

A general relativistic Thomas-Fermi treatment of neutron star core -part 2-

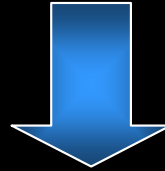
GENERALIZED FERMI ENERGIES AND BETA EQUILIBRIUM

Jorge A. Rueda and [Daniela Pugliese](#)

“Sapienza” University of Rome, Icria and Icranet

INTRODUCTION

(1) Generalized Fermi energies in a General relativistic treatment with Coulomb and nuclear interaction



(2) Modified beta equilibrium condition



(3) Constancy of the generalized Fermi energy.

- O. Klein, Rev. Mod. Phys. 21, 531 (1949).
- T. Kodama and M. Yamada, Prog. Theor. Phys. 47, 444 (1972).
- E. Olson and M. Bailyn, Phys. Rev. D 12, 3030 (1975).

GENERAL FORMULATION

$$\hbar = c = 1$$

the Latin indexes vary from 1 to 3, Greek indexes from 0 to 4

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\gamma + \mathcal{L}_{\text{int}}$$

$$\begin{aligned}\Omega_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \\ \mathcal{R}_{\mu\nu} &= \partial_\mu \rho_\nu - \partial_\nu \rho_\mu, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu,\end{aligned}$$

$$\begin{aligned}j_\omega^\mu &= \bar{\psi}_N \gamma^\mu \psi_N, \\ j_\rho^\mu &= \bar{\psi}_N \tau_3 \gamma^\mu \psi_N, \\ j_{\gamma,e}^\mu &= \bar{\psi}_e \gamma^\mu \psi_e, \\ j_{\gamma,N}^\mu &= \bar{\psi}_N \left(\frac{1 + \tau_3}{2} \right) \gamma^\mu \psi_N\end{aligned}$$

$$\begin{aligned}\mathcal{L}_g &= -\frac{1}{16\pi G} R, \\ \mathcal{L}_f &= \sum_{i=e,N} \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i, \\ \mathcal{L}_\sigma &= \frac{1}{2} \nabla_\mu \sigma \nabla^\mu \sigma - U(\sigma), \\ \mathcal{L}_\omega &= -\frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu, \\ \mathcal{L}_\rho &= -\frac{1}{4} \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu, \\ \mathcal{L}_\gamma &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \\ \mathcal{L}_{\text{int}} &= -g_\sigma \sigma \bar{\psi}_N \psi_N - g_\omega \omega_\mu j_\omega^\mu - g_\rho \rho_\mu j_\rho^\mu \\ &\quad + e A_\mu j_{\gamma,e}^\mu - e A_\mu j_{\gamma,N}^\mu,\end{aligned}$$

$$\begin{aligned}
\{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu}, \\
\{\gamma_\mu, \gamma_\nu\} &= 2g_{\mu\nu}, \\
\{\gamma^\mu, \gamma_\nu\} &= 2\delta_\nu^\mu, \\
\{\tau_i, \tau_j\} &= 2\delta_{ij}, \\
[\tau_i, \tau_j] &= 2i\epsilon_{ijk}\tau^k,
\end{aligned}$$

Einstein-Maxwell-Dirac equations with the contribution of hadronic fields

$$V_N^\mu = g_\omega \omega^\mu + g_\rho \tau \rho^\mu + e \left(\frac{1 + \tau_3}{2} \right) A^\mu$$

$$\tilde{m}_N = m_N + g_\sigma \sigma$$

$$\tilde{m}_e = m_e$$

$$T_\sigma^{\mu\nu} = \nabla^\mu \sigma \nabla^\nu \sigma - g^{\mu\nu} \left[\frac{1}{2} \nabla_\alpha \sigma \nabla^\alpha \sigma - U(\sigma) \right]$$

$$T_{(f)}^{\mu\nu} = (\mathcal{E} + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$\begin{aligned}
G_{\mu\nu} + 8\pi G T_{\mu\nu} &= 0 \\
\nabla_\mu \nabla^\mu \sigma + \partial_\sigma U(\sigma) + g_s \bar{\psi}_N \psi_N &= 0 \\
\nabla_\mu \Omega^{\mu\nu} + m_\omega^2 \omega^\nu - g_\omega j_\omega^\nu &= 0 \\
\nabla_\mu \mathcal{R}^{\mu\nu} + m_\rho^2 \rho^\nu - g_\rho j_\rho^\nu &= 0 \\
\nabla_\mu F^{\mu\nu} - e j_{\text{ch}}^\nu &= 0 \\
\left[\begin{aligned} &[\gamma_\mu (i\partial^\mu + eA^\mu) - m_e] \psi_e = 0 \\ &[\gamma_\mu (i\partial^\mu - V_N^\mu) - \tilde{m}_N] \psi_N = 0 \end{aligned} \right.
\end{aligned}$$

$$\begin{aligned}
T_\omega^{\mu\nu} &= -\Omega_\alpha^\mu \Omega^{\alpha\nu} - \frac{1}{4} g^{\mu\nu} \Omega_{\alpha\beta} \Omega^{\alpha\beta} \\
&+ m_\omega^2 \left(\omega^\mu \omega^\nu - \frac{1}{2} g^{\mu\nu} \omega_\alpha \omega^\alpha \right) \\
T_\rho^{\mu\nu} &= -\mathcal{R}_\alpha^\mu \mathcal{R}^{\alpha\nu} - \frac{1}{4} g^{\mu\nu} \mathcal{R}_{\alpha\beta} \mathcal{R}^{\alpha\beta} \\
&+ m_\rho^2 \left(\rho^\mu \rho^\nu - \frac{1}{2} g^{\mu\nu} \rho_\alpha \rho^\alpha \right), \\
T_\gamma^{\mu\nu} &= -F_\alpha^\mu F^{\alpha\nu} - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta},
\end{aligned}$$

Metric

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2$$

$$j_0^\omega = n_b u_0 = (n_n + n_p) e^{\nu/2},$$

$$j_0^\rho = n_3 u_0 = (n_p - n_n) e^{\nu/2},$$

$$j_0^{\text{ch}} = n_{\text{ch}} u_0 = (n_p - n_e) e^{\nu/2}$$

$$\epsilon_i = \sqrt{(k_i)^2 + (\tilde{m}i)^2}$$

$$e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) - \frac{1}{r^2} = -8\pi G T_0^0,$$

$$e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = -8\pi G T_1^1,$$

$$P' + \frac{\nu'}{2}(\mathcal{E} + P) = -g_\sigma n_s \sigma' - \omega' g_\omega j_\omega^0 - \rho' g_\rho j_\rho^0 - V' e j_{\text{ch}}^0,$$

$$\sigma'' + \sigma' \left[\frac{2}{r} + \frac{(\nu' - \lambda')}{2} \right] = e^\lambda [\partial_\sigma U(\sigma) + g_s n_s]$$

$$\omega'' + \omega' \left[\frac{2}{r} - \frac{(\nu' + \lambda')}{2} \right] = -e^\lambda (g_\omega j_0^\omega - m_\omega^2 \omega),$$

$$\rho'' + \rho' \left[\frac{2}{r} - \frac{(\nu' + \lambda')}{2} \right] = -e^\lambda (g_\omega j_0^\rho - m_\rho^2 \rho),$$

$$V'' + V' \left[\frac{2}{r} - \frac{(\nu' + \lambda')}{2} \right] = -e^\lambda e j_0^{\text{ch}},$$

$$n_s = \bar{\psi}_N \psi_N = \frac{2}{(2\pi)^3} \sum_{i=n,p} \int_0^{k_i^F} \frac{\tilde{m}_N}{\epsilon_i} d^3 k$$

$$T_{(f)}^{\mu\nu} = (\mathcal{E} + P)u^\mu u^\nu - P g^{\mu\nu}$$

$$\mu_i = \sqrt{(k_i^F)^2 + (\tilde{m}_i)^2}$$

$$n_i = (k_i^F)^3 / (3\pi^2)$$

$$\mathcal{E} = \frac{2}{(2\pi)^3} \sum_{i=n,p,e} \int_0^{k_i^F} \epsilon_i d^3 k$$

$$P = \frac{1}{3} \frac{2}{(2\pi)^3} \sum_{i=n,p,e} \int_0^{k_i^F} \frac{k^2}{\epsilon_i} d^3 k$$



$$\mathcal{E} + P = \sum_{i=n,p,e} n_i \mu_i$$

$$n_s = \partial \mathcal{E} / \partial \tilde{m}_N$$

$$dP = \sum_{i=n,p,e} n_i d\mu_i - g_\sigma n_s d\sigma$$

Thermodynamic relation

Generalized Gibbs-Duhem relation

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$$[\gamma_\mu(i\partial^\mu - V_N^\mu) - \tilde{m}_N] \psi_N = 0$$

$$[\gamma_\mu(i\partial^\mu + eA^\mu) - m_e] \psi_e = 0$$

$$\psi_N = \psi_N(k) e^{-ik_\mu x^\mu}$$

$$\mathcal{K}^\mu = k^\mu - V_N^\mu$$

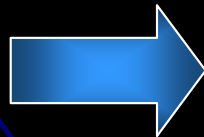
$$(\gamma_\mu \mathcal{K}^\mu - \tilde{m}_N) \psi_N(k) = 0$$

$$E_e^F = \sqrt{g_{00}}\mu_e - eV = e^{\nu/2}\mu_e - eV$$

$$E_n^F = \sqrt{g_{00}}\mu_n + g_\omega\omega - g_\rho\rho = e^{\nu/2}\mu_n + g_\omega\omega - g_\rho\rho$$

$$E_p^F = \sqrt{g_{00}}\mu_p + g_\omega\omega + g_\rho\rho + eV = e^{\nu/2}\mu_p + g_\omega\omega + g_\rho\rho + eV$$

$$E_n^F = E_p^F + E_e^F$$



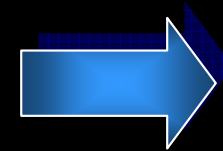
$$\mu_n = \mu_p + \mu_e + 2g_\rho\rho e^{-\nu/2}$$

CONSTANCY OF THE GENERALIZED FERMI ENERGIES

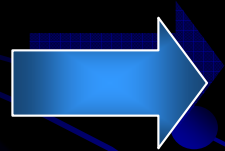
Generalized Tolman-Oppenheimer-Volkof equation

$$\nabla_\mu T_{(f)}^{\mu\nu} - g_s \bar{\psi}_N \psi_N \nabla^\nu \sigma + g_\omega \Omega_\mu^\nu j_\omega^\mu + g_\rho \mathcal{R}_\mu^\nu j_\rho^\mu + e F_\mu^\nu j_{ch}^\mu = 0$$

$$\sum_{i=n,p,e} n_i d(e^{\nu/2} \mu_i) + g_\omega n_b d\omega + g_\rho n_3 d\rho + e n_{ch} dV$$



$$\sum_{i=n,p,e} n_i dE_i^F = 0$$



$$\sum_{j=p,e} (n_j + n_n) dE_j^F = 0$$

$$E_e^F = e^{\nu/2} \mu_e - \underline{eV} = \text{constant}$$

$$E_{n,p}^F = e^{\nu/2} \mu_{n,p} + \underline{\mathcal{V}_{n,p}} = \text{constant}$$

$$\underline{\mathcal{V}_{n,p}} = V_0^N = g_\omega \omega + g_\rho \tau \rho + e \left(\frac{1 + \tau_3}{2} \right) V$$

Conclusions

- We obtained the generalized Fermi energies from the Dirac equations for nucleons and electrons (General relativistic treatment with Coulomb interaction)
- We used the generalized Fermi energies to obtain the modified beta equilibrium condition
- We showed that from the condition of minimum energy of the configuration follows the constancy of the generalized Fermi energy of each particle specie.



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