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Tortoise Coordinate Transformation on Apparent Horizon of a Dynamical Black Hole

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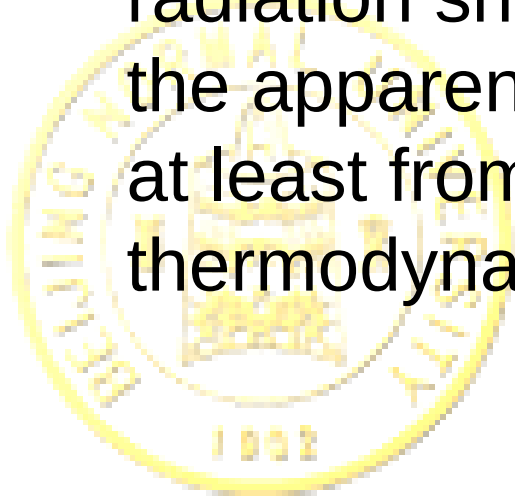
Abstract

Thinking of Hawking radiation calculation from a Schwarzschild black hole using Damour-Ruffini method, some key requirements of the tortoise coordinate transformation are pointed out. Extending these requirements to a dynamical black hole, a dynamical tortoise coordinate transformation is proposed.



Abstract

Under this new dynamical tortoise coordinate transformation, Hawking radiation from a Vaidya black hole can be got successfully using Damour-Ruffini method. Moreover, we also find that the radiation should be regarded as originating from the apparent horizon rather than the event horizon at least from the viewpoint of the first law of thermodynamics.



Spherically Symmetric Schwarzschild Vacuum Solution of Einstein Equation

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$r = \frac{2GM}{c^2}$$



The apparent horizon and event horizon
are coincident with each other.

Black Hole Physics

(1) κ is a constant over the horizon of a stationary black hole

$$(2) dM = \frac{\kappa}{8\pi} dA + \Omega_H dJ$$

(3) $\delta A \geq 0$ in any process

(4) Impossible to achieve $\kappa = 0$ by a physical process



Hawking Radiation

Under quantum theory, Hawking got the black body spectrum in 1974 as

$$N_{\omega}^2 = \frac{1}{e^{8\pi M\omega} \pm 1} = \frac{1}{e^{\omega/k_B T} \pm 1}$$

Two problems:

(1) Information loss paradox!

(2) Are there any really static or stationary black holes?



Several Intrinsic Contradictions for Black Hole Thermodynamics

- Black hole definition under classical general relativity and that under quantum theory with Hawking radiation
- Purely thermal Hawking radiation with information loss paradox and the unitary condition under quantum theory
- The black hole thermodynamics is deduced from a stationary case, but it is impossible that a black hole is stationary with Hawking radiation



Classical general relativity
and quantum theory
A exemplification of the
difficulty of quantum gravity

The Correction for Hawking Purely Thermal Spectrum

- Parikh-Wilczek method
- Hamilton-Jacobi method
- Damour-Ruffini method



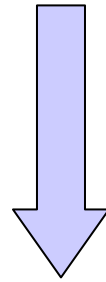
Back-Reaction of
Hawking Radiation

Parikh-Wilczek Method

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$t = \tau + f(r)$$

$$dt = d\tau + f'(r)dr$$



$$\left(1 - \frac{2M}{r}\right)^{-1} - \left(1 - \frac{2M}{r}\right)(f'(r))^2 = 1$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right)d\tau^2 + 2\sqrt{\frac{2M}{r}}d\tau dr + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Under Painlevé coordinates, the radial null geodesic equation is

$$\frac{dr}{d\tau} = \pm 1 - \sqrt{\frac{2M}{r}}$$

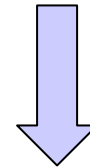


Parikh-Wilczek Method

$$\Gamma \sim \exp(-2 \operatorname{Im} I) \approx \exp(-\beta E)$$

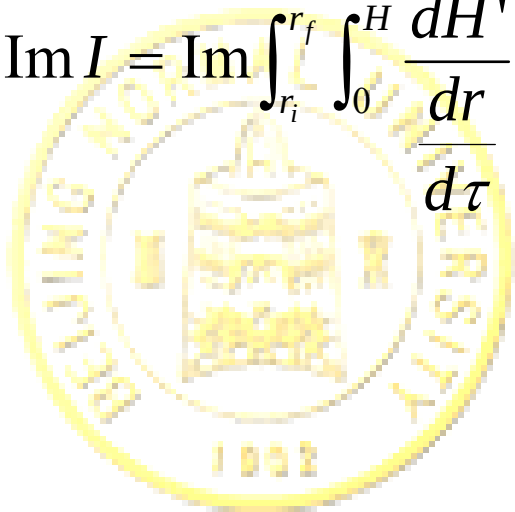
$$\operatorname{Im} I = \operatorname{Im} \int_{r_i}^{r_f} p_r dr = \operatorname{Im} \int_{r_i}^{r_f} \int_0^{p_r} dp'_r dr$$

$$\left. \frac{dH}{dp} \right|_r = \frac{\partial H}{\partial p} = \frac{dr}{d\tau}$$



$$\operatorname{Im} I = \operatorname{Im} \int_{r_i}^{r_f} \int_0^H \frac{dH'}{dr} dr = - \operatorname{Im} \int_{r_i}^{r_f} \int_0^E \frac{dr dE'}{1 - \sqrt{\frac{2(M - E')}{r}}} = 4\pi \int_0^E dE' (M - E')$$

$$\Gamma \sim \exp\left(-8\pi M E \left(1 - \frac{E}{2M}\right)\right) = \exp(\Delta S)$$



Hamilton-Jacobi Method

$$g^{\mu\nu} \partial_\mu I \partial_\nu I + m^2 = 0$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad \downarrow$$

$$-\frac{1}{1 - \frac{2M}{r}} (\partial_t I)^2 + \left(1 - \frac{2M}{r}\right) (\partial_r I)^2 + \frac{1}{r^2} ((\partial_\theta I)^2 + \frac{1}{\sin^2 \theta} (\partial_\varphi I)^2) + m^2 = 0$$

$$I = -Et + W(r) + J(\theta, \varphi)$$

$$\partial_t I = -E$$

$$\partial_r I = W'(r)$$

$$\partial_i I = J_i$$

$$I = -Et + \int \frac{dr}{1 - \frac{2M}{r}} \sqrt{E^2 - \left(1 - \frac{2M}{r}\right) \left(m^2 + \frac{1}{r^2} J_\theta^2 + \frac{1}{r^2 \sin^2 \theta} J_\varphi^2\right)} + J(\theta, \varphi)$$

Hamilton-Jacobi Method

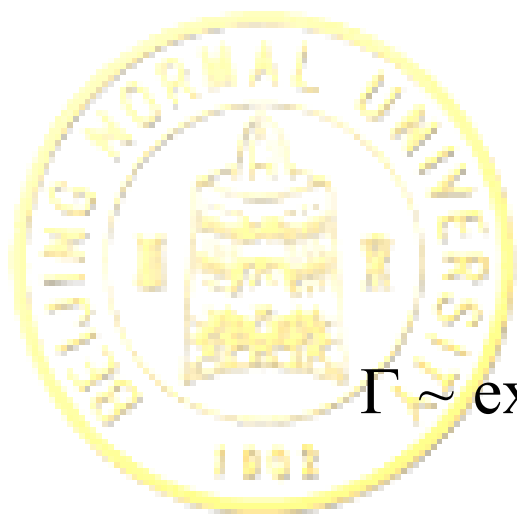
$$d\sigma^2 = \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$W(\sigma) = \int \frac{d\sigma}{\sqrt{1 - \frac{2M}{r(\sigma)}}} \sqrt{E^2 - \left(1 - \frac{2M}{r(\sigma)}\right) m^2}$$

$$\text{Im } I = \text{Im } W = 2\pi r_H E$$

Thinking of the back-reaction, we have

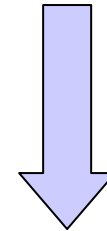
$$\Gamma \sim \exp(-2 \text{Im } I) = \exp\left(-8\pi M E \left(1 - \frac{E}{2M}\right)\right) = \exp(\Delta S)$$



Damour-Ruffini Method

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

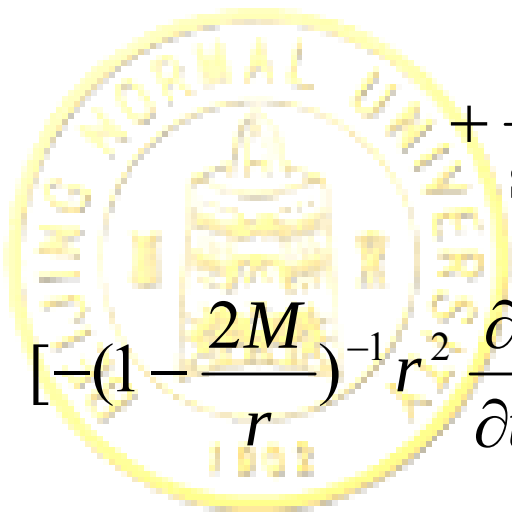
$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu}) - \mu^2 \Phi = 0$$



$$\left[-\left(1 - \frac{2M}{r}\right)^{-1} r^2 \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial r} (r^2 - 2Mr) \frac{\partial}{\partial r} \right.$$

$$\left. + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial}{\partial \theta}) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} - r^2 \mu^2 \right] \Phi = 0.$$

$$\left[-\left(1 - \frac{2M}{r}\right)^{-1} r^2 \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial r} (r^2 - 2Mr) \frac{\partial}{\partial r} - r^2 \mu^2 \right] \frac{R_\omega}{r} = -l(l+1) R_\omega / r$$



Damour-Ruffini Method

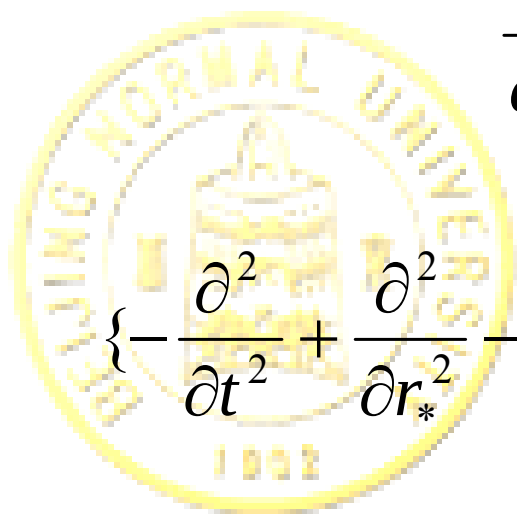
Tortoise coordinate

$$r_* = r + 2M \ln[(r - 2M)/(2M)]$$

$$dr_* = \left(1 - \frac{2M}{r}\right)^{-1} dr \quad \frac{d}{dr} = \left(1 - \frac{2M}{r}\right)^{-1} \frac{d}{dr_*}$$

$$\frac{d^2}{dr^2} = \left(1 - \frac{2M}{r}\right)^{-2} \frac{d^2}{dr_*^2} - \frac{2M}{(r - 2M)^2} \frac{d}{dr_*}$$

$$\left\{ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - \left(1 - \frac{2M}{r}\right) \left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} + \mu^2 \right] \right\} R_\omega(r, t) = 0$$



Damour-Ruffini Method

$$r \rightarrow +\infty \quad \left\{ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - \mu^2 \right\} R_\omega(r, t) = 0$$

$$r \rightarrow 2M \quad \left\{ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} \right\} R_\omega(r, t) = 0$$

Particles move near the horizon just like mass-less ones for the infinite observer.

$$R_\omega^{in} = e^{-i\omega(t+r_*)}, \quad R_\omega^{out} = e^{-i\omega(t-r_*)}$$



Damour-Ruffini Method

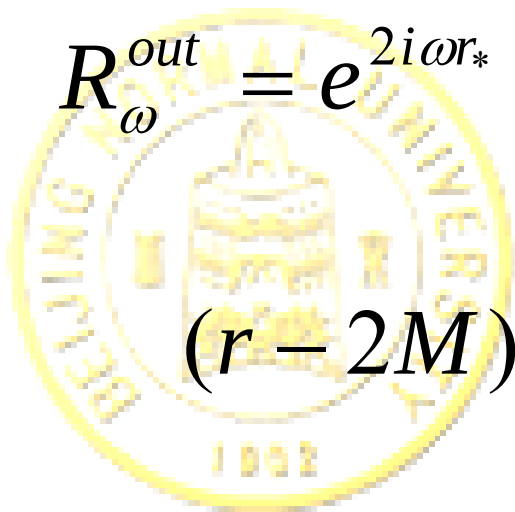
Advanced Eddington-Finkelstein coordinate

$$v = t + r_*$$

$$R_{\omega}^{in} = e^{-i\omega v}, \quad R_{\omega}^{out} = e^{2i\omega r_*} \cdot e^{-i\omega v}$$

$$R_{\omega}^{out} = e^{2i\omega r_*} \cdot e^{-i\omega v} = e^{2i\omega r} \cdot e^{-i\omega v} \cdot \left(\frac{r - 2M}{2M}\right)^{i4M\omega}$$

$$(r - 2M) \rightarrow |r - 2M|e^{-i\pi} = (2M - r)e^{-i\pi}$$



Damour-Ruffini Method

$$\begin{aligned}
 R_{\omega}^{\text{out}} &= e^{2i\omega r} \cdot e^{-i\omega v} \cdot \left(\frac{2M-r}{2M}\right)^{i4M\omega} \cdot e^{4\pi M\omega} \\
 &= e^{-i\omega v} \cdot e^{4\pi M\omega} \cdot e^{i2\omega\{r+2M \ln[(2M-r)/(2M)]\}} = e^{4\pi M\omega} \cdot e^{2i\omega r_*} \cdot e^{-i\omega v}.
 \end{aligned}$$

$$\Gamma = \left| \frac{R_{\omega}^{\text{out}}(r > 2M)}{R_{\omega}^{\text{out}}(r < 2M)} \right|^2 = e^{-8\pi M\omega},$$



$$\Gamma = \prod_i \Gamma_i = e^{-8\pi \int_0^{\omega} (M - \omega') d\omega'} = e^{-8\pi \omega (M - \omega/2)} = e^{\Delta S}$$

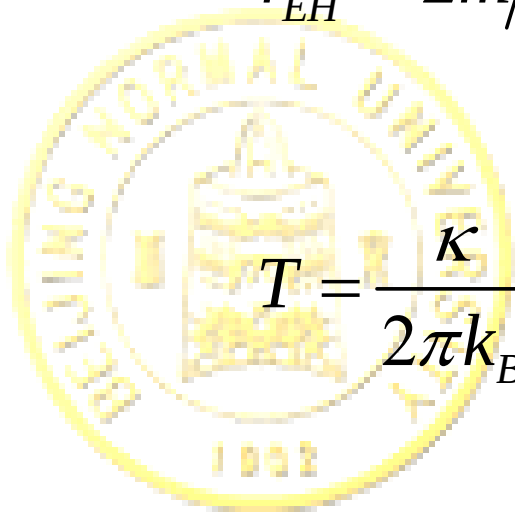
A Dynamical Vaidya Black Hole

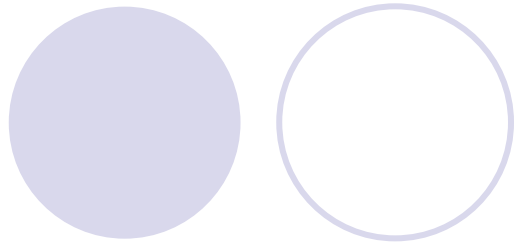
$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$r_{EH} = 2m/(1 - 2\dot{r}_{EH}) \quad r_{AH} = 2m$$

The apparent horizon and event horizon are not coincident with each other any more.

$$T = \frac{\kappa}{2\pi k_B} \quad N_\omega^2 = 1 / \left(e^{\frac{\omega}{k_B T}} \pm 1 \right)$$





event
horizon

r_{EH}



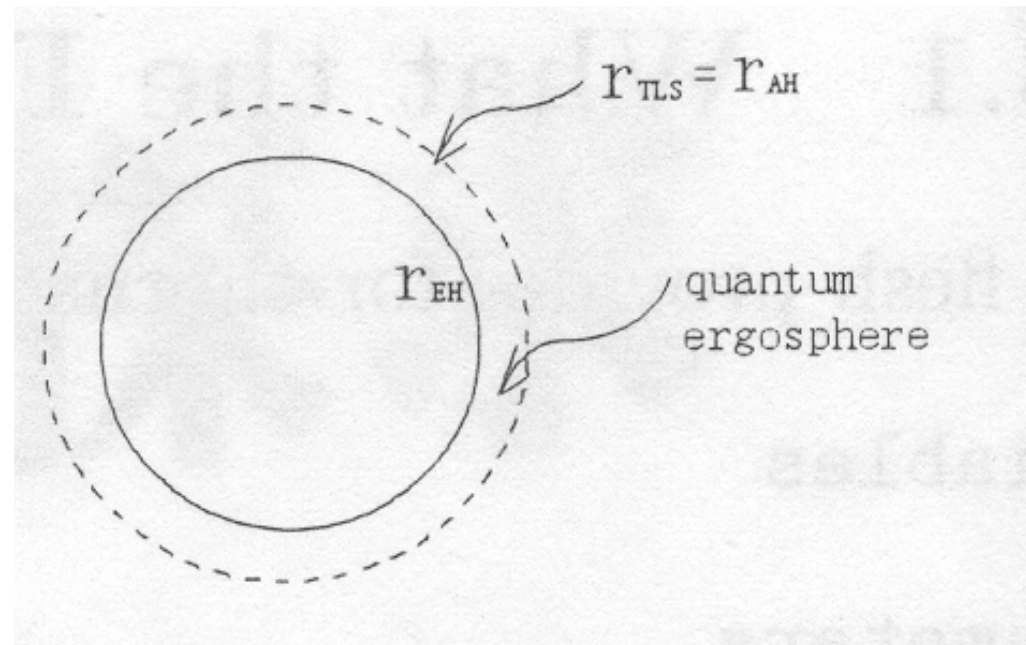
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r_{AH}

apparent
horizon

r_{TLS}

time-like limit
surface



$$r_{AH} = r_{TLS} \neq r_{EH}$$



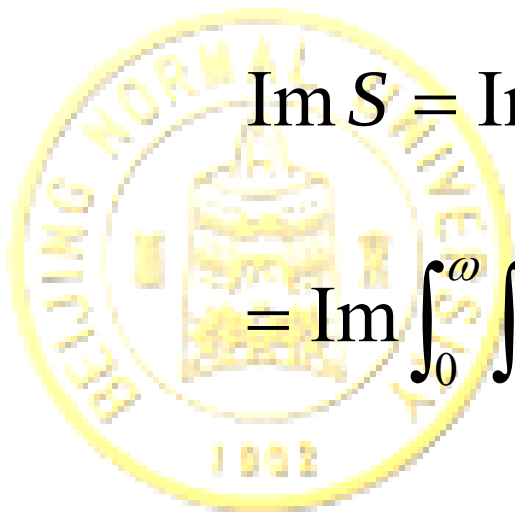
Calculating Hawking Radiation via Null Geodesics

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\dot{r} \equiv \frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{2m(v)}{r}\right)$$

$$\text{Im } S = \text{Im} \int p_r dr = \text{Im} \int \int dp_r dr = \text{Im} \iint \frac{dH}{\dot{r}} dr$$

$$= \text{Im} \int_0^\omega \int \frac{2dr}{1 - \frac{2m(v)}{r}} (-d\omega') = 4\pi m(v)\omega$$



Calculating Hawking Radiation via Null Geodesics

$$\Gamma \sim e^{-2 \operatorname{Im} S} = e^{-8\pi m(v)\omega}$$

$$T = \frac{\kappa}{2\pi} = \frac{1}{8\pi m(v)}$$

$$S = \pi r_{AH}^2 = 4\pi m^2(v)$$

$$dm(v) = TdS$$



Calculating Hawking Radiation via Null Geodesics

$$R = r - \frac{1}{2} \delta \cdot v$$

$$ds^2 = -\left(1 - \frac{2m(v)}{r} - \delta\right) dv^2 + 2dv dR + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$



$$\dot{R} \equiv \frac{dR}{dv} = \frac{1}{2} \left(1 - \frac{2m(v)}{r} - \delta\right)$$

Calculating Hawking Radiation via Null Geodesics

$$\begin{aligned}\text{Im } S &= \text{Im} \int p_R dR = \text{Im} \int \int dp_R dR = \text{Im} \iint \frac{dH}{\dot{R}} dR \\ &= \text{Im} \int_0^\omega \int \frac{2dR}{1 - \frac{2m(v)}{r} - \delta} (-d\omega') = \frac{4\pi m(v)}{(1-\delta)^2} \omega\end{aligned}$$



$$\Gamma \sim e^{-2\text{Im } S} = e^{-\frac{8\pi m(v)}{(1-\delta)^2} \omega}$$

Calculating Hawking Radiation via Null Geodesics

$$r' = 2m(v)(1 + \delta)$$

$$T' = \frac{\kappa'}{2\pi} = \frac{1}{8\pi m(v)(1 + \delta)^2}$$

$$S' = \pi r'^2 = 4\pi m^2(v)(1 + \delta)^2$$

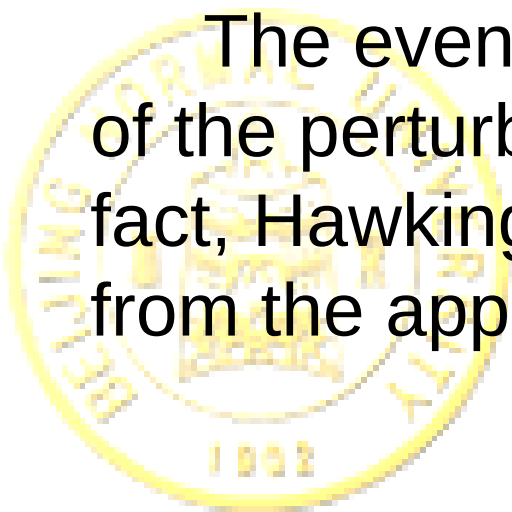
$$dm(v) = TdS$$



Calculating Hawking Radiation via Null Geodesics

If $\delta = \dot{r}_{EH}$, the new super-surface under relativistic perturbation $r' = 2m(v)(1 + \delta)$ is exactly the event horizon of a Vaidya black hole in the equations' form.

The event horizon thermodynamics is just one of the perturbations near the apparent horizon. In fact, Hawking radiation can be regarded as coming from the apparent horizon.



Black Hole and Black Body

Black Hole

$$U_1 = M$$

$$T_1 = \frac{1}{8\pi M}$$

$$S_1 = 4\pi M^2$$

$$dU_1 = T_1 dS_1$$

Black Body

$$T_2$$

$$U_2 = u_2 \cdot V = aT_2^4 \cdot V$$

$$S_2 = s_2 \cdot V = \frac{4}{3}aT_2^3 \cdot V$$

$$dU_2 = T_2 dS_2$$



Black Hole and Black Body

$$T_1 = T_2$$

$$U_1 = U_2 \quad \longrightarrow \quad M = aT_2^4 \cdot V$$

$$S_1 = S_2 \quad \longrightarrow \quad 4\pi M^2 = \frac{4}{3} a T_2^3 \cdot V$$

$$4\pi M = \frac{4}{3} \frac{1}{T_2}$$

incorrect,
inconsistent



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Black Hole and Black Body

$$T_1 = T_2$$

$$dU_1 = dU_2 \quad \longrightarrow \quad dM = 4aT_2^3 \cdot VdT_2$$

$$dS_1 = dS_2 \quad \longrightarrow \quad 8\pi M dM = 4aT_2^2 \cdot VdT_2$$

$$8\pi M = \frac{1}{T_2}$$

correct,
consistent!



Black Hole and Black Body

(1) Stationary black holes are not real thermodynamic system, they can not be treated as a usual thermodynamic system directly.

(2) Changing black holes (their quantities are changing) are real thermodynamic system, they can be treated as an ordinary thermodynamic system directly.



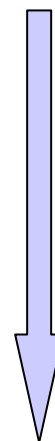
The real black hole
thermodynamics should be
dynamical! ! !

Hawking radiation from a stationary black hole

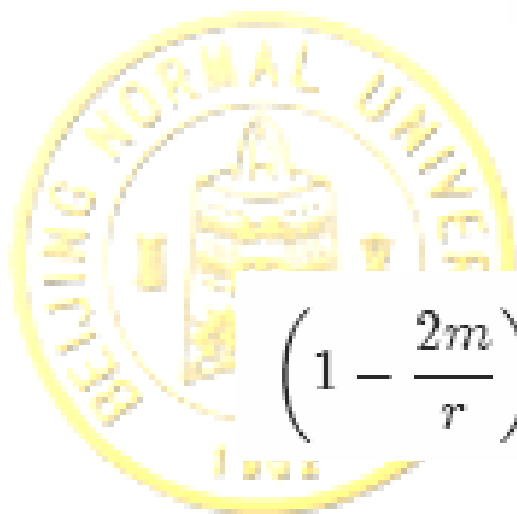
$$ds^2 = - \left(1 - \frac{2m}{r} \right) dv^2 + 2dvdr + r^2 d\Omega^2$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu}) \Phi = 0.$$

$$\Phi = \frac{1}{r} \rho(r, v) Y_{lm}(\theta, \varphi)$$



$$\left(1 - \frac{2m}{r} \right) \frac{\partial^2 \rho}{\partial r^2} + 2 \frac{\partial^2 \rho}{\partial r \partial v} + \frac{2m}{r^2} \frac{\partial \rho}{\partial r} - \left[\frac{2m}{r^3} + \frac{l(l+1)}{r^2} \right] \rho = 0.$$

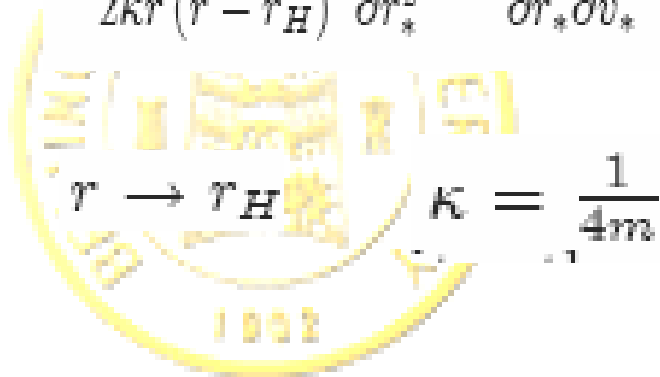


Hawking radiation from a stationary black hole

tortoise coordinate transformation

$$\begin{cases} r_* = \frac{1}{2\kappa} \ln \left(\frac{r-r_H}{r_H} \right), \\ v_* = v, \end{cases}$$

$$\left[\frac{r-2m}{2\kappa r(r-r_H)} \right] \frac{\partial^2 \rho}{\partial r_*^2} + 2 \frac{\partial^2 \rho}{\partial r_* \partial v_*} + \left[\frac{2m}{r^2} - \frac{r-2m}{r(r-r_H)} \right] \frac{\partial \rho}{\partial r_*} - 2\kappa(r-r_H) \left[\frac{2m}{r^3} + \frac{l(l+1)}{r^2} \right] \rho = 0.$$



$$r_H = 2m.$$

$$\frac{\partial^2 \rho}{\partial r_*^2} + 2 \frac{\partial^2 \rho}{\partial r_* \partial v_*} = 0$$

Hawking radiation from a stationary black hole

$$\rho_{\omega}^{out} = e^{-i\omega(v_* - 2r_*)}, \quad \rho_{\omega}^{in} = e^{-i\omega v_*}.$$

$$\rho_{\omega}^{out} = e^{-i\omega v} \left(\frac{r - r_H}{r_H} \right)^{i4m\omega}, \quad \rho_{\omega}^{in} = e^{-i\omega v}.$$

Extend the outgoing wave function into the horizon by turning $(-\pi)$ angle through the negative half complex plane

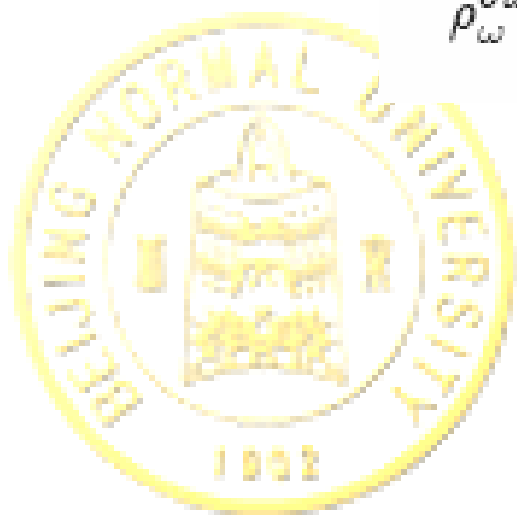
$$r - 2m \rightarrow |r - 2m|e^{-i\pi} = (2m - r)e^{-i\pi}$$



Hawking radiation from a stationary black hole

$$\rho_{\omega}^{\text{out}}(r < 2m) = e^{-i\omega v} e^{4m\pi\omega} \left(\frac{2m - r}{2m} \right)^{i4m\omega}$$

$$\rho_{\omega}^{\text{out}}(r > 2m) = e^{-i\omega v} \left(\frac{r - 2m}{2m} \right)^{i4m\omega}$$



$$\Gamma = \frac{|\rho_{\omega}^{\text{out}}(r > 2m)|^2}{|\rho_{\omega}^{\text{out}}(r < 2m)|^2} = e^{-8m\pi\omega}$$

Some properties of the tortoise coordinate near the event horizon

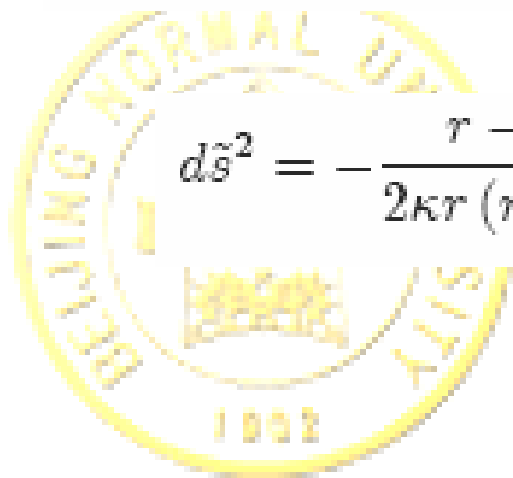
$$\left(\frac{\partial}{\partial v_*}\right)^a = \left(\frac{\partial}{\partial v}\right)^a, \quad \left(\frac{\partial}{\partial r_*}\right)^a = 2\kappa(r - r_H)\left(\frac{\partial}{\partial r}\right)^a.$$

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dv_*^2 + 2\kappa(r - r_H)2dv_*dr_* = 2\kappa(r - r_H)\left\{-\frac{r - 2m}{2\kappa r(r - r_H)}dv_*^2 + 2dv_*dr_*\right\}.$$

$$d\tilde{s}^2 = -\frac{r - 2m}{2\kappa r(r - r_H)}dv_*^2 + 2dv_*dr_*,$$

$$ds^2 = \Omega^2 d\tilde{s}^2.$$

$$d\tilde{s}^2|_{r \rightarrow r_H} = -dv_*^2 + 2dv_*dr_*.$$



Some properties of the tortoise coordinate near the event horizon

It is obvious that the space time described by the new line element is **an asymptotic flat space time near the horizon**, which means that Schwarzschild space time is homeomorphic to an asymptotic flat space time near the horizon. Under the condition of near-horizon approximation, the tortoise coordinate transformation has the same quality as the conformal transformation has. The quality of conformal transformation is also a special requirement for the tortoise transformation and it guarantees that the Klein-Gordon equation can be simplified to the standard wave equation near the event horizon.

Dynamical tortoise coordinate transformation

$$\left\{ \begin{array}{l} \left(\frac{\partial}{\partial v_*} \right)^a_{r \rightarrow r_H(v)} = \left(\frac{\partial}{\partial v} \right)^a_{r \rightarrow r_H(v)}, \\ \left(\frac{\partial}{\partial r_*} \right)^a = 2\kappa(v)(r - r_H(v)) \left(\frac{\partial}{\partial r} \right)^a, \end{array} \right.$$

$$\left\{ \begin{array}{l} dr_* = \frac{1}{2\kappa(v)(r - r_H(v))} dr + Q(r, v) dv, \\ dv_* = dv, \end{array} \right.$$

$$\lim_{r \rightarrow r_H(v)} Q(r, v) = 0$$



Conformal quality of the dynamical tortoise coordinate transformation

$$ds^2 = - \left(1 - \frac{2m(v)}{r} \right) dv^2 + 2dvdr.$$

$$\begin{aligned} ds^2 &= - \left(1 - \frac{2m(v)}{r} \right) dv_*^2 + 2\kappa(v)(r - r_H(v)) [2dv_*dr_* - 2Q(r, v)dv_*^2] \\ &= 2\kappa(v)(r - r_H(v)) \left\{ - \left[\frac{r - 2m(v)}{2\kappa(v)r(r - r_H(v))} + 2Q(r, v) \right] dv_*^2 + 2dv_*dr_* \right\} \end{aligned}$$

$$d\tilde{s}^2 = - \left[\frac{r - 2m(v)}{2\kappa(v)r(r - r_H(v))} + 2Q(r, v) \right] dv_*^2 + 2dv_*dr_*.$$



Conformal quality of the dynamical tortoise coordinate transformation

$$ds^2 = \dot{\Omega}^2 d\tilde{s}^2$$

$$\dot{\Omega}^2 = 2\kappa(v)(r - r_H(v)) = e^{2\kappa(v)(r_* - \frac{1}{2}v_*)}$$



$$d\tilde{s}^2|_{r \rightarrow r_H(v)} = -dv_*^2 + 2dv_* dr_*$$

Hawking radiation from a Vaidya black hole

$$\left(1 - \frac{2m(v)}{r}\right) \frac{\partial^2 \rho}{\partial r^2} + 2 \frac{\partial^2 \rho}{\partial r \partial v} + \frac{2m(v)}{r^2} \frac{\partial \rho}{\partial r} - \left[\frac{2m(v)}{r^3} + \frac{l(l+1)}{r^2} \right] \rho = 0$$

$$\frac{\partial}{\partial r} = \frac{1}{2\kappa(v)(r - r_H(v))} \frac{\partial}{\partial r_*}, \quad \frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} + Q(r, v) \frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial r^2} = \left[\frac{1}{2\kappa(v)(r - r_H(v))} \right]^2 \frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(v)(r - r_H(v))^2} \frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial r \partial v} = \frac{1}{2\kappa(v)(r - r_H(v))} \left[\frac{\partial^2}{\partial r_* \partial v_*} + Q(r, v) \frac{\partial^2}{\partial r_*^2} \right] + \left[\frac{\dot{r}_H(v)}{2\kappa(v)(r - r_H(v))^2} - \frac{\dot{\kappa}(v)}{2\kappa^2(v)(r - r_H(v))} \right] \frac{\partial}{\partial r_*}.$$

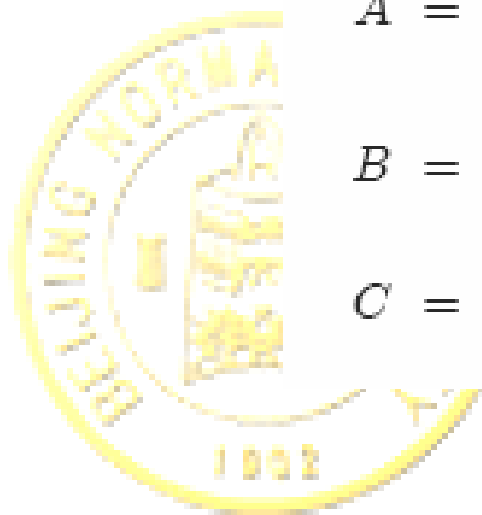
Hawking radiation from a Vaidya black hole

$$A \frac{\partial^2 \rho}{\partial r_*^2} + 2 \frac{\partial^2 \rho}{\partial r_* \partial v_*} + B \frac{\partial \rho}{\partial r_*} - C \rho = 0$$

$$A = \frac{r - 2m(v)}{2\kappa(v)r(r - r_H(v))} + 2Q(r, v),$$

$$B = \frac{2\dot{r}_H(v)}{r - r_H(v)} - \frac{2\dot{\kappa}(v)}{\kappa(v)} + \frac{2m(v)}{r^2} - \frac{1}{r - r_H(v)} \frac{r - 2m(v)}{r},$$

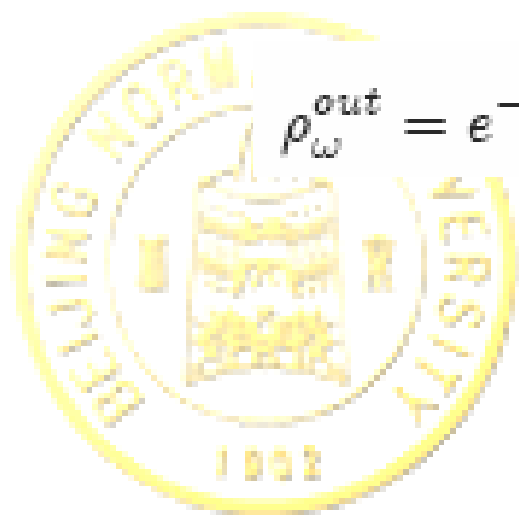
$$C = 2\kappa(v)(r - r_H(v)) \left[\frac{2m(v)}{r^3} + \frac{l(l+1)}{r^2} \right].$$



Hawking radiation from a Vaidya black hole

Near the apparent horizon the radial equation can be written as

$$\frac{\partial^2 \rho}{\partial r_*^2} + 2 \frac{\partial^2 \rho}{\partial r_* \partial v_*} + 2\kappa(v_*) \frac{\partial \rho}{\partial r_*} = 0.$$



$$\rho_{\omega}^{\text{out}} = e^{-i\omega(v_* - 2r_*) + L(v_*)}, \quad \rho_{\omega}^{\text{in}} = e^{-i\omega v_* + L(v_*)}.$$

$$L(v_*) = - \int_0^{v_*} \kappa(v_*) dv_*$$

Hawking radiation from a Vaidya black hole

The radiating particles are assumed to tunnel through the horizon in a null radial geodesic

$$\left. \frac{dr}{dv} \right|_{r=r_H(v)} = \frac{r_H(v) - 2m(v)}{2r_H(v)}$$

$$r_* = \frac{1}{2\kappa(v)} \ln \left(\frac{r - r_H(v)}{r_H(v)} \right) + \frac{1}{2}v, \quad v_* = v.$$

Hawking radiation from a Vaidya black hole

$$\rho_{\omega}^{\text{out}}(r > 2m(v)) = e^{i \frac{\omega}{\kappa(v)} \ln \left(\frac{r - r_H(v)}{r_H(v)} \right)} + L(v)$$

$$\rho_{\omega}^{\text{out}}(r < 2m(v)) = e^{\frac{\omega \pi}{\kappa(v)}} \cdot e^{i \frac{\omega}{\kappa(v)} \ln \left(\frac{r_H(v) - r}{r_H(v)} \right)} + L(v)$$

$$\Gamma = \frac{|\rho_{\omega}^{\text{out}}(r > 2m(v))|^2}{|\rho_{\omega}^{\text{out}}(r < 2m(v))|^2} = e^{-2\pi\omega/\kappa(v)}$$

$$T = \frac{\kappa(v)}{2\pi} = \frac{1}{8\pi m(v)}$$

References

1. **Wenbiao Liu**. New coordinates for BTZ black hole and Hawking radiation via tunnelling. Physics Letters B 634 (2006) No.5-6, 541.
2. Xiaokai He, **Wenbiao Liu**. Modified Hawking radiation in a BTZ black hole using Damour-Ruffini method. Physics Letters B 653 (2007) No.2-4, 330.
3. Shiwei Zhou, **Wenbiao Liu**. Charged Dirac particles' Hawking radiation from a Kerr-Newman black hole. Phys. Rev. D 77 (2008) No.10, 104021.
4. Shiwei Zhou, **Wenbiao Liu**. Apparent horizon and event horizon of a Vaidya black hole. Modern Physics Letters A 24 (2009) No.26, 2099.
5. Xianming Liu, **Wenbiao Liu**. Where does Hawking radiation of a dynamical black hole come from? International Journal of Theoretical Physics 49 (2010) No.5, 1088.
6. Xianming Liu, Zheng Zhao, **Wenbiao Liu**. Tortoise coordinate transformation on apparent horizon of a dynamical black hole. Classical and Quantum Gravity (to be published, in refereeing).

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Thanks

Welcome the questions!



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