

Quantum Noise in the Mirror-Field System: A Field Theoretic Approach

Da-Shin Lee

Department of Physics
National Dong Hwa University
Hualien, Taiwan

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Motivations

- ▶ Quantum noise in a laser interferometer detector arises from the quantum nature of the light directly via the photon number fluctuations (shot noise) or indirectly via random motion of the mirror under a fluctuating force (radiation pressure fluctuations).
- ▶ To minimize the uncertainty by assuming no correlation between two noises may give the SQL when an input power is appropriately chosen. (Caves 1980,1981)
- ▶ The field theoretic approach is a necessity for microscopically considering the coupling between the mirror and the fields, and also constructing the field operator to describe the photon flow that incorporates not only shot noise but also noise from the random motion of the mirror under a fluctuating environment by which their correlations can be naturally established to possibly reduce an overall noise.

References:

- ▶ W. G. Unruh, in *Quantum Optics, Experimental Gravitation, and Measurement Theory*, edited by P. Meystre and M. O. Scully (Plenum, New York, 1982), p. 647.
- ▶ C.-H. Wu and L.H. Ford, Phys. Rev. D 64, 045010 (2001).
- ▶ C. M. Caves, Rev. Lett. 45, 75 (1980); Phys. Rev. D 23, 1693 (1981).

The standard quantum limit (SQL) (a simple example)

Consider the measurement of the position of a free quantum object with mass m .

Suppose that the position has an initial uncertainty $\Delta x(0)$. Then, this gives the uncertainty of the momentum about $\Delta p(0) \geq \hbar/2\Delta x(0)$. The square of the uncertainty on the position at time t later increases to (**Braginsky and Vorontsov 1974**)

$$\begin{aligned} (\Delta x)^2(t) &= (\Delta x)^2(0) + (\Delta p)^2(0)t^2/m^2 \\ &\geq 2\Delta x(0)\Delta p(0)t/m \geq \hbar t/m \quad (" = " \text{ corresponds to SQL}). \end{aligned}$$

However there are corrections between Δx and Δp . It was argued that the square of the uncertainty in the position should be (**Yuen 1983**)

$$\begin{aligned} (\Delta x)^2(t) &= (\Delta x)^2(0) + ((\Delta p)^2(0)t^2/m^2 \\ &+ [\langle \Delta x(0)\Delta p(0) + \Delta p(0)\Delta x(0) \rangle - 2\langle x(0) \rangle \langle p(0) \rangle]t/m. \end{aligned}$$

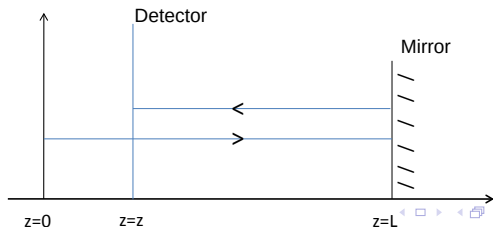
Negative correlations may result in the uncertainty even smaller than SQL.

The idea

We consider that a single mirror with perfect reflection is illuminated by a massless scalar field propagating along the z direction that gives motion of the mirror. The mirror of mass m and area A is originally placed at the $z = L$ plane. Thus, the boundary condition on the field evaluated at the mirror surface can be expressed in the specific form:

$$\phi(\mathbf{x}_{\parallel}, L + q(t), t) = 0,$$

where $q(t)$ is the displacement along the z -direction from its original position at $z = L$.



The model

The approximate solution to the field equation subject to the boundary condition if we assume slow motion ($\dot{q} \ll 1$) is that

$$\phi = \phi^+ + \phi^-,$$

where the positive (negative) energy solution ϕ^+ (ϕ^-) is respectively given by (Unruh 1982)

$$\begin{aligned} \phi^+(x, t; L + q(t)) &= \int \frac{d\mathbf{k}_{\parallel}}{(2\pi)^2} \int_0^{\infty} \frac{dk_z}{(2\pi)} \frac{1}{k} a_k e^{-ikt} \\ &\times e^{i\mathbf{k}_{\parallel} \cdot \mathbf{x}_{\parallel}} (e^{ik_z z} - e^{-ik_z(z-2L-2q(t-(L-z)))}) \end{aligned}$$

for $L^2 \gg A$.

The force acting on the mirror is given by the area integral of the $z - z$ component of the stress tensor in terms of field operators:

$$F(t) = \int_A d\mathbf{x}_{\parallel} T_{zz}(\mathbf{x}_{\parallel}, z = L, t),$$

where

$$T_{zz} = \frac{1}{2} \left[(\partial_t \phi)^2 + (\partial_z \phi)^2 - (\partial_x \phi)^2 - (\partial_y \phi)^2 \right].$$

Thus, the equation of motion for the position operator is then (Wu & Ford 2001)

$$q(t) = \frac{1}{m} \int_0^t d\tau \int_0^{\tau} dt' \int_A d\mathbf{x}'_{\parallel} T_{zz}(x') \big|_{z'=L}.$$

A proper renormalization on the position operator can be done by absorbing its vacuum expectation into the redefinition of L , namely,

$$q(t) + L = q(t) - \langle q \rangle_0 + (L + \langle q \rangle_0) = q_R(t) + L_R$$

Here we assume that the scalar particle detection is based upon the processes of stimulated absorption by the detector due to the coupling between the scalar field and the monopole moment of the detector. By standard perturbation theory, the transition rate between states of the detector can be given by

$$P(E_1 \rightarrow E_2 (E_1 < E_2)) = |\langle E_2 | \text{monopole operator} | E_1 \rangle|^2 \times \Pi_\phi(E_2 - E_1).$$

The response function is defined as

$$\Pi_\phi(E) = \delta(E - \omega_0) \int dt \langle \phi^-(x) \phi^+(x) \rangle_\alpha,$$

where we have assumed that the incident field is in a single-mode quantum state, α , with frequency ω_0 .

Thus, the quantity of interest is obtained by further integration over the area located at an arbitrary $z = z$ plane as

$$I_T(z, t; q + L) = \int_0^t dt' \int d\mathbf{x}_\parallel I(\mathbf{x}_\parallel, z, t'; q + L),$$

where

$$I(x) = \phi^-(x) \phi^+(x).$$

Consider that a single-mode coherent state is incident. This quantum state can be described as:

$$|\alpha\rangle = D(\alpha) |0\rangle,$$

where

$$D(\alpha) = \exp[\alpha a_{\omega_0}^+ - \alpha^* a_{\omega_0}].$$

Under small q approximation consistent with slow motion and large $|\alpha|$ (the particle number in the coherent state), we then expand the term $\Delta I_T(z, t; q + L)$ ($\Delta O = O - \langle O \rangle_\alpha$) up to q^2 terms as:

$$\begin{aligned} \Delta I_T(z, t; q + L) &= \Delta I_T(z, t; L) + \Delta q(t - (L - z)) \langle \partial_L I_T(z, t; L) \rangle_\alpha \\ &\quad + \Delta \partial_L I_T(z, t; L) \langle q(t - (L - z)) \rangle_\alpha \\ &\quad + \Delta q(t - (L - z)) \langle q(t - (L - z)) \rangle_\alpha \langle \partial_L^2 I_T(z, t; L) \rangle_\alpha \\ &\quad + \frac{1}{2} \Delta \partial_L^2 I_T(z, t; L) \langle q(t - (L - z)) \rangle_\alpha^2, \end{aligned}$$

where we also have assumed that

$$\frac{\langle (\Delta O)^2 \rangle_\alpha}{\langle O \rangle_\alpha^2} \approx \frac{1}{|\alpha|^2 \frac{A}{\Omega} t} \approx \frac{1}{Pt/\omega_0} \ll 1 \quad (P = \text{power})$$

The overall uncertainty on the relative distance between the detector and the mirror can be defined as

$$\Delta q_{\text{eff}} = \frac{\Delta I_T(z, t; q + L)}{\langle \partial_L I_T(z, t; L) \rangle_\alpha};$$

$$\begin{aligned} \Delta q_{\text{eff}} = & \frac{\Delta I_T(z, t; L)}{\langle \partial_L I_T(z, t; L) \rangle_\alpha} + \Delta q(t - (L - z)) \\ & + \frac{\langle q(t - (L - z)) \rangle_\alpha}{\langle \partial_L I_T(z, t; L) \rangle_\alpha} \Delta \partial_L I(z, t; L) \\ & + \frac{\langle q(t - (L - z)) \rangle_\alpha \langle \partial_L^2 I_T(z, t; L) \rangle_\alpha}{\langle \partial_L I_T(z, t; L) \rangle_\alpha} \Delta q(t - (L - z)) \\ & + \frac{1}{2} \frac{\langle q(t - (L - z)) \rangle_\alpha^2}{\langle \partial_L I_T(z, t; L) \rangle_\alpha} \Delta \partial_L^2 I_T(z, t; L) \end{aligned}$$

The first term arises from a direct effect of fluctuations of the fields (shot noise). The other terms are due to an indirect effect either via random motion of the mirror or via the modified field fluctuations as a result of the motion of the mirror. The motion of the mirror is due to radiation pressure.

The square of the position uncertainty: $\langle (\Delta q_{\text{eff}})^2 \rangle_\alpha$

To compute the square of the position uncertainty, we may use the following identity:

$$\begin{aligned} \phi_1 \phi_2 \phi_3 \phi_4 &= : \phi_1 \phi_2 \phi_3 \phi_4 : + : \phi_1 \phi_2 : \langle \phi_3 \phi_4 \rangle_0 + : \phi_1 \phi_3 : \langle \phi_2 \phi_4 \rangle_0 \\ &+ : \phi_1 \phi_4 : \langle \phi_2 \phi_3 \rangle_0 + : \phi_2 \phi_3 : \langle \phi_1 \phi_4 \rangle_0 + : \phi_2 \phi_4 : \langle \phi_1 \phi_3 \rangle_0 \\ &+ : \phi_3 \phi_4 : \langle \phi_1 \phi_2 \rangle_0 + \langle \phi_1 \phi_2 \rangle_0 \langle \phi_3 \phi_4 \rangle_0 + \langle \phi_1 \phi_3 \rangle_0 \langle \phi_2 \phi_4 \rangle_0 \\ &+ \langle \phi_1 \phi_4 \rangle_0 \langle \phi_2 \phi_3 \rangle_0, \end{aligned}$$

The first term is fully normal ordered term, the next six terms are cross terms and the final three terms are pure vacuum terms.

For a coherent state (Wu & Ford 2001),

$$\begin{aligned} \langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle_\alpha - \langle \phi_1 \phi_2 \rangle_\alpha \langle \phi_3 \phi_4 \rangle_\alpha &= \langle : \phi_1 \phi_3 : \rangle_\alpha \langle \phi_2 \phi_4 \rangle_0 + \text{cross terms} \\ &+ \langle \phi_1 \phi_2 \rangle_0 \langle \phi_3 \phi_4 \rangle_0 + \text{pure vacuum terms} \end{aligned}$$

The fully normal terms cancel. However, cross terms and pure vacuum terms may give singularity as the fields in the end will be evaluated in the same point.

The spacetime average on the pure vacuum term leads to the result that varies as an inverse power of the size of the spacetime. However, the cross terms depends on the particle number in the coherent state. Thus, for a large value of the particle number, the cross terms can dominate over the pure vacuum terms. We thus obtain

$$\langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} \approx \langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} \text{ cross term}$$

Shot noise: direct effect from quantum field fluctuations:

$$\begin{aligned}
\langle (\Delta I_T)^2 \rangle_\alpha &= \langle I_T^2 \rangle_\alpha - \langle I_T \rangle_\alpha^2 \\
&= \int_0^t dt_1 \int d\mathbf{x}_1 \int_0^t dt_2 \int d\mathbf{x}_2 \langle \phi^-(\mathbf{x}_1, z, t_1; L) \phi^+(\mathbf{x}_2, z, t_2; L) \rangle_\alpha \\
&\quad \times \langle \phi^+(\mathbf{x}_1, z, t_1; L) \phi^-(\mathbf{x}_2, z, t_2; L) \rangle_0 \\
&= 16 \frac{|\alpha|^2}{\Omega} \frac{A}{2\omega_0} \sin^2[\omega_0(L-z)] \int_0^t dt_1 \int_0^t dt_2 \int_0^\infty \frac{dk}{2\pi} \frac{1}{2k} \sin^2[k(L-z)] \\
&\quad \times e^{-i(k-\omega_0)t_1} e^{i(k-\omega_0)t_2} \\
&\approx 8 \frac{|\alpha|^2}{\Omega} \frac{A}{2\omega_0} \sin^4[\omega_0(L-z)] \times t \quad (\text{for } t \gg 1/\omega_0)
\end{aligned}$$

The normalization term:

$$\langle \partial_L I_T \rangle_\alpha = 2 \frac{|\alpha|^2}{\Omega} A \sin[2\omega_0(L-z)] \times t$$

Thus,

$$\frac{\langle (\Delta I_T)^2 \rangle_\alpha}{\langle \partial_L I_T \rangle_\alpha^2} \approx \frac{1}{P\omega_0 t} \frac{1}{4} \tan^2[\omega_0(L-z)] \geq 0$$

Noise from random motion of the mirror due to radiation pressure fluctuations: (Wu & Ford 2001)

$$\begin{aligned}
 \langle (\Delta q)^2 \rangle_\alpha &= \langle q^2 \rangle_\alpha - \langle q \rangle_\alpha^2 \\
 &= \int_0^t d\tau_1 \int_0^{\tau_1} dt_1 \int d\mathbf{x}_1 \int_0^t d\tau_2 \int_0^{\tau_2} dt_2 \int d\mathbf{x}_2 \\
 &\quad (\langle : T_{zz}(x_1) :: T_{zz}(x_2) : \rangle_\alpha - \langle : T_{zz}(x_1) : \rangle_\alpha \langle : T_{zz}(x_2) : \rangle_\alpha) \\
 &\approx \frac{1}{m^2} \frac{|\alpha|^2}{\Omega} A \omega_0^2 \times t^3 \\
 &= \frac{P \omega_0 t^3}{m^2} \geq 0 \quad (\text{for } t \gg 1/\omega_0; (L - z))
 \end{aligned}$$

Noise from modified field fluctuations due to motion of the mirror under radiation pressure:

$$\frac{\langle q \rangle_\alpha^2}{\langle \partial_L I_T \rangle_\alpha^2} \langle \Delta \partial_L I \rangle_\alpha^2 \approx \frac{P \omega_0 t^3}{m^2} \geq 0 \quad (\text{for } t \gg 1/\omega_0; (L - z))$$

Correlation between shot noise and noise from random motion of the mirror (and noise from modified field fluctuations):

$$\begin{aligned}
 & \frac{1}{\langle \partial_L I_T \rangle_\alpha} (\langle \Delta I_T \Delta q \rangle_\alpha + \langle \Delta q \Delta I_T \rangle_\alpha) \\
 & \approx \frac{\langle q \rangle_\alpha}{\langle \partial_L I_T \rangle_\alpha^2} (\langle \Delta I_T \Delta \partial_L I_T \rangle_\alpha + \langle \Delta \partial_L I_T \Delta I_T \rangle_\alpha) \\
 & \approx \frac{t}{m} 2 \tan[\omega_0(L-z)] \leq \text{or} \geq 0 \quad (\text{for } t \gg 1/\omega_0; (L-z))
 \end{aligned}$$

Correlation between noises from the q^2 terms:

$$\begin{aligned}
 & \frac{\langle q \rangle_\alpha}{\langle \partial_L I_T \rangle_\alpha} (\langle \Delta \partial_L I_T \Delta q \rangle_\alpha + \langle \Delta q \Delta \partial_L I_T \rangle_\alpha) \\
 & + \frac{\langle q \rangle_\alpha \langle \partial_L^2 I_T \rangle_\alpha}{\langle \partial_L I_T \rangle_\alpha^2} (\langle \Delta I_T \Delta q \rangle_\alpha + \langle \Delta q \Delta I_T \rangle_\alpha) \\
 & + \frac{1}{2} \frac{\langle q \rangle_\alpha^2}{\langle \partial_L I_T \rangle_\alpha^2} (\langle \Delta \partial_L I_T \Delta I_T \rangle_\alpha + \langle \Delta I_T \Delta \partial_L I_T \rangle_\alpha) \\
 & \approx \frac{P \omega_0 t^3}{m^2} \left(\frac{9}{2} - \frac{3}{2} \tan^2[\omega_0(L-z)] \right) \quad (\text{for } t \gg 1/\omega_0; (L-z))
 \end{aligned}$$

Putting together all terms up to order q^2 gives:

$$\begin{aligned}\langle(\Delta q_{\text{eff}})^2\rangle_{\alpha} &= \frac{\langle(\Delta I_T)^2\rangle_{\alpha}}{\langle\partial_L I_T\rangle_{\alpha}^2} \\ &= \frac{1}{P\omega_0 t} \frac{x^2}{4} + \frac{t}{m} 2x + \frac{P\omega_0 t^3}{m^2} \left(\frac{11}{2} - \frac{3}{2}x^2 \right),\end{aligned}$$

where $x = \tan[\omega_0(L - z)]$.

To make the result sensible for large P , it is required that

$$\frac{11}{2} - \frac{3}{2}x^2 > 0$$

leading to

$$x^2 \leq \frac{11}{3},$$

the region of interest.

Assume that the frequency of the coherent state follows the distribution function e.g. in a form of Lorentzian:

$$f(\omega, \omega_0; \sigma_0) = \frac{\sigma_0}{\pi} \frac{1}{(\omega - \omega_0)^2 + \sigma_0^2}.$$

Thus, the further average is taken as follows:

$$\langle \langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} \rangle_f = \frac{\langle \langle (\Delta I_T)^2 \rangle_{\alpha} \rangle_f}{\langle \langle \partial_L I_T \rangle_{\alpha}^2 \rangle_f},$$

where

$$\langle O \rangle_f = \int_{-\infty}^{\infty} f(\omega, \omega_0; \sigma_0) O(\omega) d\omega.$$

In the case of $\omega_0 \gg \sigma_0$ (the single-mode approximation) and $\omega_0(L - z) \gg \sigma_0(L - z) \gg 1$, we obtain (Caves 1980,1981)

$$\langle\langle(\Delta q_{\text{eff}})^2\rangle_\alpha\rangle_f = \frac{3}{4} \frac{1}{P\omega_0 t} + \frac{P\omega_0 t^3}{m^2}.$$

The averaged result of the correlation terms gives no contribution to the position uncertainty squared. Minimizing $\langle\langle(\Delta q_{\text{eff}})^2\rangle_\alpha\rangle_f$ by choosing an optimal value of the power given by

$$P_{\text{opt}} = \frac{\sqrt{3}}{2} \frac{m}{\omega_0 t^2},$$

leads to

$$\langle\langle(\Delta q_{\text{eff}})^2\rangle_\alpha\rangle_{f;\text{min}} = \sqrt{3} \frac{t}{m}.$$

This gives a bound. Notice that $\langle q \rangle_\alpha \partial_L \sim \langle q \rangle_\alpha \times \omega_0 \mid_{P_{\text{opt}}} \sim 0.86$. The small q approximation can be barely satisfied where the terms of higher powers of q will give sizable corrections.

Now go back to the single-mode case where

$$\langle\langle(\Delta q_{\text{eff}})^2\rangle_{\alpha}\rangle_f = \frac{1}{P\omega_0 t} \frac{x^2}{4} + \frac{t}{m} 2x + \frac{P\omega_0 t^3}{m^2} \left(\frac{11}{2} - \frac{3}{2}x^2 \right).$$

For $x > 0$, the minimum value of $\langle\langle(\Delta q_{\text{eff}})^2\rangle_{\alpha}\rangle_{f;\min}$ is found to be

$$\langle\langle(\Delta q_{\text{eff}})^2\rangle_{\alpha}\rangle_{f;\min} = x \left(2 + \sqrt{\frac{11}{2} - \frac{3}{2}x^2} \right) \frac{t}{m}$$

with an optimal value of the power given by

$$P_{\text{opt}} = \frac{x}{\sqrt{22 - 6x^2}} \frac{m}{\omega_0 t^2}.$$

Recall that the condition for having negligible fluctuations of the quantum operator with respect to a coherent state as compared with its expectation value gives a small parameter

$$\delta = \frac{\omega_0}{Pt} \big|_{P_{\text{opt}}} \approx \frac{\omega_0^2 t}{m} \ll 1.$$

Then, we obtain

$$\delta \left(\frac{t}{m} \right) \ll \langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} f; \min \leq \left(\frac{1}{2} \sqrt{\frac{111}{2} + 32\sqrt{3}} \approx 5.27 \right) \left(\frac{t}{m} \right),$$

where

$$\delta \frac{m}{\omega_0 t^2} \ll P_{\text{opt}} \leq \left(\sqrt{\frac{9 + 4\sqrt{3}}{78 - 24\sqrt{3}}} \approx 0.66 \right) \frac{m}{\omega_0 t^2}.$$

Note that $\langle q \rangle_{\alpha} \partial_L \sim \langle q \rangle_{\alpha} \times \omega_0 |_{P_{\text{opt}} \sim 0.66 - \delta} < 1$. The small q approximation can be justified.

For $x < 0$, the minimum value of $\langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} \rangle_{f;\min}$ is found to be

$$\langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} \rangle_{f;\min} = x \left(2 - \sqrt{\frac{11}{2} - \frac{3}{2}x^2} \right) \frac{t}{m}$$

with an optimal value of the power given by

$$P_{\text{opt}} = \frac{-x}{\sqrt{22 - 6x^2}} \frac{m}{\omega_0 t^2}.$$

Then, we obtain

$$\delta \left(\frac{t}{m} \right) \ll \langle (\Delta q_{\text{eff}})^2 \rangle_{\alpha} \rangle_{f;\min} \leq \left(\frac{1}{2} \sqrt{\frac{111}{2} - 32\sqrt{3}} \approx 0.14 \right) \left(\frac{t}{m} \right),$$

where

$$\delta \frac{m}{\omega_0 t^2} \ll P_{\text{opt}} \leq \left(\sqrt{\frac{9 - 4\sqrt{3}}{78 + 24\sqrt{3}}} \approx 0.13 \right) \frac{m}{\omega_0 t^2}.$$

The small q approximation can be satisfied.

SQL can be beaten.

Discussion on the Langevin equation with backreaction effects

Consider that the mirror is described by a wavepacket. The Langevin equation to describe the motion of the center of the wavepacket can be derived from the method of Influence Functional as: (Wu & Lee 2005)

$$m \ddot{q}(t) - \int dt' i \Theta(t - t') \langle [F(t), F(t')] \rangle q(t') = F(t),$$

For the small q approximation, the backreaction term is an expression of:

$$\begin{aligned} & - \int dt' i \Theta(t - t') \langle [F(t), F(t')] \rangle q(t') \\ &= \frac{6}{\pi} \frac{|\alpha|^2}{\Omega} \omega_0^2 A \left(\sin[\omega_0(t - L)] \left(\frac{\pi}{2} \sin[\omega_0(t - L)] \right. \right. \\ & \quad \left. \left. - \left(\frac{1}{2} + \ln[\omega_0 \epsilon] + \gamma_E \right) \cos[\omega_0(t - L)] q(t) \right. \right. \\ & \quad \left. \left. - \frac{1}{2\omega_0^2} \cos[\omega_0(t - L)] \ddot{q}(t) - \frac{1}{2\omega_0^2} \cos[\omega_0(t - L)] \ddot{\ddot{q}}(t) \right) \right) \end{aligned}$$

Renormalization is needed to absorb the divergence. The dissipation effect is given by the third order time derivative of q .

Here, when the mirror is illuminated by a coherent state, the dissipative force then is given by the third order time derivative.

However, quantum noise for the mirror-field system when the mirror undergoes some non-inertial motion can be further studied from the formalism we develop here with a more complete Langevin equation that incorporates all backreaction (dissipation/fluctuations) effects.

Conclusions

- ▶ Field theoretic approach is employed to study quantum noise in the mirror-field system where the mirror as a reflector is illuminated by a coherent state.
- ▶ Various sources of quantum noises, which are from shot noise, random motion of mirror due to the radiation pressure fluctuations, and modified field fluctuations given by the motion of mirror, are all incorporated consistently with an emphasis on their correlation effects.
- ▶ Overall noise can be decreased (increased) as a result of the negative (positive) correlations between shot noise and noise from random motion of mirror that can be achieved by tuning the parameters in the problem.
- ▶ To beat SQL, the order of magnitude of $P_{\text{opt}} \approx m/\omega_0^2 t$ is about 10^{10} Watt for the mass of the mirror ≈ 10 kg ; wavelength of laser $\approx 10^{-6} m$; and duration $10^{-7} s$. Unfortunately it is much larger than typical laser power for the interferometer about 10^3 Watt . (LIGO-II parameters).

Collaborators

Jen-Tsung Hsiang (National Dong Hwa University)

Tai-Hung Wu (National Dong Hwa University)