

Symmetries and Singularities

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Based on joint work with
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(in various combinations)

A few words about AEI ...

An institution based in Potsdam and Hannover, devoted to research on all aspects of **Einstein's General Relativity**, from pure theory to experiment:

- Mathematical Relativity and Geometric Analysis
- Quantum Gravity and Unified Theories
- Astrophysical and Numerical Relativity
- Gravitational Waves (GEO600)

Two Graduate Schools (IMPRS):

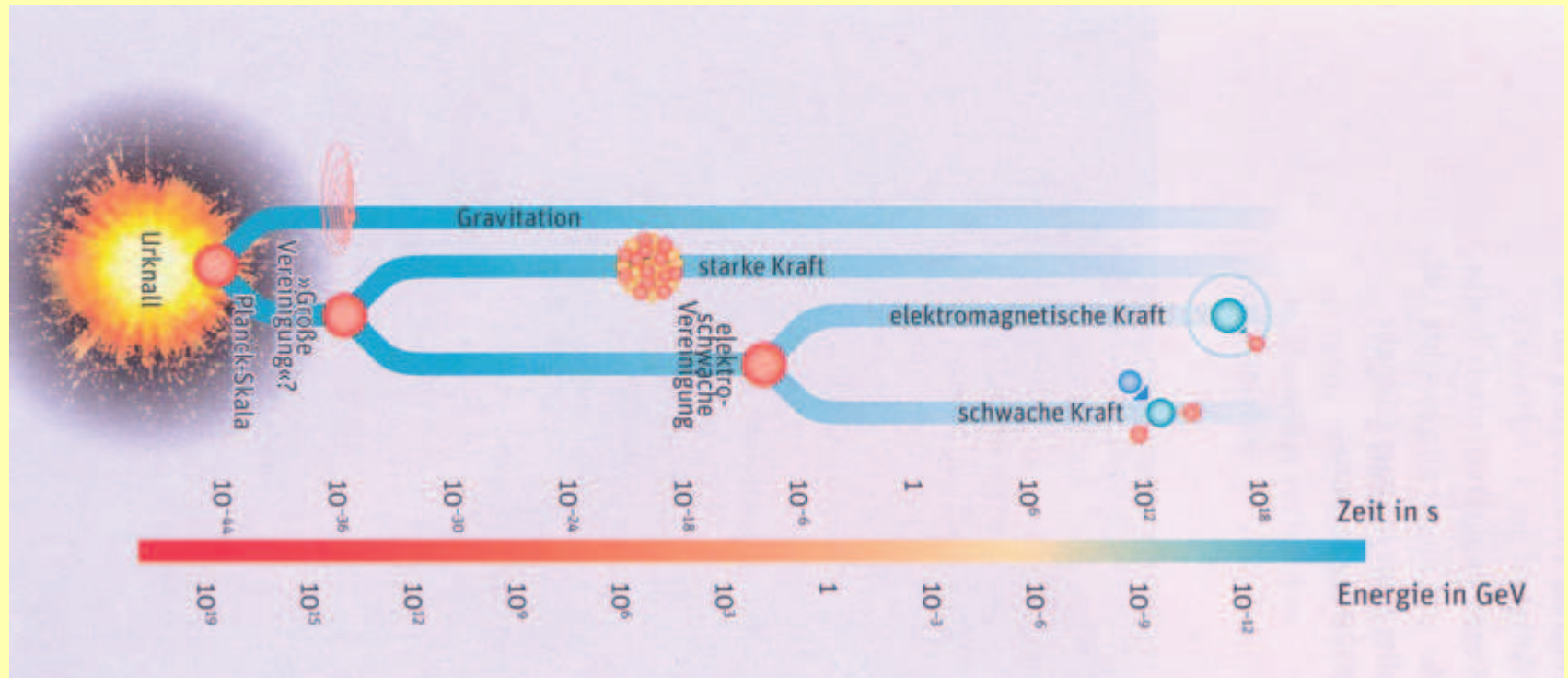
- Geometric Analysis, Gravitation and String Theory
- Gravitational Wave Astronomy

Main theme: Symmetry

... arguably the most successful principle of physics!

- Space-time symmetries
 - Rotations and translations in Newtonian physics
 - Special relativity and the Poincaré group
 - General relativity and general covariance
- Internal symmetries
 - Isospin $SU(2)$ symmetry: $m_{\text{neutron}} = 1.00135m_{\text{proton}}$ [Heisenberg]
 - Flavor symmetry $SU(3)$ and the strong interactions
 - Standard model and $SU(3)_c \times SU(2)_w \times U(1)_Y$
- The two fundamental theories of modern physics, General Relativity and the Standard Model of Particle Physics, are based on and largely determined by symmetry principles!

Symmetry and Unification



Like a ferromagnet: symmetry is broken more and more with decreasing temperature as universe expands.

But where do we go from here?

Idea: **symmetry enhancement** as a guiding principle!

- Grand Unification $\rightarrow$ quark lepton unification,...

$$SU(3)_c \times SU(2)_w \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6 \subset \dots?$$

- Higher dimensions: gauge group from isometries?
- Supersymmetry: relates Bosons $\leftrightarrow$ Fermions, or:
Forces (vector bosons) $\leftrightarrow$ Matter (quarks & leptons)?
- Duality symmetries [Dirac, 1931]

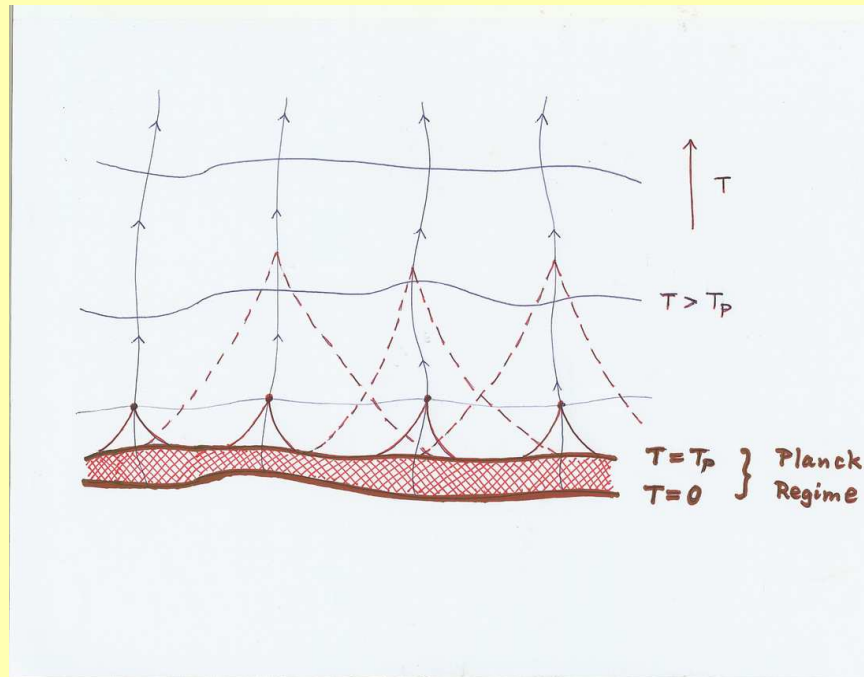
$$\mathbf{E} + i\mathbf{B} \rightarrow e^{i\alpha}(\mathbf{E} + i\mathbf{B}) \quad , \quad q + ig \rightarrow e^{i\alpha}(q + ig)$$

- ‘Fusion’ of space-time and internal symmetries?
- Quantum symmetry and quantum space-time?
- Expectation: **new symmetry concepts needed at ℓ_{Pl} !**

Quantum gravity and hidden symmetries

- Basic question: what is the configuration space (or ‘moduli space’) $\mathcal{M}$ of quantum gravity, and what are the symmetries acting on it?
- Hint from supergravity: *symmetry increases* with decreasing number of dimensions. In particular:
- Maximal supergravity: $E_{n(n)}$ in $11 - n$ dimensions.
[Cremmer, Julia, 1979]
- An unexpected link with the exceptional Lie groups of **Killing and Cartan**: G_2, F_4, E_6, E_7, E_8 !
- $E_{9(9)} \equiv E_8^{(1)}$: a **solution generating symmetry** acting on $\mathcal{M} = E_{9(9)}/K(E_9)$ = moduli space of colliding plane wave solutions of maximal supergravity.
- ... suggests $E_{10(10)}$ **for** $D = 1$: no space, only time?!?

BKL and Spacelike Singularities (I)



Hypothesis: for $T \rightarrow 0$ spatial points decouple and the system is effectively described by a continuous *superposition of one-dimensional systems* $\rightarrow$ **effective dimensional reduction to $D = 1$!** [Belinski,Khalatnikov,Lifshitz (1972)]

BKL and Spacelike Singularities (II)

Near cosmological singularity parametrize metric as

$$ds^2 = -N^2 dt^2 + g_{mn} dx^m dx^n, \quad g_{mn} = e_m^a e_{na}$$

Iwasawa decomposition of spatial zehnbain $e_m^a \equiv e_m^a(t, \mathbf{x})$

$$e_m^a = e^{-\beta^a} \theta_m^a, \quad \det \theta_m^a = 1$$

From classical BKL analysis we know that:

[Belinski,Khalatnikov,Lifshitz (1972); Misner (1969); Chitre (1972); DHN (2003)]

- Dynamics near singularity is dominated by logarithmic scale factors $\beta^a \rightarrow \infty$ and effective potential which results from ‘integrating out’ non-diagonal metric and matter degrees of freedom.
- $\Rightarrow$ off-diagonal metric components θ_m^a and matter degrees of freedom ‘freeze’ as $T \rightarrow 0$.

Walls and Roots

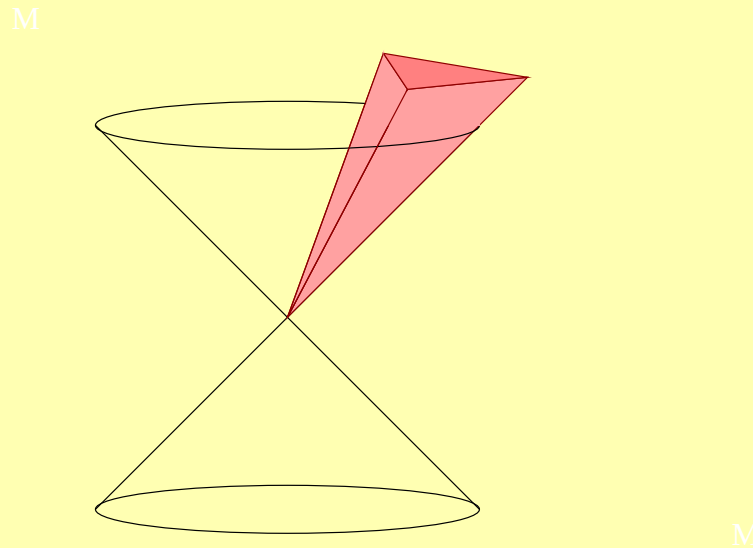
- ‘Integrating out’ non-diagonal degrees of freedom leads to an effective description in terms of *cosmological billiards* in β -space of scale factors:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}n^{-1}G_{ab}\dot{\beta}^a\dot{\beta}^b + V_{\text{eff}}(\beta)$$

with *Lorentzian* DeWitt metric G_{ab} .

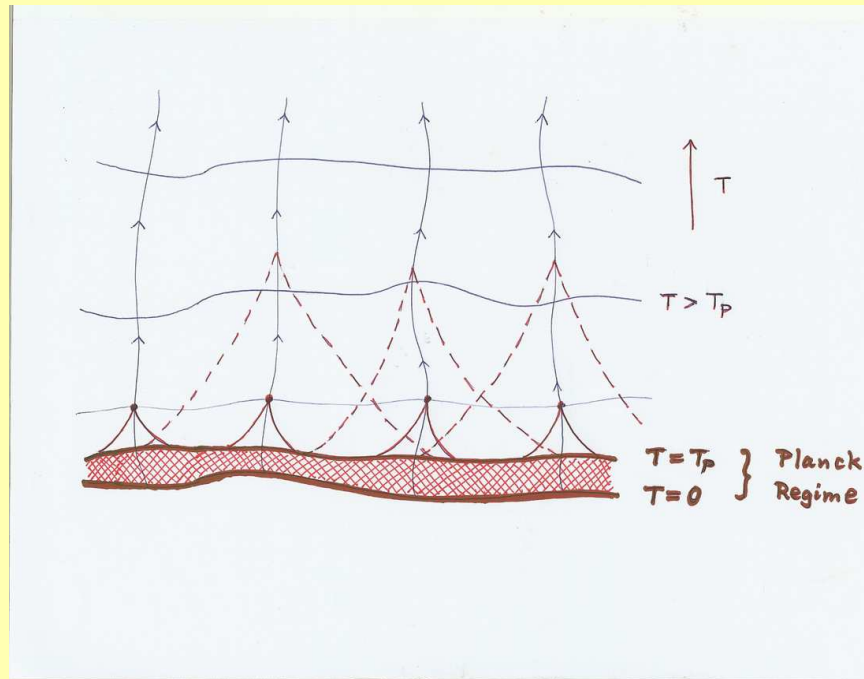
- The *Lie algebra connection* : identify space of logarithmic scale factors $\{\beta^a\}$ with the *Cartan subalgebra* of some indefinite *Kac Moody algebra*.
- Effective potential $V_{\text{eff}}(\beta)$: sharp walls from *spacelike normal vectors* = *simple roots* of this algebra.
- For maximal supergravity this Kac Moody algebra is the *maximally extended hyperbolic algebra* E_{10} .
- Half-maximal theories: $DE_{10} \subset E_{10}$ and BE_{10} .

Cosmobilliards in ' β -spacetime'



The 'Kasner billiard ball' moves in the '**billiard chamber**' on lightlike straight lines ('free Kasner flights'), bouncing off the walls of the chamber ('Kasner bounces'). *Chaotic oscillations* of metric if chamber is contained within forward lightcone, otherwise 'AVD' behavior.

E_{10} : The Basic Picture

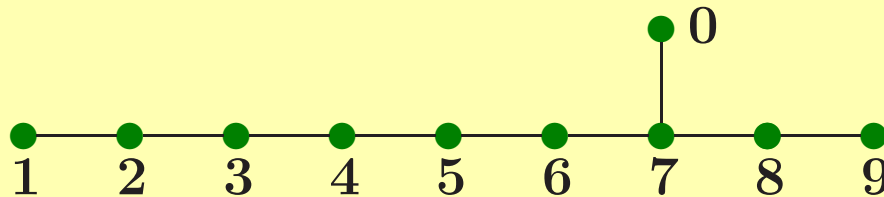


Conjecture: for $0 < T < T_P$ space-time ‘de-emerges’, and space-time based (quantum) field theory is replaced by (quantized) $E_{10}/K(E_{10})$ σ -**model** [Cf. DN, 0705.2643]

What is E_{10} ?

(No one knows, really....)

E_{10} is the ‘group’ associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram [e.g. Kac]

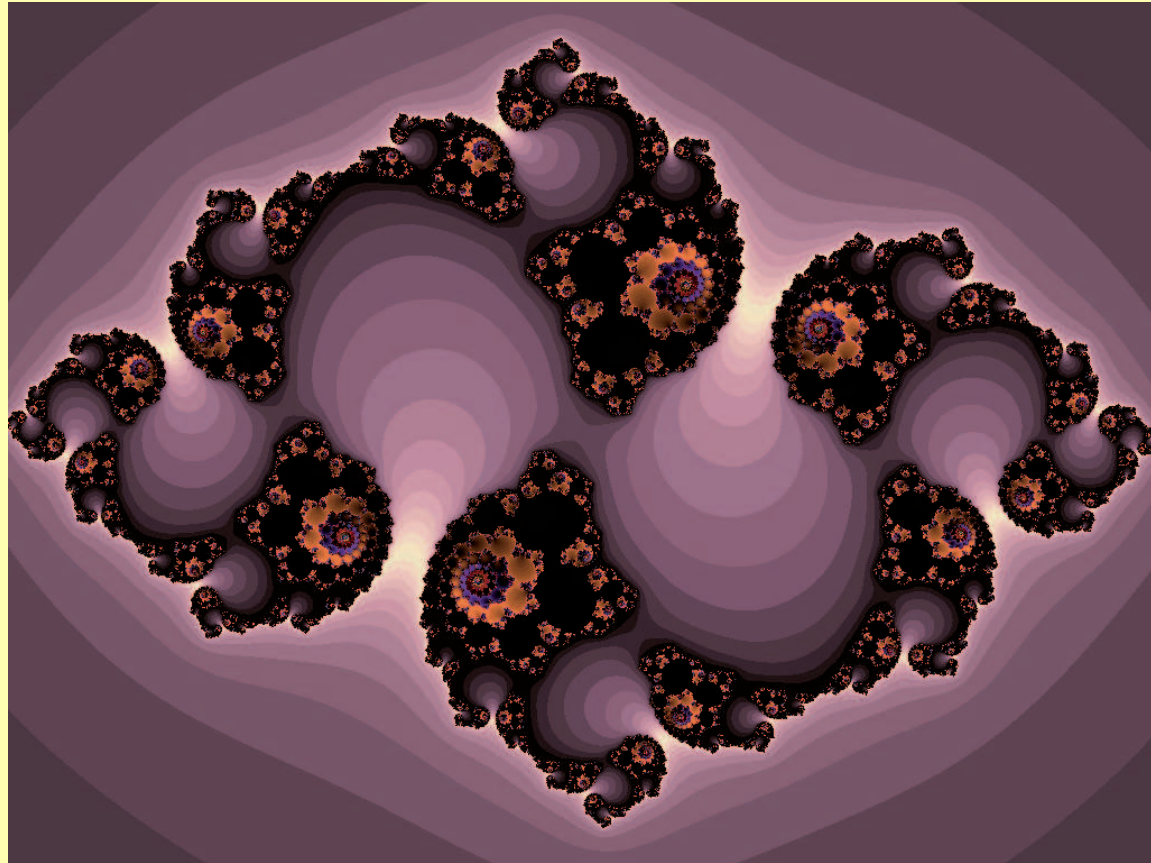


Defined by **generators** $\{e_i, f_i, h_i\}$ and **relations** via **Cartan matrix** A_{ij} (‘Chevalley-Serre presentation’)

$$\begin{aligned} [h_i, h_j] &= 0, & [e_i, f_j] &= \delta_{ij} h_i, \\ [h_i, e_j] &= A_{ij} e_j, & [h_i, f_j] &= -A_{ij} f_j, \\ (\text{ad } e_i)^{1-A_{ij}} e_j &= 0 & (\text{ad } f_i)^{1-A_{ij}} f_j &= 0. \end{aligned}$$

$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations $\rightarrow$ infinite dimensional as A_{ij} is *indefinite* $\rightarrow$ Lie algebra of *exponential growth* !

Infinite Complexity from simple recursion



A Mandelbrot set generated from $z_{n+1} = f_c(z_n)$.

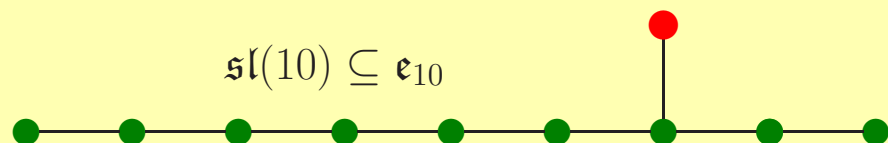
Why E_{10} is very special

- E_{10} occupies a uniquely distinguished place among all infinite-dimensional Lie algebras (much like E_8 among the finite-dimensional Lie algebras)
- In BKL approximation, classical dynamics of SUGRA_{11} near the initial singularity is well approximated by cosmological billiards in Weyl chamber of E_{10}
- E_{10} may provide *Lie-algebraic mechanism* for the ‘de-emergence’ of space and (upon quantization) time near the singularity (that is, for $0 < T < T_P$)
- E_{10} ‘knows all’ about maximal supersymmetry:
 - Different ‘ slicings ’ of the E_{10} algebra yield correct supermultiplets for maximal supergravities (SUGRA_{11} , mIIA, IIB, ...)
 - $E_{10}/K(E_{10})$ σ -model dynamics at low levels matches with respective equations of motion when truncated to first order spatial gradients

Outlook

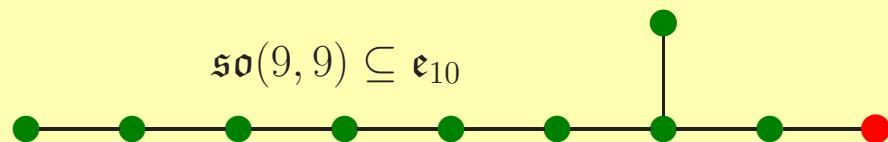
- Symmetry by no means exhausted as a *guiding principle of physics* but many open questions remain.
- Analysis of cosmological singularities *à la* BKL for maximal supergravity reveals key role of E_{10} .
- E_{10} is a *uniquely distinguished* Lie algebra, but to find a manageable representation for it remains an outstanding mathematical challenge (after 40 years).
- E_{10} ‘knows all’ about maximal supersymmetry and unifies many known $(S, T, U, \dots)$ dualities.
- Exponentially increasing complexity of E_{10} algebra $\rightarrow$ an element of *non-computability* for $T \rightarrow 0$?

E_{10} Versatility



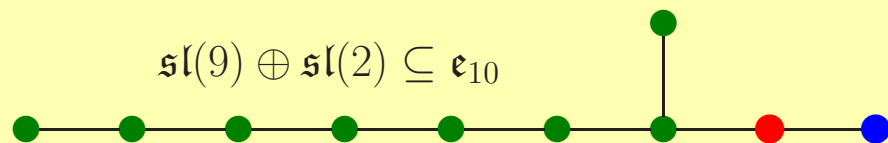
$$\mathfrak{sl}(10) \subseteq \mathfrak{e}_{10}$$

$D = 11$ SUGRA



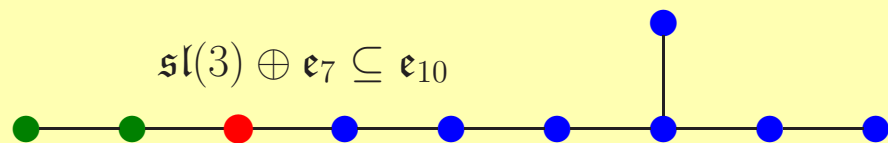
$$\mathfrak{so}(9, 9) \subseteq \mathfrak{e}_{10}$$

mIIA $D = 10$ SUGRA



$$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq \mathfrak{e}_{10}$$

IIB $D = 10$ SUGRA



$$\mathfrak{sl}(3) \oplus \mathfrak{e}_7 \subseteq \mathfrak{e}_{10}$$

$\mathcal{N} = 8, D = 4$ SUGRA