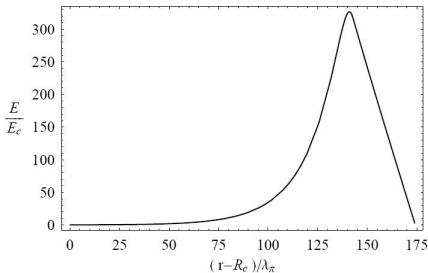
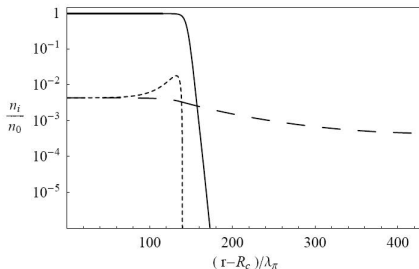
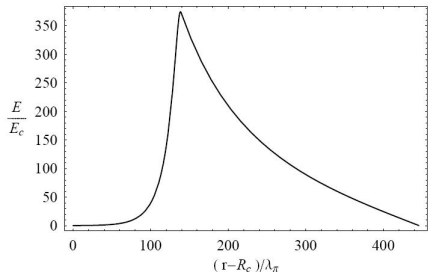


Figure: Electric field and particle densities at the transition surface for $E_e^F = 0.2 m_\pi$ and $E_e^F = 0.35 m_\pi$.

Summary II

- 1 The neutron star equilibrium configurations are parameterized by the central density as well as by the generalized Fermi energy of the electrons of the core, which determines uniquely the electrostatic structure of the star.
- 2 The correct boundary conditions (**continuity of the generalized particle Fermi energies**) leads to a new family of neutron stars with crusts of smaller mass and smaller thickness, and consequently to neutron stars with alternative mass-radius relation.
- 3 Electric fields above the critical value appears in the core-crust interface extending on a surface shell of thickness $\sim \lambda_e$.

Open questions II

- 1 Rotation $\Rightarrow$ Magnetic Fields ! (see Boshkayev's talk, Rohollah's talk and Xue's talk)
- 2 Stability against radial oscillations (see Haney's talk)
- 3 Inclusion of strong interaction (see next talk)

THANKS !