

Pair winds from compact objects with an application to GRB

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Abstract

An spherically symmetrical outflowing wind or a fireball consisting of electron-positron pairs, photons, and baryons is considered. We do not assume thermal equilibrium, and include the two-body processes that occur in such plasma: Moller and Bhaba scattering of pairs, Compton scattering, two-photon pair annihilation, two-photon pair production, and pe , pp Coulomb scattering, together with their radiative three-body variants: bremsstrahlung, double Compton scattering, and three-photon pair annihilation, with their inverse processes. We solve the relativistic kinetic Boltzmann equations for distribution functions of pairs, protons, and photons numerically. By the calculating of the timescale of the thermalization we can prove the possibility of using the hydrodynamical approximation (HD). Also the purpose of our investigations is to describe the plasma correctly in all cases, and then the plasma is not in thermal equilibrium and cannot be described in the frame of HD.

Motivation of the work

- We study kinetic properties of γ , $e^\pm$, p plasma, relevant, for example, for GRB phenomenon (the fireshell) and compact astrophysical sources (the wind).
- We checked assumptions used for the investigation of such phenomenon: the hydrodynamical approximation, timescales, optical depths. In some cases the plasma is not in the thermal equilibrium, and we need to use the kinetic description. In some cases it is possible to use the HD description, but not in the whole region of the solution.
- In publications 2004, 2005, 2007 we considered γ , $e^\pm$ plasma evolution in the frame of kinetic Boltzmann equations. In 2009, 2010 we expanded the kinetic approach for plasma with the baryonic loading. Baryons in progenitors of GRB's (massive stars, NS) are important for the bursts time duration and for the spectrum. We made the description in the frame the kinetic, in the hydrodynamic.
- The development of computational methods for such tasks is also an important problem.

Parameters of the plasma

- To exclude the other particles except γ , $e^\pm$ the temperature range at the thermal equilibrium is

$$0.1 \text{ MeV} \lesssim T_{\text{th}} \lesssim 10 \text{ MeV}.$$

In the case of protons the low temperature limit is arbitrary. From the GRB parameters

$$10^{48} \text{ erg} \leq E_0 \leq 10^{54} \text{ erg}, 10^6 \text{ cm} \leq R_0 \leq 10^8 \text{ cm},$$

one can estimate the temperature in the thermal equilibrium given by first formula. The nonthermal plasma flow from the compact object $R \approx 10\text{km}$, $L \gtrsim 10^{38} \text{ ergs} \cdot \text{s}^{-1}$.

- The baryonic loading $B = \frac{n_p M c^2}{\rho}$.

Parameters important for calculations

- Mildly relativistic plasma, average energy per particle $0.1 \text{ MeV} \lesssim \epsilon_{\text{th}} \lesssim 10 \text{ MeV}$.
- The non ideality plasma parameter $(nr_D^3)^{-1} \ll 1 \sim 10^{-3}$, where $d = \sqrt{\frac{k_B T}{4\pi e^2 n_e}} = \frac{c}{\omega} \sqrt{\theta_e}$ is the Debye length. The plasma can be described by 1-particles distribution functions $f_i(\mathbf{p}, \mathbf{r}, t)$.

$$\frac{\partial f_i}{\partial t} + v^\beta \frac{\partial f_i}{\partial x^\beta} = c(\tilde{\eta}_i^q - \chi_i^q f_i). \quad (1)$$

- The classicality parameter $\varkappa = e^2 / (\hbar v_r) = \alpha / \beta_r < 1$ — quantum description of plasma.
- The Coulomb logarithm $\Lambda = \mathcal{M} dv_r / \hbar$, $\mathcal{M}$ is reduced mass.
- Intensity of interactions between protons and other particles the depth $\tau = n\sigma R$.
- The degeneracy $\theta_F = \left[\left(\frac{\hbar}{mc} \right)^2 (3\pi^2 n_e)^{2/3} \right]^{1/2} - 1$. The plasma is non-degenerate $\theta > \theta_F$ except the upper bound of the temperature region; Parameters for perturbative expansions are α and m/M .

- Pair production, Compton and electron-electron scattering:
 $t_{ee} \sim (\sigma_T n c \Lambda)^{-1}$, $t_{\gamma e} \sim (\sigma_T n c)^{-1}$; The thermal equilibrium establishing $t_{br} = t_{3p} = \alpha^{-1} t_c$;
- Proton-proton:
 $(n_p t_{pp})^{-1} \approx \sqrt{\frac{m}{M}} (n_- t_{ee})^{-1}$, $v_p \approx \sqrt{\frac{m}{M}} v_e$, $v_e \approx c$;
 Electron-proton: $t_{ep}^{-1} \approx \frac{\epsilon_{\pm}}{Mc^2} t_{ee}^{-1}$, $\epsilon_{\pm} \ll \epsilon_p$; Proton Compton scattering is not important:
 $(n_p t_{\gamma p})^{-1} \approx \left(\frac{\epsilon}{Mc^2}\right)^2 (n_- t_{\gamma e})^{-1}$, $\epsilon \geq mc^2$.
- Expansion timescale: $t_{\text{hyd}} \sim c/R_0$;

The depth and HD approximation

- $\tau = \int_r^\infty n\sigma dr'$. In the region with high $\tau_e, \tau_p, \tau_\gamma \gg 1$ we can expect that HD approximation is working well.
- In the local system with $\mathbf{v} = 0$ the energy-momentum tensor is

$$T^{ik} = \text{diag}(\rho\epsilon, P, P, P).$$

For the any system using four-velocity $u^i = \gamma(1, \mathbf{v}/c)$ we have the generalization of T^{ik} in any system

$$T^{ik} = (\rho\epsilon + P)u^i u^k - P g^{ik}.$$

- HD equations are the conservation laws of energy and momentum

$$\frac{\partial T^k_i}{\partial x^k} = 0$$

- The conservation if particles

$$\frac{\partial n u^i}{\partial x^i} = 0$$

Kinetic approach and HD equations

- Kinetic Boltzmann equations
- The Lorentz transformation of 4-vectors
- Local kinetic equilibrium DF in the co-moving coordinate system K' with T and φ .

The Lorentz transformation of 4-vectors

Let's write hydrodynamic equations for pairs. We start from the kinetic equation (1). Let coordinate system K' move with the velocity $\mathbf{V}$ in the laboratory system K . The Lorentz transformation of electron energy-momentum is (Mihalas)

$$\epsilon = \Gamma (\epsilon' + V \mathbf{N} \cdot \mathbf{p}') , \quad \mathbf{p} = \mathbf{p}' + \left[(\Gamma - 1) \mathbf{N} \cdot \mathbf{p}' + \Gamma \frac{V}{c} \frac{\epsilon'}{c} \right] \mathbf{N}, \quad (2)$$

and

$$t = \Gamma (t' + V \mathbf{N} \cdot \mathbf{r}') , \quad \mathbf{r} = \mathbf{r}' + \left[(\Gamma - 1) \mathbf{N} \cdot \mathbf{r}' + \Gamma \frac{V}{c} \frac{t'}{c} \right] \mathbf{N}, \quad (3)$$

where

$$\Gamma = \frac{1}{\sqrt{1 - (V/c)^2}}, \quad \mathbf{N} = \frac{\mathbf{V}}{V}.$$

The energy and the momentum are related with the mass and velocity in K

$$\epsilon^2 = m^2 c^4 + p^2 c^2, \quad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - (v/c)^2}}.$$

$$d\mathbf{p} = \frac{\epsilon}{\epsilon'} d\mathbf{p}'.$$

Another relation is the invariant DF due to the relation $d\mathbf{r}d\mathbf{p} = d\mathbf{r}'d\mathbf{p}'$ and invariant number of particles $f(\mathbf{p})d\mathbf{r}d\mathbf{p} = f'(\mathbf{p}')d\mathbf{r}'d\mathbf{p}'$:

$$f(\mathbf{p}) = f'(\mathbf{p}').$$

Assumptions and the HD parameters

We want to describe electrons by only 3 parameters: density, velocity and energy.

$$f'(\mathbf{p}') = \frac{2}{(2\pi\hbar)^3} \exp\left(-\frac{\epsilon' - \varphi}{kT}\right), \quad \epsilon' = \sqrt{m^2 c^4 + (\mathbf{p}'c)^2}.$$

Let's now define hydrodynamic parameters of electrons. The particles concentration and the density are

$$n \equiv \int f' d\mathbf{p}', \quad \rho \equiv mn, \quad (4)$$

pressure is

$$P \equiv c^2 \int f' \frac{\mathbf{p}'^2}{3\epsilon'} d\mathbf{p}', \quad (5)$$

and the specific energy density is

$$\varepsilon \equiv \frac{1}{\rho} \int f' \epsilon' d\mathbf{p}'. \quad (6)$$

From Boltzmann equation we obtain hydrodynamic equations: $m \int d\mathbf{p} \times$ equation (1):

$$\frac{\partial(\Gamma\rho)}{\partial t} + \frac{\partial}{\partial x^\alpha}(\Gamma\rho V^\alpha) = m \int d\mathbf{p} c(\tilde{\eta} - \chi f),$$

$\int d\mathbf{p} \mathbf{p} \times$ equation (1):

$$\frac{\partial}{\partial t}(\Gamma\rho\Gamma h V^\alpha) + \frac{\partial}{\partial x^\beta}(\Gamma\rho\Gamma h V^\alpha V^\beta + \delta^{\alpha\beta}P) = \int d\mathbf{p} p^\alpha c(\tilde{\eta} - \chi f),$$

$\int d\mathbf{p} \epsilon \times$ equation (1):

$$\frac{\partial}{\partial t}\left(\Gamma\rho\left(\Gamma c^2 h - \frac{P}{\Gamma\rho}\right)\right) + \frac{\partial}{\partial x^\alpha}(\Gamma\rho V^\alpha \Gamma c^2 h) = \int d\mathbf{p} \epsilon c(\tilde{\eta} - \chi f),$$

where the specific dimensionless enthalpy is

$$h = \frac{1}{c^2} \left(\epsilon + \frac{P}{\rho} \right).$$

Transformation properties of Intensity, emission, absorption coefficients

We have invariants for the emitted and the absorbed particles

$$\frac{\eta d\omega d\epsilon d\mathbf{r} dt}{\epsilon}, \frac{\chi I d\omega d\epsilon d\mathbf{r} dt}{\epsilon},$$

the invariants $d\mathbf{r} dt$, invariant DF

$$f = \frac{I(\epsilon, \mu)}{\epsilon^2 p} = \frac{I'(\epsilon', \mu')}{\epsilon'^2 p'},$$

and the invariant

$$\frac{d\mathbf{p}}{\epsilon} = \frac{d\epsilon d\mu d\phi \epsilon^2 \beta}{c^3 \epsilon}.$$

For emission and absorption coefficients we can make the invariant values

$$\frac{\eta(\epsilon, \mu)}{\epsilon p} = \frac{\eta'(\epsilon', \mu')}{\epsilon' p'}, \quad \frac{\chi(\epsilon, \mu)}{\epsilon'} = \frac{\chi'(\epsilon', \mu')}{\epsilon}.$$

Reaction rates calculations

From v , T_e , φ_e we calculate f_e in the laboratory frame. f_γ in the laboratory frame is known. We know η , χ in the laboratory frame from the exact expressions of the rates of 2-particles reactions. We will get exact conservation laws of particles, energy in numerical calculations for such reactions. But we have two limitations in such approach.

1. To reduce calculations we can calculate 2p reactions in the co-moving coordinates where electrons distribution functions are isotropic.
2. For 3-particles reactions rates converge to the thermal equilibriums expressions with the multipliers $\exp^{\phi_i/(kT)}$ in the isotropic and the non degenerate case. Due to the fact that 2p reactions are faster than the 3p ones, the plasma is in kinetic equilibrium with $T_\gamma = T_e$, and $\varphi_\gamma = \varphi_e$ (ref. 2007).

We can define E'_e , E'_γ in the co-moving frame, to calculate η' , χ' in the co-moving frame, and to define $\eta(\epsilon, \mu) = \frac{\epsilon p}{\epsilon' p'} \eta'(\epsilon')$, $\chi(\epsilon, \mu) = \frac{\epsilon'}{\epsilon} \chi'(\epsilon')$ in the laboratory frame.

The scheme

For the spectral energy density for the phase space in the spherical-symmetric case (ϵ, μ, r, t)

$(\int d\epsilon d\mu E_i = \rho \epsilon_i = \int d\mathbf{p} \epsilon f_i = \int d\epsilon d\mu d\phi \epsilon^2 \beta / c^3 \epsilon f_i): E_i = \frac{2\pi \epsilon^3 \beta_i f_i}{c^3} = \frac{2\pi}{c} I_i$
we have the kinetic equation in the conservative form

$$\frac{1}{c} \frac{\partial E_i}{\partial t} + \mu \frac{\partial}{r^2 \partial r} (r^2 \beta E_i) + \frac{\partial}{\partial \mu} \left((1 - \mu^2) \frac{\beta}{r} E_i \right) = \sum_q (\eta_i^q - \chi_i^q f_i).$$

- The computational grid in the phase space, and the second order finite difference scheme. The second order provides transition to diffusion approximation in the nontransparent region.
- Stiff ODE's system
- Implicit Gear's method and the cyclic reduction to inverse Jacobi matrix.

The local thermodynamic equilibrium and the steady solution

To estimate the energy of escaping photons we can use the temperature in the photosphere where $\tau_\gamma \approx 1$. We also neglect the radiation transport due to the photons diffusion. We adopt the distribution function of photons in the moving coordinate system K'

$$f'_\gamma(\mathbf{p}) = \frac{2}{(2\pi\hbar)^3} \exp\left(-\frac{\epsilon'}{kT}\right),$$

where $\epsilon' = p'c$. We adopt

$$\rho = \rho^{\text{eq}}. \quad (7)$$

In the momentum and energy conservation equations we should use

$$P = P_e + P_\gamma, \quad h = h_e + h_\gamma$$

$$\frac{\partial}{\partial t} (\Gamma\rho\Gamma h V^\alpha) + \frac{\partial}{\partial x^\beta} (\Gamma\rho\Gamma h V^\alpha V^\beta + \delta^{\alpha\beta} P) = 0, \quad (8)$$

$$\frac{\partial}{\partial t} \left(\Gamma\rho \left(\Gamma c^2 h - \frac{P}{\Gamma\rho} \right) \right) + \frac{\partial}{\partial x^\alpha} (\Gamma\rho V^\alpha \Gamma c^2 h) = 0.$$

Let $\rho = \rho^{\text{eq}}$ and $\partial_t = 0$. Then one will get the couple between variables Γ and θ

$$\Gamma^2 \rho h V \frac{\partial V}{\partial r} + \frac{\partial P}{\partial r} = 0,$$

or

$$\frac{\partial \ln \Gamma}{\partial r} + \frac{1}{\rho \varepsilon + P} \frac{\partial P}{\partial r} = 0, \text{ or } \Delta(\ln \Gamma + \mathcal{P}(\theta)) = 0,$$

where

$$\mathcal{P}(\theta) \equiv \int_1^\theta \frac{dP}{\rho \varepsilon + P} = \int_1^\theta \frac{d\theta \cdot \partial_\theta P}{\rho \varepsilon + P} = \ln \theta.$$

At any temperature one has

$$\Gamma \theta = \text{const} = \Gamma_0 \theta_0,$$

$$r^2 \Gamma^2 \rho c^2 h V = \text{const} = r_0^2 \Gamma_0^2 \rho_0 c^2 h_0 V_0 = f_E,$$

$$\frac{\partial P}{\partial r} = \rho c^2 h \frac{\partial \ln \theta}{\partial r}.$$

The steady pair wind by Grimsrud and Wasserman 1998

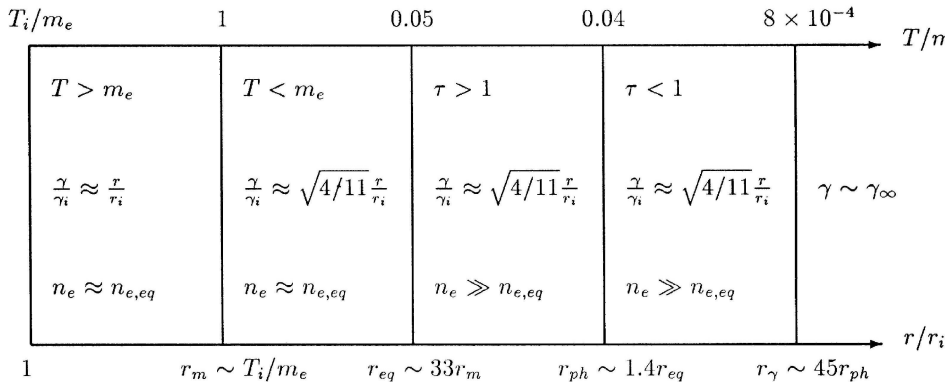
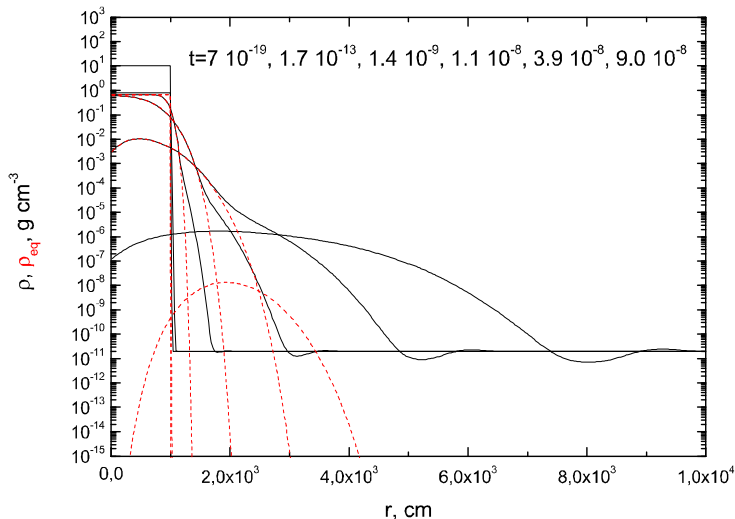
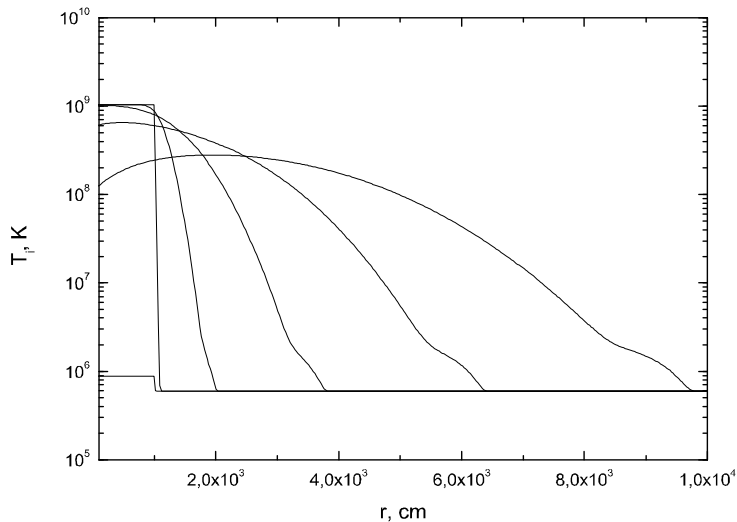


Figure 1. Summary of analytical model with γ_i and $r_i/10^6$ cm both of order unity.

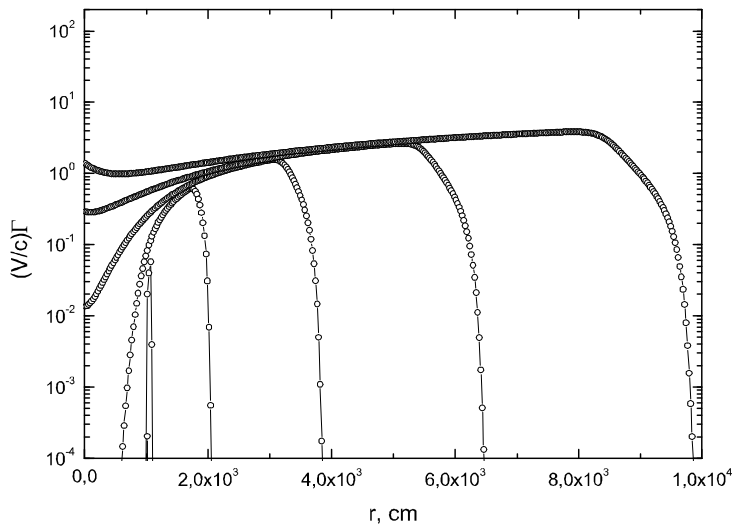
Pair plasma HD. Density profiles.



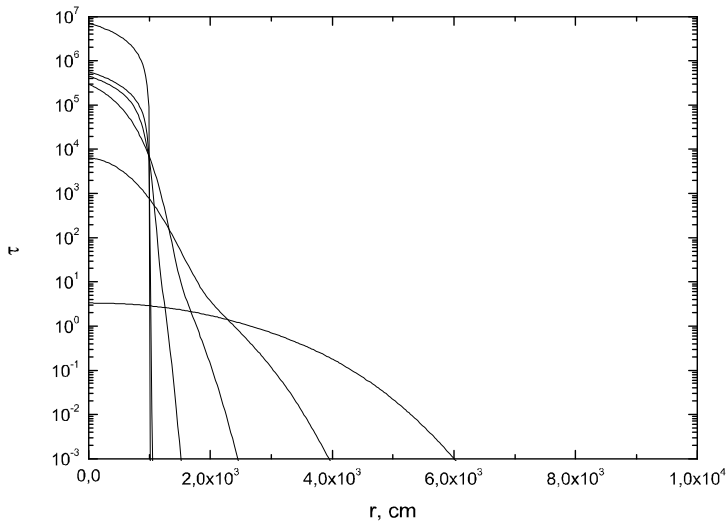
Temperature profiles.



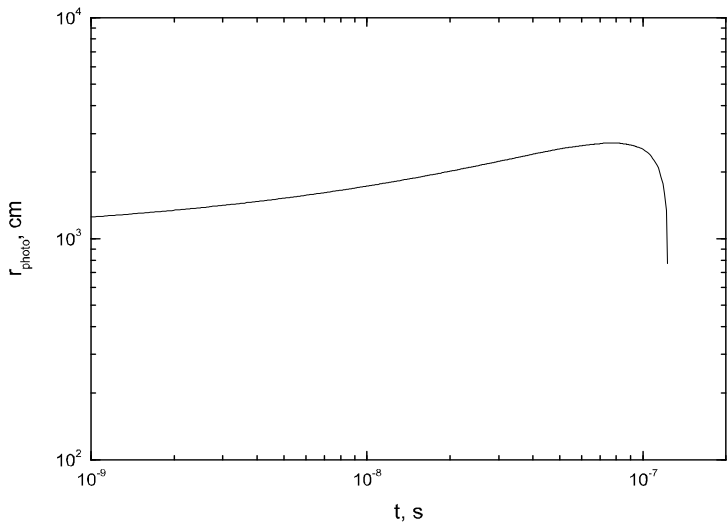
Pair plasma. Velocity profiles.



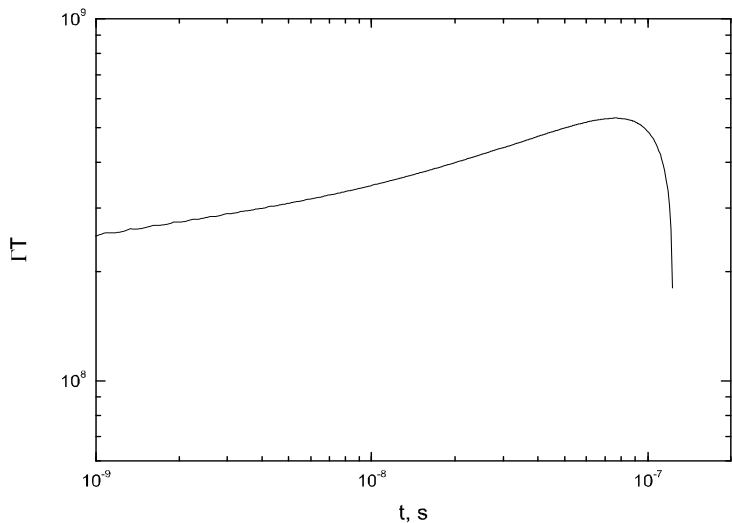
Pair plasma τ .



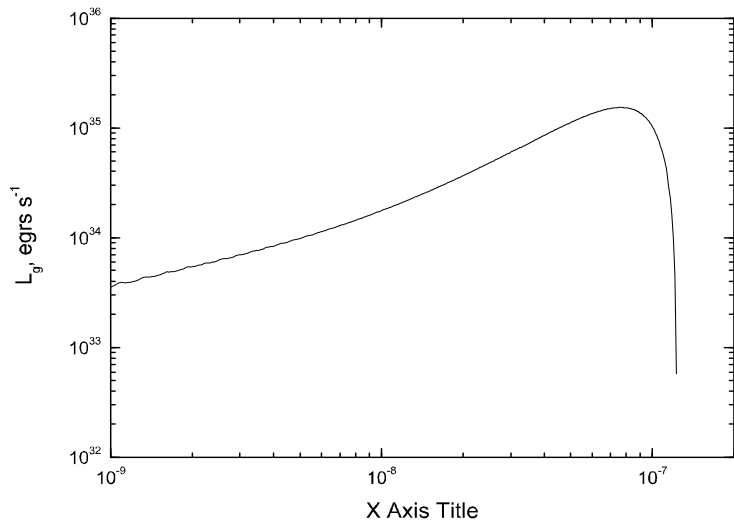
Pair plasma photosphere radius.



Pair plasma photosphere γT .

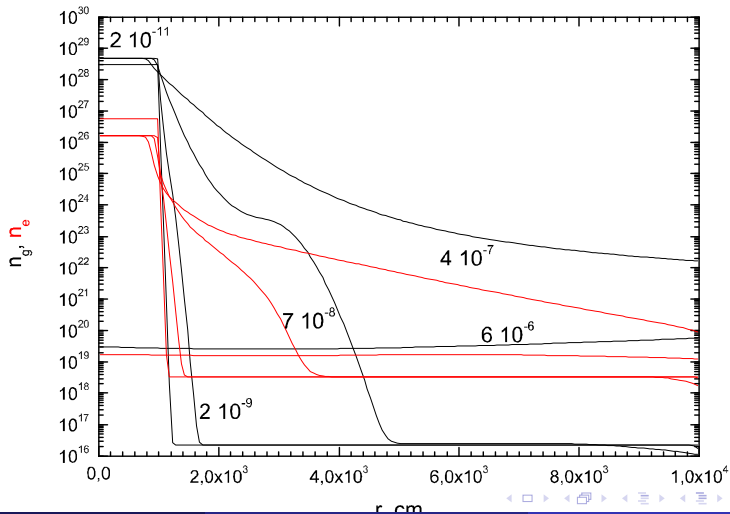


Pair plasma luminosity.

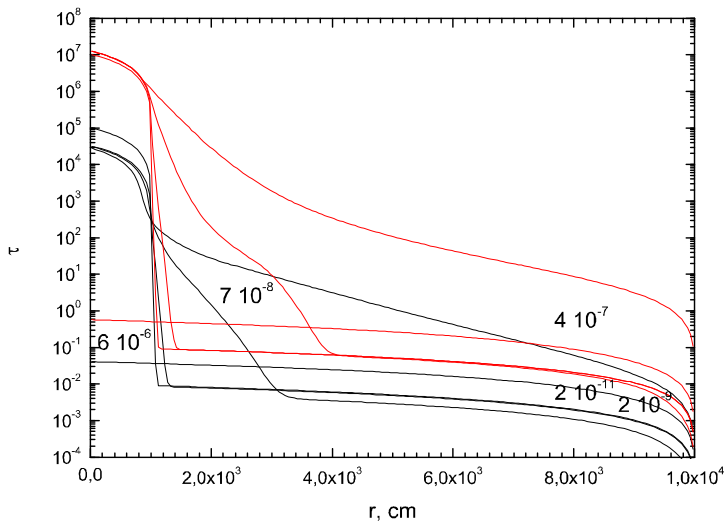


Kinetics of the pair plasma. Concentrations profiles

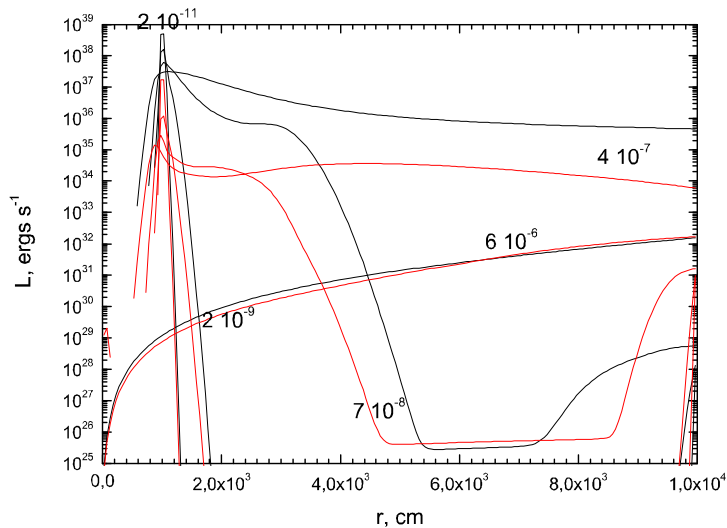
$t = 2.2 \cdot 10^{-11}, 1.9 \cdot 10^{-9}, 6.7 \cdot 10^{-8}, 4.3 \cdot 10^{-7}, 6 \cdot 10^{-6}$ s.



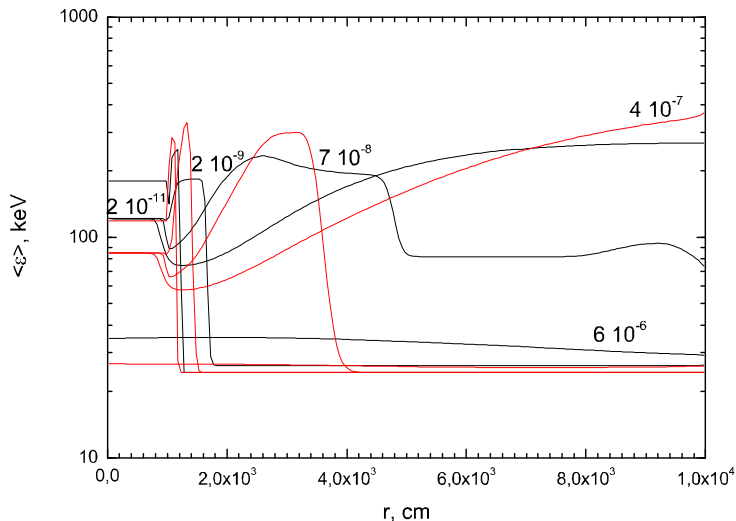
τ .



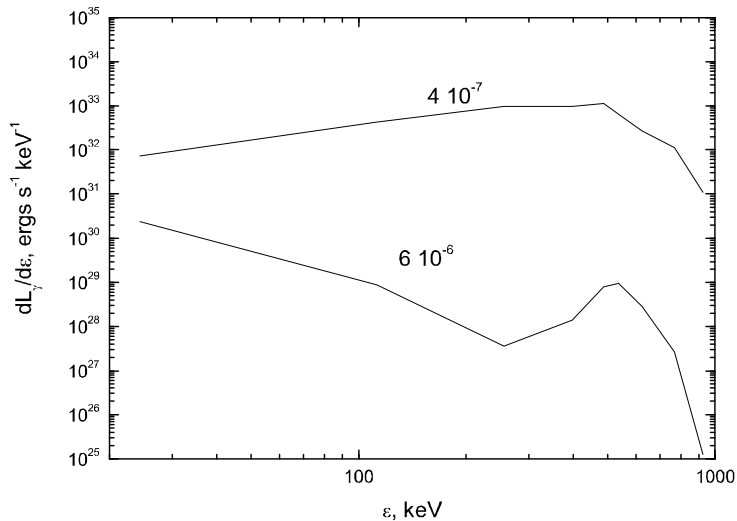
Luminosities



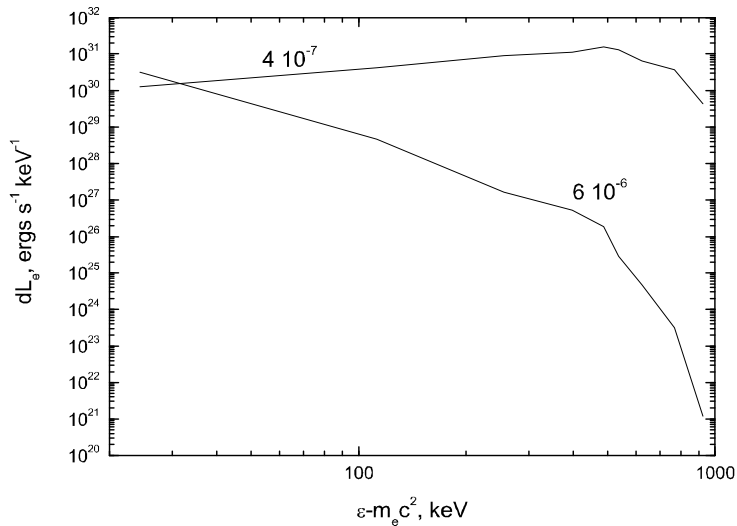
Average energies



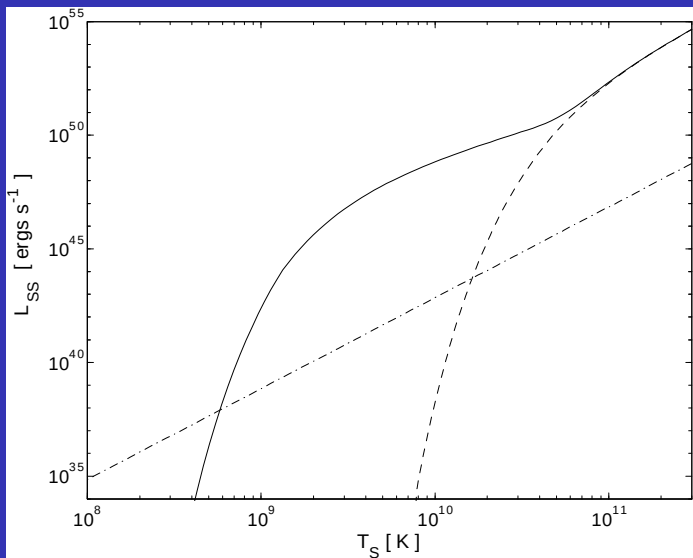
Photon spectra



Electron spectra

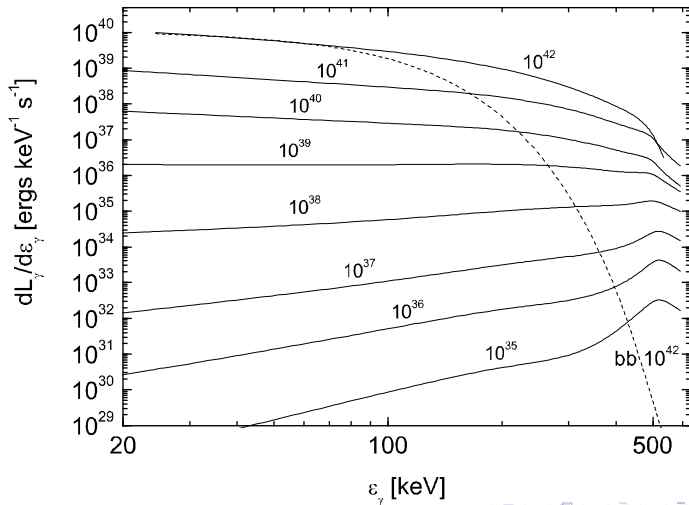


SS radiation in pairs (dotted) and in thermal photons (dashed curve).



SS spectrum for different luminosities

(near the curve in ergs s^{-1}).



Conclusions and future prospects

- We studied spherically symmetric γ , $e^\pm$, p plasma using HD equations and kinetic equations. Kinetic equations are used far away from the thermal equilibrium. The developed codes are published.
- We considered the nonuniform spherically-symmetric case winds in the kinetic approach for the mildly relativistic case for both optically thin and thick cases with enough large $\tau_\gamma \sim 10^5$, $\tau_e \sim 10^7$. We can integrate DF for any radius. We can study important details in the photons spectra at $\tau_\gamma \lesssim 10^3$. We did not assume HD for electrons, and found it is not applicable for high τ_e , if photons spectra differ from kinetic equilibrium $\exp((\epsilon - \phi_\gamma)/kT_\gamma)$!
- In order to study real GRB sources we need better resolution in the spatial grid to resolve small angles in the laboratory frame due to the ultrarelativistic motion.
- Our approach in calculations should be also carried in the case of degenerate particles. 3p interactions is the main difficulty in that case.
- At high temperature we need to take into account neutrinos.