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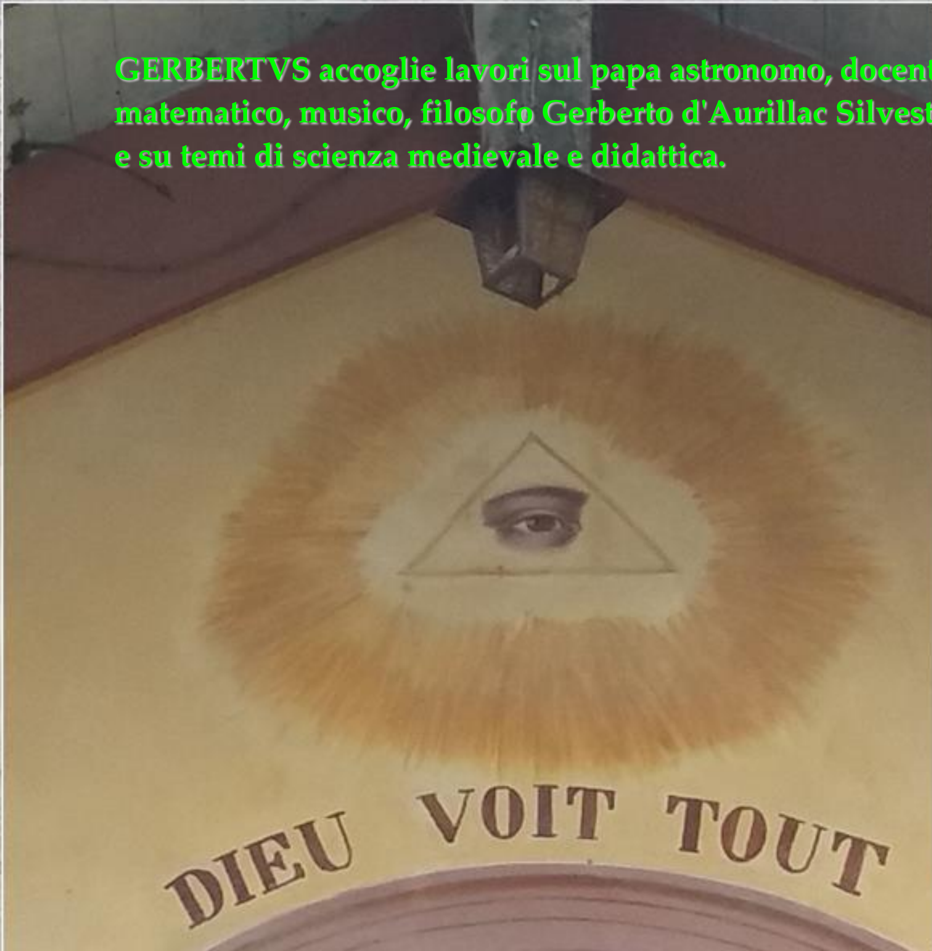

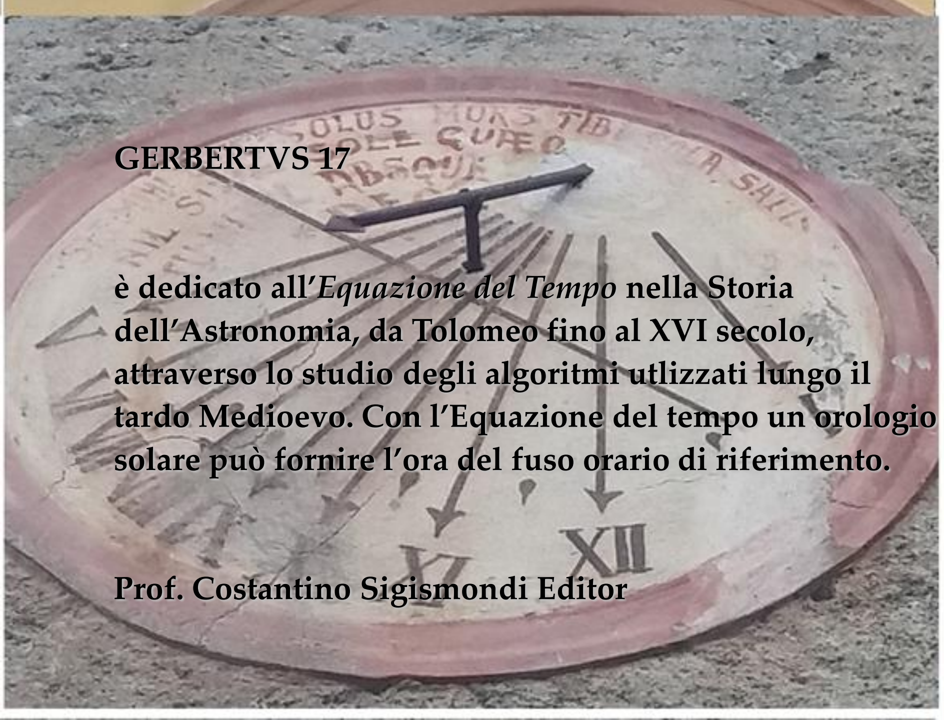

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è dedicato all'*Equazione del Tempo* nella Storia dell'Astronomia, da Tolomeo fino al XVI secolo, attraverso lo studio degli algoritmi utlizzati lungo il tardo Medioevo. Con l'Equazione del tempo un orologio solare può fornire l'ora del fuso orario di riferimento.

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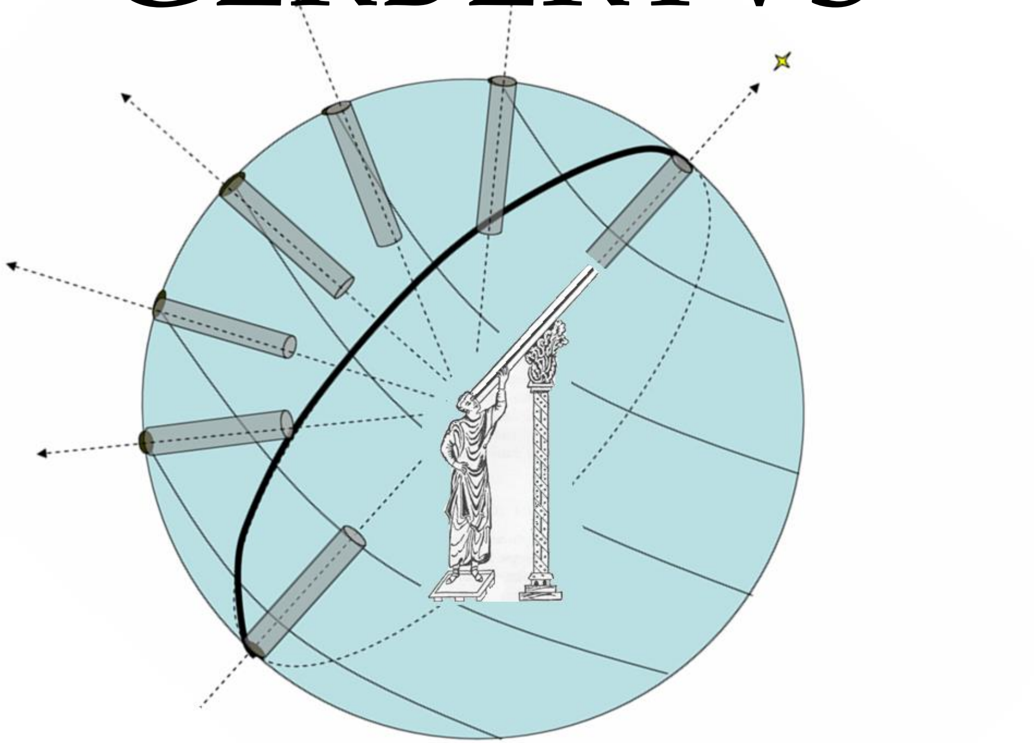

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Con il contributo di

A Survey on the evolution of the equation of time during the centuries

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Abstract
Throughout time not very many astronomical tables kept evolving. However, some did, as is the case of the table for the equation of time.
We will offer a general survey of the evolution of the equation of time through different traditions, starting from Greece arriving to the pre-Copernican period.
We have some indication, in the primary sources, on how the equation of time table was recomputed but only few secondary literature, such as editions or translations of the primary sources, is available. In particular we have a secondary literature on Ptolemy (II century) and on Levi ben Gerson (XIV century) that we used as a guide and as a term of comparison for the algorithm and for the quality of the results.
In this survey Alfonsine astronomy provides examples of the equation of time, which will be the object of our research.
We will show the mathematical analyses on some equation of time tables already recomputed and on others never investigated before.
We started from Ptolemy (c. 100 – c. 175) and then, following a chronological order, we analyzed al-Khwārizmī (c. 780 – c. 850), al-Battānī (c. 858 – 929), John of Murs (1290 - 1355), Peter of Saint Omer, John of Lignères (1290 -), Levi ben Gerson (1288 – 1344), John Holbroke (- 1437), Giovanni Bianchini (1410 - c. 1469), Georg Puerbach (1423-1461) and Abraham Zacut (1452-1515).
At the beginning we started with a modern recomputation, then we passed to a recomputation by the use of tables only for a selection of astronomers.
Methods and algorithms will be shown.
This is a preliminary work on tables that allow us to draw only some conclusion and some suggestion for further investigation.
An initial version of this work was prepared in the context of the ERC project ALFA (CoG 723085).

Sommario

Nel corso del tempo sono state poche le tavole astronomiche che si sono evolute. Tuttavia in alcune tavole questa evoluzione si è verificata, come nel caso della tavola dell'equazione del tempo.

In questo lavoro verrà fornita una panoramica generale dell'evoluzione dell'equazione del tempo attraverso le diverse tradizioni, a partire dalla Grecia fino al periodo pre-copernicano.

Abbiamo alcune indicazioni, nelle fonti primarie, su come la tavola dell'equazione del tempo sia stata ricalcolata, ma a disposizione abbiamo poche fonti secondarie, come ad esempio riedizioni o traduzioni delle fonti primarie. In particolare, abbiamo a disposizione un'ampia letteratura secondaria su Tolomeo (II secolo) e su Levi ben Gerson (XIV secolo) che abbiamo utilizzato come linea guida e come termine di paragone per quanto riguarda l'algoritmo e per la validità dei risultati.

In quest'indagine l'astronomia Alfonsina fornisce esempi dell'equazione del tempo, che saranno oggetto della nostra ricerca.

Verrà mostrata l'analisi matematica di alcune tavole dell'equazione del tempo già precedentemente ricalcolate e su altre finora mai indagate.

Siamo partiti da Tolomeo (c. 100 – c. 175) e poi, seguendo un ordine cronologico, abbiamo analizzato al-Khwārizmī (c. 780 – c. 850), al-Battānī (c. 858 – 929), Jean de Murs (1290 - 1355), Peter of Saint Omer, Jean de Lignères (1290 -), Levi ben Gerson (1288 – 1344), John Holbroke (- 1437), Giovanni Bianchini (1410 - c. 1469), Georg Puerbach (1423-1461) and Abraham Zacut (1452-1515).

All'inizio si è partiti da un algoritmo con formule matematiche odierne, poi si è passati a un calcolo con l'uso stesso delle tavole solo per una selezione di astronomi. Verranno mostrati nel dettaglio metodi e algoritmi.

Complessivamente è stato svolto un lavoro preliminare sulle tavole che ci permette di trarre solo alcune conclusioni e suggerimenti per ulteriori indagini.

Una prima versione di questo lavoro è stata preparata nell'ambito del progetto ERC ALFA (CoG 723085).

Keywords: history, history of astronomy, astronomy, astronomical tables, equation of time, sun, sun motion, solar day, mean day, Ptolemy, al-Khwārizmī, al-Battānī, Jean de Murs, Peter of Saint Omer, Jean de Lignères, Levi ben Gerson, John Holbroke, Giovanni Bianchini, Georg Puerbach, Abraham Zacut.

A SURVEY ON THE EVOLUTION OF THE EQUATION OF TIME DURING THE CENTURIES

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June 2022

Abstract

Throughout time not very many astronomical tables kept evolving. However, some did, as is the case of the table for the equation of time.

In the ALFA project we will offer a general survey of the evolution of the equation of time through different traditions, starting from Greece arriving to the pre-Copernican period. We have some indication, in the primary sources, on how the equation of time table was recomputed but only few secondary literature, such as editions or translations of the primary sources, is available. In particular we have a secondary literature on Ptolemy (II century) and on Levi ben Gerson (XIV century) that we used as a guide and as a term of comparison for the algorithm and for the quality of the results. In this survey Alfonsine astronomy provides examples of the equation of time, which will be the object of our research. We will show the mathematical analyses on some equation of time tables already recomputed and on others never investigated before. We started from Ptolemy and then, following a chronological order, we analyzed al-Battānī, al-Khwārizmī, John of Murs, Peter of Saint Omer, John of Lignères, Levi ben Gerson, John Holbroke, Giovanni Bianchini, Georg Puerbach and Abraham Zacut.

At the beginning we started with a modern recomputation, then we passed to a recomputation by the use of tables only for a selection of astronomers.

Methods and algorithms will be shown in the following sections.

This is a preliminary work on tables that allow us to draw only some conclusion and some suggestion for further investigation.

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Contents

1 The Equation of Time **13**

2 The Algorithm **15**
2.0.1 Recomputation by modern method 17
2.0.2 Recomputation by the use of Tables 17

3 Ptolemy **18**
3.1 Overview 18
3.1.1 The Almagest 18
3.1.2 The Handy Tables 20
3.2 Presentation of the table 21
3.3 Analysis of the Table 30
3.3.1 Results for the recomputation by modern method 30
3.3.2 Recomputation by the use of Tables 31
3.4 Conclusions 42
3.5 List of Manuscripts 42

4 Al-Khwārizmī **43**
4.1 Overview 43
4.1.1 Sindhind Zīj 43
4.2 Presentation of the table 44
4.3 Analyses of the Table 45
4.3.1 Results by the use of modern method 45
4.3.2 Results by the use of Tables 46
4.4 Conclusions 56
4.5 List of Manuscripts 56

5 Al-Battānī **57**
5.1 Overview 57
5.1.1 Sābi'Zīj 57
5.2 Presentation of the table 57
5.3 Analyses of the table 64
5.3.1 Recomputation by modern method 64
5.3.2 Recomputation by the use of tables 65
5.4 Conclusions 75
5.5 List of Manuscripts 75

6 John of Murs **75**
6.1 Overview 76
6.2 Presentation of the table 76
6.3 Analyses of the table 79
6.3.1 Recomputation by the use of tables 86
6.4 Conclusions 93
6.5 List of Manuscripts 93

7 Peter of Saint Omer & John of Lignères **94**
7.1 Overview . . . 94
7.1.1 Peter of Saint Omer . . . 94
7.1.2 John of Lignères . . . 94
7.2 Presentation of the table . . . 94
7.3 Analyses of the table . . . 97
7.3.1 Recomputation by modern method . . . 97
7.3.2 Recomputation by the use of tables . . . 97
7.4 Conclusions . . . 107
7.5 List of Manuscripts . . . 107

8 Levi ben Gerson **108**
8.1 Overview . . . 108
8.2 Presentation of the table . . . 109
8.3 Vatican Library . . . 111
8.3.1 Vatican, Latin, 3380 . . . 111
8.3.2 Vatican Latin, 3098 . . . 112
8.4 Analyses of the table . . . 112
8.4.1 Recomputation by the use of tables . . . 112
8.5 Conclusions . . . 122
8.6 List of Manuscripts . . . 122

9 John Holbroke **123**
9.1 Overview . . . 123
9.1.1 Egerton 889 . . . 123
9.2 Presentation of the table . . . 123
9.3 Analyses of the table . . . 124
9.3.1 Recomputation by modern method . . . 124
9.4 Conclusions . . . 134
9.5 List of Manuscripts . . . 134

10 Giovanni Bianchini **134**
10.1 Overview . . . 134
10.2 Presentation of the table . . . 135
10.3 Analyses of the table . . . 140
10.3.1 Recomputation by modern method . . . 140
10.3.2 Recomputation by the use of tables . . . 141
10.4 Conclusions . . . 151
10.5 List of Manuscripts . . . 152

11 Georg Puerbach **152**
11.1 Overview . . . 152
11.2 Presentation of the Table . . . 152
11.3 Analyses of the table . . . 153
11.3.1 Recomputation by modern method . . . 153
11.4 Conclusions . . . 164

11.5 List of manuscripts . . . 164

12 Abraham Zacut **164**
12.1 Overview . . . 164
12.2 Presentation of the table . . . 165
12.3 Analyses of the table . . . 166
12.4 Recomputation by modern method . . . 166
12.5 Conclusions . . . 166
12.6 List of Manuscripts . . . 166

13 Topics For Further Research **167**

Appendices **169**

A Ptolemy **170**
A.1 Primary and secondary sources . . . 171
A.2 How to get the Equation of Time . . . 171
A.3 Analysis of the equation of time table by Benno van Dalen . . . 172
A.3.1 Mathematical Analysis . . . 173
A.3.2 Final Conclusions . . . 174
A.3.3 Some considerations on the table for the equation of time in the papyrus P. London 1278 . . . 175
A.3.4 Statistics . . . 176

B Al-Khwārizmī **177**
B.1 Primary and secondary sources . . . 178
B.2 Brief Description of some Set of Tables . . . 178
B.3 Suter's Survey . . . 179
B.4 Neugebauer's Survey . . . 179
B.5 Benno van Dalen's Survey . . . 179
B.5.1 Reconstruction of the Underlying Right Ascension and Solar Equation . . . 180
B.6 Conclusions . . . 182

C Al-Battānī **183**
C.1 Primary and secondary sources . . . 184
C.2 Ratdolt: Tabule Astronomice illustrissimi Alfontij regis castelle (1483) . . . 184
C.3 Nallino: Al-Battani Opus Astronomicum (1903) . . . 184
C.4 Pedersen: The Toledan Tables (2002) . . . 185
C.4.1 About Right Ascension and Equation of Time Table . . . 186

D John of Murs **187**
D.1 Primary and secondary sources . . . 188
D.2 Chabás and Goldstein: John of Murs revisited: the *Kalendarium Solis et Lune* for 1321 . . . 188
D.3 Chabás and Goldstein: John of Murs's tables of 1321 . . . 189

D.3.1 John of Murs's tables of 1321 189
D.3.2 Patefit 191

E Peter of Saint Omer & John of Lignères **192**
E.1 Primary and secondary sources 193
E.2 Peter of Saint Omer 193
E.3 John of Lignères 193

F Levi ben Gerson **195**
F.1 Primary and secondary sources 196
F.1.1 The equation of time's Table 196

G John Holbroke **197**
G.1 Primary and secondary sources 198

H Giovanni Bianchini **199**
H.1 Primary and secondary sources 200
H.2 Tabulae astronomiae 200

I Georg Puerbach **201**
I.1 Primary and secondary sources 202

J Abraham Zacut **203**
J.1 Primary and secondary sources 204
J.2 The equation of time table in the *Almanach perpetuum* 204

List of Tables

1 Standard deviations σ and averages E_M (given in seconds) . . . 32
2 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part I. 33
3 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part II. 34
4 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part III. 35
5 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part IV. 36
6 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part V. 37
7 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part VI. 38
8 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part VII. 39
9 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part VIII. 40

10 Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part IX. 41
11 Al-Khwārizmī: data from manuscript, recomputation and differences, part I. 47
12 Al-Khwārizmī: data from manuscript, recomputation and differences, part II. 48
13 Al-Khwārizmī: data from manuscript, recomputation and differences, part III. 49
14 Al-Khwārizmī: data from manuscript, recomputation and differences, part IV. 50
15 Al-Khwārizmī: data from manuscript, recomputation and differences, part V. 51
16 Al-Khwārizmī: data from manuscript, recomputation and differences, part VI. 52
17 Al-Khwārizmī: data from manuscript, recomputation and differences, part VII. 53
18 Al-Khwārizmī: data from manuscript, recomputation and differences, part VIII. 54
19 Al-Khwārizmī: data from manuscript, recomputation and differences, part IX. 55
20 Al-Battānī: data from manuscript, recomputation and differences, part I. 66
21 Al-Battānī: data from manuscript, recomputation and differences, part II. 67
22 Al-Battānī: data from manuscript, recomputation and differences, part III. 68
23 Al-Battānī: data from manuscript, recomputation and differences, part IV. 69
24 Al-Battānī: data from manuscript, recomputation and differences, part V. 70
25 Al-Battānī: data from manuscript, recomputation and differences, part VI. 71
26 Al-Battānī: data from manuscript, recomputation and differences, part VII. 72
27 Al-Battānī: data from manuscript, recomputation and differences, part VIII. 73
28 Al-Battānī: data from manuscript, recomputation and differences, part IX. 74
29 John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in λ, part I. 87
30 John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in λ, part II. 88
31 John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in days with λ from MS Vat. Lat. 3116, part I. 89

32 John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in days λ from MS Vat. Lat. 3116, part II. 90
33 John of Murs data from manuscript, recomputation and differences for Kalendarium with λ from MS Vat. Lat. 3116. 91
34 Standard deviations σ and averages E_M (given in minutes of degrees) for our recomputation with the estimation of λ_A and E_0. . 97
35 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part I. 98
36 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part II. 99
37 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part III. 100
38 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part IV. 101
39 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part V. 102
40 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part VI. 103
41 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part VII. 104
42 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part VIII. 105
43 Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part IX. 106
44 Levi ben Gerson data from manuscript, recomputation and differences, part I. 113
45 Levi ben Gerson data from manuscript, recomputation and differences, part II. 114
46 Levi ben Gerson data from manuscript, recomputation and differences, part III. 115
47 Levi ben Gerson data from manuscript, recomputation and differences, part IV. 116
48 Levi ben Gerson data from manuscript, recomputation and differences, part V. 117
49 Levi ben Gerson data from manuscript, recomputation and differences, part VI. 118
50 Levi ben Gerson data from manuscript, recomputation and differences, part VII. 119
51 Levi ben Gerson data from manuscript, recomputation and differences, part VIII. 120
52 Levi ben Gerson data from manuscript, recomputation and differences, part IX. 121
53 John Holbroke data from manuscript, recomputation and differences, part I. 125

54 John Holbroke data from manuscript, recomputation and differences, part II. . . . 126
55 John Holbroke data from manuscript, recomputation and differences, part III. . . . 127
56 John Holbroke data from manuscript, recomputation and differences, part IV. . . . 128
57 John Holbroke data from manuscript, recomputation and differences, part V. . . . 129
58 John Holbroke data from manuscript, recomputation and differences, part VI. . . . 130
59 John Holbroke data from manuscript, recomputation and differences, part VII. . . . 131
60 John Holbroke data from manuscript, recomputation and differences, part VIII. . . . 132
61 John Holbroke data from manuscript, recomputation and differences, part IX. . . . 133
62 Standard deviations σ and averages E_M given in seconds of time for our recomputation with the estimation of λ_A and E_0. . . . 141
63 Giovanni Bianchini: data from manuscript, recomputation and differences, part I. . . . 142
64 Giovanni Bianchini: data from manuscript, recomputation and differences, part II. . . . 143
65 Giovanni Bianchini: data from manuscript, recomputation and differences, part III. . . . 144
66 Giovanni Bianchini: data from manuscript, recomputation and differences, part IV. . . . 145
67 Giovanni Bianchini: data from manuscript, recomputation and differences, part V. . . . 146
68 Giovanni Bianchini: data from manuscript, recomputation and differences, part VI. . . . 147
69 Giovanni Bianchini: data from manuscript, recomputation and differences, part VII. . . . 148
70 Giovanni Bianchini: data from manuscript, recomputation and differences, part VIII. . . . 149
71 Giovanni Bianchini: data from manuscript, recomputation and differences, part IX. . . . 150
72 Georg Puerbach: data from manuscript, recomputation and differences, part I. . . . 154
73 Georg Puerbach: data from manuscript, recomputation and differences, part II. . . . 155
74 Georg Puerbach: data from manuscript, recomputation and differences, part III. . . . 156
75 Georg Puerbach: data from manuscript, recomputation and differences, part IV. . . . 157
76 Georg Puerbach: data from manuscript, recomputation and differences, part V. . . . 158

77 Georg Puerbach: data from manuscript, recomputation and differences, part VI. . . . 159
78 Georg Puerbach: data from manuscript, recomputation and differences, part VII. . . . 160
79 Georg Puerbach: data from manuscript, recomputation and differences, part VIII. . . . 161
80 Georg Puerbach: data from manuscript, recomputation and differences, part IX. . . . 162

List of Figures

1 Ptolemaic eccentric model for the Sun. . . . 13
2 True and Mean Sun. . . . 15
3 Astronomical parameters. . . . 17
4 Ptolemy's equation of time table in the Tihon's Edition (part I). 22
5 Ptolemy's equation of time table in the Tihon's Edition (part II). 23
6 Ptolemy's equation of time table in the Tihon's Edition (part III). 24
7 Ptolemy's equation of time table in the Tihon's Edition (part IV). 25
8 Ptolemy's equation of time table in the Stahlman's thesis (part I). 26
9 Ptolemy's equation of time table in the Stahlman's thesis (part II). . . . 27
10 Ptolemy's equation of time table in the Stahlman's thesis (part III). . . . 28
11 Ptolemy's equation of time table in the Stahlman's thesis (part IV). . . . 29
12 Ptolemy's equation of time in the *Handy Tables* as a function of λ. . . . 30
13 Sample of our Excel file. . . . 31
14 Ptolemy's equation of time differences, given in seconds. . . . 42
15 Al-Khwārizmī's equation of time table, part I. . . . 44
16 Al-Khwārizmī's equation of time table, part II. . . . 45
17 Al-Khwārizmī's equation of time differences, given in seconds. . 56
18 Al-Battānī's equation of time table part I in Nallino. . . . 58
19 Al-Battānī's equation of time table part II in Nallino. . . . 59
20 Al-Battānī's equation of time table part III in Nallino. . . . 60
21 Al-Battānī's equation of time table part IV in Nallino. . . . 61
22 Al-Battānī's equation of time table part I in Ratdolt. . . . 62
23 Al-Battānī's equation of time table part II in Ratdolt. . . . 63
24 Al-Battānī's equation of time table part III in Ratdolt. . . . 64
25 Al-Battānī's equation of time differences, given in minutes of degrees. Table shifted by 90 degrees for the recomputation. . . . 75
26 John of Murs' equation of time table in Kalendarium. . . . 77
27 John of Murs' equation of time, in Tables of 1321, with the argument given in days of the year. . . . 78

28 John of Murs' equation of time, in Tables of 1321, with λ as argument. 79
29 Verus locus, for the month of January, from MS Vat. Lat. 3116. 80
30 Verus locus, for the month of February, from MS Vat. Lat. 3116. 81
31 Verus locus, for the month of March, from MS Vat. Lat. 3116. . 81
32 Verus locus, for the month of April, from MS Vat. Lat. 3116. . 82
33 Verus locus, for the month of May, from MS Vat. Lat. 3116. . . 82
34 Verus locus, for the month of June, from MS Vat. Lat. 3116. . . 83
35 Verus locus, for the month of July, from MS Vat. Lat. 3116. . . 83
36 Verus locus, for the month of August, from MS Vat. Lat. 3116. 84
37 Verus locus, for the month of September, from MS Vat. Lat. 3116. 84
38 Verus locus, for the month of October, from MS Vat. Lat. 3116. 85
39 Verus locus, for the month of November, from MS Vat. Lat. 3116. 85
40 Verus locus, for the month of December, from MS Vat. Lat. 3116. 86
41 John of Murs' equation of time differences, given in seconds, in *Tables of 1321 in* λ. 92
42 John of Murs' equation of time differences, given in seconds, in *Tables of 1321 in days*. 92
43 John of Murs' equation of time differences, given in minutes of time, in *Kalendarium*. 93
44 Equation of time table in Vatican, Biblioteca Apostolica, MS Pal. Lat. 1412, 101v 96
45 Peter of St. Omer/John of Lignères' equation of time differences, given in minutes of degree. 107
46 Levi ben Gerson's equation of time in Paris, hebr. 724, 117b and Paris, hebr. 725, 91a. 110
47 Levi ben Gerson's Tabula equationis solis ultimata, Vat. Lat. 3380, 152r. 111
48 Levi ben Gerson's equation of time differences given in minutes of degree. 122
49 John Holbroke's equation of time in Egerton 889. 124
50 John Holbroke's equation of time differences, given in minutes of degrees. 134
51 Giovanni Bianchini's equation of time table part I in Edition 1526. 136
52 Giovanni Bianchini's equation of time table part II in Edition 1526. 137
53 Giovanni Bianchini's equation of time table in Pal. Lat. 1375, 258v. 138
54 Giovanni Bianchini's equation of time table in Pal. Lat. 1375, 90r. 139
55 Giovanni Bianchini's equation of time differences, given in seconds. 140
56 Giovanni Bianchini's equation of time differences, given in seconds. 151
57 Georg Puerbach's equation of time in Tabulae eclipsium. 153

58 Georg Puerbach's equation of time differences, given in seconds. 163
59 Differences between Georg Puerbach and Levi ben Gerson (multiplied by 4)'s equation of time tables, given in minutes. 163
60 Abraham Zacut's equation of time in Almanach perpetuum. . . 165
61 Abraham Zacut's equation of time differences, given in minutes. 166

1 The Equation of Time

In ancient and medieval times the system used was geocentric with the Sun moving along an orbit called ecliptic. Ptolemy described this system in the *Almagest* and it was in use until the sixteenth century.

In Fig.1 we can see the *eccentric* model for the Sun [1][Campanus of Novara 1971, p. 42] where

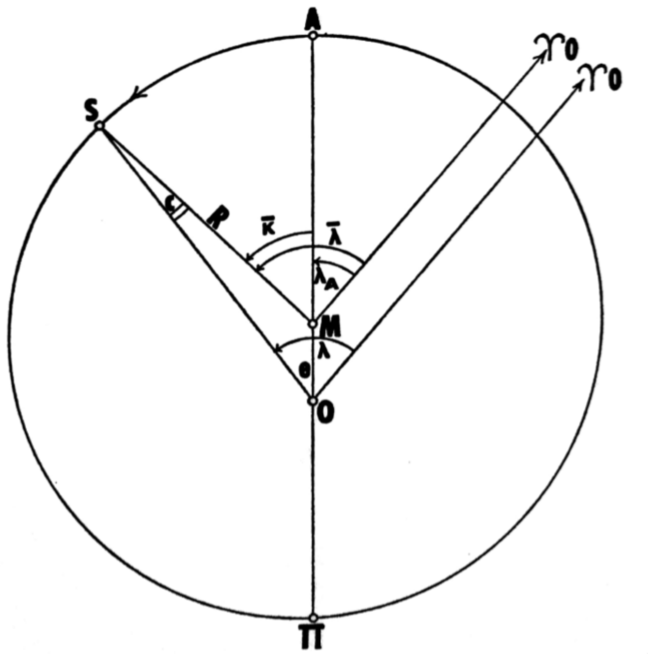


Figure 1: Ptolemaic eccentric model for the Sun.

- O is the observer on the Earth;
- S is the Sun moving on the eccentric with uniform velocity about its center M;
- M is the center of the eccentric (that is the ecliptic);
- the distance MO is called e that is the *eccentricity*;
- MS is the radius R of the eccentric circle;
- A is the *apogee* (*aux*);
- Π is the *perigee* (*augis*);

- λ is the *true longitude*, the position of the Sun measured at O and it is the angular distance along the ecliptic from the vernal equinox Ari 0° indicated with γ_0. It indicates the position of the true Sun;

- $\bar{\lambda}$ is the *mean longitude*, the position of the Sun measured at M and it is the angular distance along the ecliptic from the vernal equinox Ari 0° indicated with γ_0. It indicates the position of the mean Sun;

- λ_A is the longitude of the apogee;

- the angle AMS is the *mean anomaly* $\bar{\kappa}$ (*argumentum medium*);

- the angle MSO is the *solar equation* $c(\bar{\kappa})$ (*equatio*);

To calculate the true solar longitude we need to know the ratio $e : R$, λ_A (considered tropically fixed), the mean motion (*medius motus*) that is the rate at which $\bar{\lambda}$ increases and the "epoch" or initial constant E_0, (see next section). With these parameters we can compute $c(\bar{\kappa})$. The mean anomaly (*argumentum solis*) is given by $\bar{\kappa} = \bar{\lambda} - \lambda_A$ and the true longitude is given by $\lambda = \bar{\lambda} \pm c(\bar{\kappa})$ where we must subtract the solar equation for $0° \leq \bar{\kappa} \leq 180°$ and we must add it for $180° \leq \bar{\kappa} \leq 360°$.

The annual solar motion along the ecliptic is not uniform because of the variation of the speed of the Sun along its orbit according the Kepler's Second Law and because of the obliquity of the ecliptic ε (the inclination between the ecliptic and the celestial equator). The consequence is that the true solar day is not constant during the year. The ancients were aware about that and they introduced a mean ecliptical Sun (moving along the ecliptic but with uniform speed) to define a mean time based on a mean day of a constant length. The true Sun reaches the maximum angular speed along the ecliptic at the perigee, the minimum speed at the apogee (Kepler's Second Law). The mean ecliptical Sun, that moves with constant speed along the ecliptic, eliminates the irregularity of the true Sun. The mean equatorial Sun moves along the celestial equator at constant speed and crosses the vernal point at the same moment together with the mean ecliptical Sun. The equatorial mean Sun gives a correction to the irregularity of the Sun motion due to the fact that it does not moves along the equator. With the equatorial mean Sun we can define the mean solar day and the mean solar time.

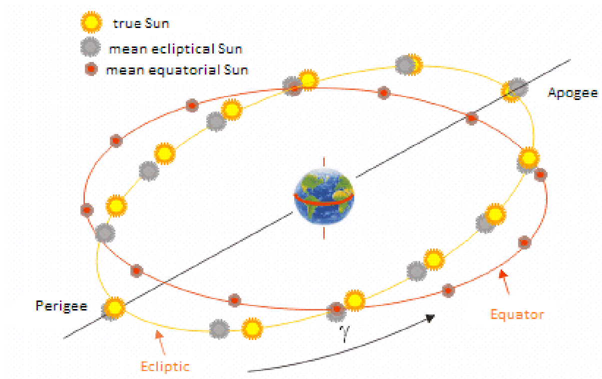


Figure 2: True and Mean Sun.

The difference between the true solar day (the time between two consecutive meridian transits of the Sun that the ancients could read from a sundial) and the mean solar day (the day counted from noon to noon) is the equation of time[2][Neugebauer 2004, Vol. 1, p. 61].

The difference is at most around 30 minutes but it is important to take it into account in the calculation of the time of an eclipse in which we need the exact position of the Moon, for example.

The conversion from true solar time to mean solar time (or vice versa) is given by the addition or by the subtraction of the equation of time.

The Arabic name was *ta'dīl al-'ayyām bilayālīhā* (correction of the days with their nights) translated then into Latin as *equatio dierum cum noctibus suis*. In many ancient handbooks and manuscripts we can find a table for the equation of time as a function of the true solar position or, sometimes, of the mean solar position. Discovering the argument used is one of the aim of this research project. We have also to consider the annual effects (the Sun, after one year, do not come back exactly at the vernal point but in advance because of the precession of the equinoxes).

2 The Algorithm

Our knowledge of the methods of computation used in ancient and medieval astronomical tables is limited and most of the times we do not know the values of the astronomical parameters underlying such tables. The algorithm to recompute a table and their relatives underlying parameters are specific to the astronomer who created the table. Trying to find them out is a way to determine the origin of the table itself and a way to define the astronomical and mathematical knowledge belonging to the period in which that astronomer lived.

The choice of the algorithm is made according to primary and secondary sources on Ptolemy and on Levi ben Gerson. More precisely we followed some indication given by Benno van Dalen's analyses [3][van Dalen 1993, p. 104] on Ptolemy and by Goldstein's analyses [4] on Levi ben Gerson, making some adjustment in the procedure and in the choice of the underlying parameters.

For the Alfonsine period we focused our research on Peter of Saint Omer, as an example of a table with entries given to the minutes of degrees, on Giovanni Bianchini, as an example of a table with entries given to the seconds of time and of John of Murs that presents the equation of time table in three different ways as we will see in the dedicated section.

We recompute equation of time, as a function of the true solar longitude, according to formula

$$E(\lambda) = \frac{1}{D}(\alpha(\lambda) - \lambda \pm c(\lambda) + E_0) \tag{1}$$

and in the case of the mean solar longitude we use

$$\bar{E}(\bar{\lambda}) = \frac{1}{D}(\alpha(\bar{\lambda} \pm \bar{c}(\bar{\lambda})) - \bar{\lambda} + E_0). \tag{2}$$

where

- λ is the true solar longitude;
- $\bar{\lambda}$ is the mean solar longitude;
- $c(\lambda)$ is the solar equation as a function of λ linked to the true anomaly by the relation $\kappa = \lambda - \lambda_A$ where λ_A is the solar apogee;
- $\bar{c}(\bar{\lambda})$ is the solar equation as a function of $\bar{\lambda}$ and it is linked to the mean solar anomaly by the relation $\bar{\kappa} = \bar{\lambda} - \lambda_A$;
- $\alpha(\lambda)$ is the right ascension as a function of λ;
- $\alpha(\bar{\lambda})$ is the right ascension as a function of $\bar{\lambda}$;
- E_0 is the initial constant, calculated for the minimum of the equation of time; it is the starting point for the tables of planetary motion and it is used to make all the equation of time values positive;
- D is the conversion factor: it is equal to 15 ($^\circ$/h) (because $24^h = 360^\circ$) if we want to express the equation of time in hours, otherwise D is taken to be equal to 1, if we want to express it in degrees.

All the quantities are expressed in degrees.

In modern astronomy this initial constant E_0 is taken to be zero but in ancient astronomers used to fix an "epoch" in order to determine it.

An accurate conversion factor is $(360; 0 + 0; 59, 8)/24 \approx 15; 2, 28^\circ$ because the daily motion of the Sun on the ecliptic is about $0; 59, 8, 17, 13, 12, 31^\circ$/day but we use 15 in our calculations, all the details of our choices will be given in the following sections.

In Fig. 3 we can see some astronomical parameters listed above:

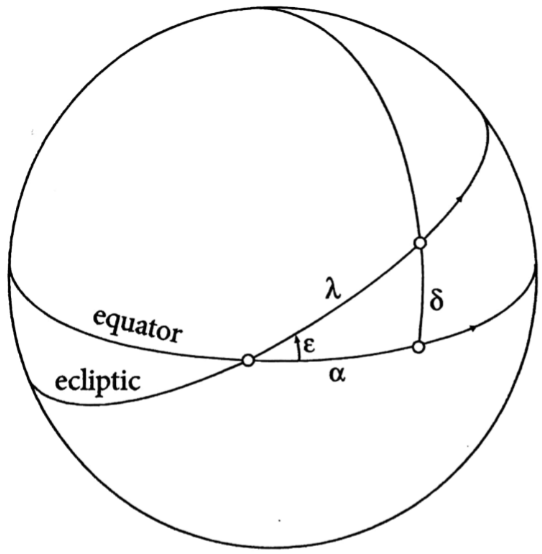


Figure 3: Astronomical parameters.

where ε is the obliquity of the ecliptic and δ is the solar declination.

2.0.1 Recomputation by modern method

In our first analyses we simply recomputed the equation of time, for all the astronomers listed in the introduction, according to the modern formulas [5][van Dalen 2013, p. 102]:

$$c(\lambda) = \arcsin\left(\frac{e}{60}\sin(\lambda - \lambda_A)\right) \tag{3}$$

where e is the solar eccentricity given in parts.

For the right ascension we use, for $\lambda \in [0°, 90°]$ [5][van Dalen 2013, p. 102]:

$$\alpha(\lambda) = \arctan(\cos\varepsilon \tan\lambda) \tag{4}$$

and for $\lambda \in (90°, 360°]$ we use, for reasons of symmetry, $\alpha(180° - \lambda) = 180° - \alpha(\lambda)$ and $\alpha(180° + \lambda) = 180° + \alpha(\lambda)$.

We perform a least square estimation with a software created by Benno van Dalen in order to find out the astronomical parameters and we made some historical considerations in order to make a final choice for our set of parameters.

2.0.2 Recomputation by the use of Tables

Here we provide the procedure for our recomputation using the right ascension table and the solar equation table available in the primary sources.

The solar equation table is given as a function of the mean solar anomaly $\bar{\kappa}$ and we proceed in two different ways:

- by approximating the original table $c(\bar{\kappa})$ as a function of the true solar anomaly κ;
- by performing linear interpolation from the original table to reconstruct $c(\kappa)$ using the relation $\kappa = \bar{\kappa} \mp c(\bar{\kappa})$ to change the argument.

After this choice, in both cases we have to perform a linear interpolation in order to apply the shift due to the solar apogee.

In our Excel file we insert the solar equation table with the mean anomaly $\bar{\kappa} = \bar{\lambda} - \lambda_A$ as argument from 1° to 180° and the entries are given in degrees and minutes of degree. We remind that we must subtract the solar equation for $0° \leq \bar{\kappa} \leq 180°$ and we must add it for $180° \leq \bar{\kappa} \leq 360°$ and that the following symmetry holds: $c(360° - \bar{\kappa}) = -c(\bar{\kappa})$.

Then we insert the right ascension table, reminding that the right ascension tabulated is the *normed right ascension*:

$$\alpha' = \alpha + 90°. \tag{5}$$

we choose a value for E_0 (by least mean square, by primary sources when available or by "hand computation"), we recompute each entry of the equation of time table by Eq. 1 if the argument is λ or by Eq. 2 if the argument is $\bar{\lambda}$.

3 Ptolemy

The Greek scientist Ptolemy lived in the II century in Alexandria of Egypt, he was the greatest astronomer of antiquity and he influenced all the other astronomical traditions after him. His main works are the *Almagest*, based on the Era Nabonassar (Year 1 of Era Nabonassar, month Thoth of the Egyptian calendar, day 1 corresponds to Feb. 26, 747 BC), and the *Handy Tables*, based on Era Philip (year 1 of the Era Philip, month Thoth, day 1 corresponds to Nov. 12, 323 BC). We remind that at the time chronology was important to report observations that were given with a date in some calendar [6][Chabás and Goldstein 2012, p.13].

The equation of time is tabulated only in the *Handy Tables*, probably for the first time, but in the *Almagest* we have some indications on how was computed: for example the tables which use the Era Nabonassar have the value zero of the equation of time in Aquarius. From this information we know that the Era used for the computation of the equation of time table is not the Era Philip, typical for the *Handy Tables*, but indeed the Era Nabonassar.

3.1 Overview

3.1.1 The Almagest

Ptolemy [7][Toomer 1984, pp. 131-172] talks about the length of the year intended as the return of the Sun to the same equinox or solstice and he finds that it exceeds 365 days by less than a quarter day but its return to one of the fixed stars is greater than 365 1/4 days [7][Toomer 1984, p. 131]. Hipparcus believed that the fixed stars had a slow motion towards the rear with respect to the revolution of the daily motion but Ptolemy said that this suspicion derived from some mistaken observations he did so he concludes that the length of the year is constant. Also from the observations of Hipparcus he finds that the solstices and equinoxes occur earlier than 1/4 days by one day in 300 years [Toomer 1984, p. 137] [7]. The closest approximation from data for the length of the year is 365;14,48 days, the daily motion of the Sun is $0;59,8,17,13,12,31°$ and the hourly motion is $0;2,27,50,43,3,1°$ [7][Toomer 1984, p. 140]. Then Ptolemy demonstrates the apparent anomaly of the Sun explaining that the motion of bodies is uniform and circular by two hypotheses: that the uniform motion takes place on a circle that is not concentric with the Universe or on another circle carried by the first circle (the *epicycle*) [7][Toomer 1984, p. 141]. In order to have a correction for any given position he established a table where the argument is the mean anomaly and the function is the equation of the anomaly that corresponds to the arc of mean motion; he divided the quadrants near the apogee into 15 subdivisions (each one of 6°) and the quadrants near the perigee into 30 subdivisions (each one of 3°) [7][Toomer 1984, pp. 165-166] because of the difference between two equations of anomaly are greater near the perigee than near the apogee. He found for the epoch in mean motion in the first year of Nabonassar, Thoth I in the Egyptian calendar, noon, that the Sun's distance in mean motion is $265;15°$ to the rear of the apogee; its mean position is Psc $0;45°$ (literally 45 minutes of the first degree of Pisces) [7][Toomer 1984, p. 169].

Ptolemy also said about the inequality of the solar day. A solar day is the return of the Sun from one point on the horizon or on the meridian to the same point so it is the period comprising the passage of the 360 time-degree of one revolution of the equator plus 0;59 time-degrees (that is the amount of the mean motion of the Sun during that period); an anomalistic solar day is the period comprising the passage of the 360 time-degree of one revolution of the equator plus that stretch of the equator which rises with (or crosses the meridian with) the anomalistic motion of the Sun [7][Toomer 1984, p. 170]. The stretch cannot be a constant because of the Sun's apparent anomaly and because equal sections of the ecliptic do not cross the horizon or the meridian in equal times. The greatest accumulated difference occurs between the two positions of the Sun where its true speed equals its mean speed. The Sun's anomalistic position is about Psc $3;8°$ [7][Toomer 1984, p. 172].

He established the beginning of solar day from the meridian-crossing of the Sun because if we calculate it from its rising or setting the time difference with respect to the horizon it can reach several hours and it is not the same everywhere because of the latitudes, whereas the time difference with respect to the meridian is the same everywhere on the Earth.

The true solar day is defined as a day counted from noon to noon or from midnight to midnight.

3.1.2 The Handy Tables

Stahlman, in his PhD thesis in 1959 [8][The Astronomical Tables of Codex Vaticanus Graecus 1291], explained the main differences of right ascension tables, between the *Almagest* and the *Handy Tables*. First he introduced the concept of *ascension* or *rising time*, the *right ascension* and the configuration of *right sphere*, the *oblique ascension* and the configuration of *oblique sphere*. In the *Almagest* some problems, such as the length of the day and the seasonal hour, were solved thorough the use of ascension tables but in the *Handy Tables* the length of the day is computed by the use of separate tables in order to simplify the procedure of calculus.

In the *Almagest* we find, for the ascensional arcs for the sphaera recta, a first column for the times of rising of each decan (intervals of $10°$) of every zodiacal sign (in grades and minutes); the second column is a list of the cumulative total times starting from Ari $1°$.

In the *Handy Tables* the argument is listed in intervals of 1 degree in order to eliminate some interpolation. The column of total times for all latitudes are normed and begins with Ari $1°$ but for the sphaera recta the counting is renormed so that starts with Cap $1°$. In the *Handy Tables* so we have the introduction of a new right ascension such that $\alpha' = \alpha + 90°$, in this way the right ascension of a culminating point equals the oblique ascension of the rising point and this reduces the calculations. There are also two more columns: one in the table for sphaera recta (the equation of time as a function of the solar longitude) and one for the tables of oblique ascension (each climate in equatorial degrees and minutes as a function of the solar longitude).

The critical edition and the mathematical analysis of the Handy Tables have been taken up by Tihon and Mercier [9][Ptolemaiou procheiroikanones by Tihon and Mercier] in a project that will include six volumes. Only the first volume, divided in two parts (volume 1a by Tihon for the philological part and volume 1b by Mercier for the mathematical part), is available.

In volume 1a we find the purpose of the "Handy Tables" that consists of 22 tables: 20 astronomical, one geographical and one chronological. We can find right and oblique ascension tables. In this volume we find the reproduction of the tables from one manuscript in Florence, Biblioteca Medicea Laurenziana, MS 28/26.

In volume 1b we find a transcription of right and oblique ascension tables for the seven climates an oblique ascension from Byzantium (not belonging to the original Handy Tables). We find that in the Handy Tables the epoch is the era of Philip and not the era of Nabonassar as for the Almagest. Mercier found that the value for the solar longitude of the apogee ins $66°$ instead of $65;30°$ in the Almagest.

A History of Ancient Mathematical Astronomy by Neugebauer [2] is a book in 3 volumes on mathematical astronomy, numerical, geometrical and graphical methods to explain the planetary system.

We know that there is no equation of time table in the *Almagest* but in [2] we find some indication on how it was computed.

In [2][vol 2, pp. 948-949] we find out, in an anonymous treatise preserved in Cod. Par. gr. 2841 which contains a commentary of the unknown Artemidoros, that there is a recomputation of the true longitudes of the sun for Nabonassar 958X28 (=A.D.211 Apr. 25) and the results obtained for the sun in the *Handy Tables* are $0;2°$ ahead the ones in the *Almagest* and this discrepancy is caused by the equation of time for an inaccurate rounding. In the *Handy Tables* the table for solar declination are presented with and interval of 3 degrees for the argument and are calculated from solstice to solstice. We find the value of $\varepsilon = 23;51°$ for the obliquity of the ecliptic.

3.2 Presentation of the table

The original table is in Greek. In the first column we find, as argument, the true solar longitude λ given in intervals of one degree. For the entries we have 2 columns for each zodiacal sign (the table begins at Cap 1°): one for the right ascension α, expressed in degrees, and one for the equation of time, given in time to minutes and seconds. The highest maximum is 0;33,23h at Aqr 18° and the other maximum is 0;16,21h at Leo 9°, the lowest minimum is 0;0,0h at Lib 30°- Sco 3° and the other minimum is 0;6,12h at Tau 30°.

Here we can see, in Fig. 4, 5, 6 and 7, the table reproduced by Tihon and Mercier [9][Vol. 1 a, pp. 97-100] (Laurentianus Graecus, ff.55r-56v, transcription on pag.10-13, Vol. 1b):

A1 (I) : ascensions de la sphère droite (F, f. 55)

Tit : ὀρθὴ σφαῖρα σὺν μεσουρανήματι πανταχῇ[2].

♂	Αἰγοκέρου				Ὑδριχόου				Ἰχθύων			
μοιρ αι	ἀναφορ μ°	ξ′	ωρ χρ ξ ξ′	ξ′	ἀναφορ μ°	ξ′	ωρ χρ ξ ξ′	ξ′	ἀναφορ μ°	ξ′	ωρ χρ ξ ξ ξ′	
α	α	ς	ιη	μ	λγ	ιη	λα	ις	ξγ	ζ	λβ	ιγ
β	β	ιβ	ιθ	ια	λδ	κ	λα	κη	ξδ	δ	λβ	ō
γ	γ	ιη	ιθ	ιβ	λε	κβ	λα	μ	ξε	α	λα	μζ
δ	δ	κδ	κ	ιγ	λς	κδ	λα	νβ	ξε	νη	λα	λγ
ε	ε	λ	κ	μδ	λζ	κς	λβ	δ	ξς	νε	λα	κ
ς	ς	λε	κα	ιε	λη	κη	λβ	ιε	ξζ	να	λα	ς
ζ	ζ	μ	κα	με	λθ	κθ	λβ	κδ	ξη	μζ	λ	ν
η	η	με	κβ	ιδ	μ	λ	λβ	λβ	ξθ	μγ	λ	λδ
θ	θ	ν	κβ	μδ	μα	λα	λβ	μα	ο	λθ	λ	ιη
ι	ι	νε	κγ	ιγ	μβ	λβ	λβ	με	οα	λε	λ	β
ια	ιβ	ō	κγ	μβ	μγ	λβ	λβ	νη	οβ	λα	κθ	μς
ιβ	ιγ	ε	κδ	ια	μδ	λβ	λγ	ς	ογ	κζ	κθ	λ
ιγ	ιδ	ι	κδ	λη	με	λβ	λγ	θ	οδ	κγ	κθ	ια
ιδ	ιε	ιε	κε	δ	μς	λβ	λγ	ιβ	οε	ιθ	κη	να
ιε	ις	κ	κε	λα	μζ	λβ	λγ	ιε	ος	ιε	κη	λβ
ις	ιζ	κε	κε	νζ	μη	λβ	λγ	ιη	οζ	ι	κη	ιβ
ιζ	ιη	λ	κς	κδ	μθ	λβ	λγ	κ	οη	ε	κζ	νγ
ιη	ιθ	λδ	κς	ν	ν	λβ	λγ	κγ	οθ	ō	κζ	λγ
ιθ	κ	λη	κζ	ιγ	να	λα	λγ	κ	οθ	νε	κζ	ια
κ	κα	μβ	κζ	λς	νβ	λ	λγ	ιζ	π	ν	κς	μθ
κα	κβ	μς	κζ	νη	νγ	κη	λγ	ιδ	πα	με	κς	μζ
κβ	κγ	ν	κη	κ	νδ	κς	λγ	ιδ	πβ	μ	κς	ε
κγ	κδ	νδ	κη	μβ	νε	κδ	λγ	η	πγ	λε	κε	μγ
κδ	κε	νη	κθ	δ	νς	κβ	λγ	ε	πδ	λ	κε	κα
κε	κζ	α	κθ	κδ	νζ	κ	λβ	νθ	πε	κε	κδ	νη
κς	κη	δ	κθ	μδ	νη	ιη	λβ	νβ	πς	κ	κδ	λδ
κζ	κθ	ζ	λ	δ	νθ	ις	λβ	μδ	πζ	ιε	κδ	ια
κη	λ	ι	λ	κδ	ξ	ιδ	λβ	λε	πη	ι	κγ	μη
κθ	λα	ιγ	λ	μδ	ξα	ιβ	λβ	λγ	πθ	ε	κγ	κδ
λ	λβ	ις	λα	δ	ξβ	ι	λβ	κς	Ϛ	ō	κγ	ō
	λβ	ις			κθ	νδ			κζ	ν		

Figure 4: Ptolemy's equation of time table in the Tihon's Edition (part I).

A1 (II) : Ascensions de la sphère droite (F, f. 55v)

Tit : ὀρθὴ σφαῖρα σὺν μεσουρανήματ(ι)[3] πανταχή.[4]

σ'	Κριοῦ				Ταύρου				Διδύμων			
μ°	ἀναφ		ω° χ° ξ ξ		ἀναφ		ω° χ° ξ ξ		ἀναφ		ω° χ° ξ ξ	
α	ϟ	νε	κβ	λς	ριη	μη	ι	νζ	ρμη	μζ	ς	ις
β	ϟα	ν	κβ	ια	ριθ	μς	ι	μ	ρμθ	ν	ς	ιθ
γ	ϟβ	με	κα	μζ	ρκ	μδ	ι	κδ	ρν	νγ	ς	κγ
δ	ϟγ	μ	κα	κβ	ρκα	μβ	ι	ζ	ρνα	νς	ς	κς
ε	ϟδ	λε	κ	νη	ρκβ	μ	θ	να	ρνβ	νθ	ς	λ
ς	ϟε	λ	κ	λγ	ρκγ	λη	θ	λε	ρνδ	γ	ς	λδ
ζ	ϟς	κε	κ	θ	ρκδ	λς	θ	κα	ρνε	ς	ς	λθ
η	ϟζ	κ	ιθ	μδ	ρκε	λδ	θ	ς	ρνγ	ι	ς	μδ
θ	ϟη	ιε	ιθ	ιθ	ρκς	λβ	η	νβ	ρνζ	ιδ	ς	ν
ι	ϟθ	ι	ιη	νδ	ρκζ	λ	η	λζ	ρνη	ιη	ς	νε
ια	ρ	ε	ιη	κθ	ρκη	κθ	η	κγ	ρνθ	κβ	ζ	α
ιβ	ρα	ō	ιη	δ	ρκθ	κη	η	η	ρξ	κς	ζ	ς
ιγ	ρα	νε	ιζ	μ	ρλ	κη	ζ	νη	ρξα	λ	ζ	ιε
ιδ	ρβ	ν	ιζ	ις	ρλα	κη	ζ	μζ	ρξβ	λε	ζ	κδ
ιε	ργ	με	ις	να	ρλβ	κη	ζ	λζ	ρξγ	μ	ζ	λγ
ις	ρδ	μα	ις	κζ	ρλγ	κη	ζ	κζ	ρξδ	με	ζ	μβ
ιζ	ρε	λζ	ις	β	ρλδ	κη	ζ	ιζ	ρξε	ν	ζ	να
ιη	ρς	λγ	ιε	λη	ρλε	κη	ζ	ζ	ρξς	νε	η	ō
ιθ	ρζ	κθ	ιε	ις	ρλς	κη	ζ	α	ρξη	ō	η	ια
κ	ρη	κε	ιδ	νδ	ρλζ	κη	ς	νε	ρξθ	ε	η	κβ
κα	ρθ	κα	ιδ	λα	ρλη	κθ	ς	ν	ρο	ι	η	λγ
κβ	ρι	ιζ	ιδ	θ	ρλθ	λ	ς	μδ	ροα	ιθ	η	μδ
κγ	ρια	ιγ	ιγ	μς	ρμ	λα	ς	λθ	ροβ	κ	η	νς
κδ	ριβ	θ	ιγ	κδ	ρμα	λβ	ς	λγ	ρογ	κε	θ	η
κε	ριγ	ε	ιγ	β	ρμβ	λδ	ς	λ	ροδ	λ	θ	κ
κς	ριδ	β	ιβ	μα	ρμγ	λς	ς	κς[5]	ροε	λς	θ	λβ
κζ	ριδ	νθ	ιβ	ιθ	ρμδ	λη	ς	κγ	ρος	μβ	θ	με
κη	ριε	νς	ια	νζ	ρμε	μ	ς	ιθ	ροζ	μη	θ	νζ
κθ	ρις	νγ	ια	λε	ρμς	μβ	ς	ις	ροη	νδ	ι	ι
λ	ριζ	ν	ια	ιγ	ρμζ	μδ	ς	ιβ	ρπ	ō	ι	κγ
	κζ	ν			κθ	νδ			λβ	ις		

Figure 5: Ptolemy's equation of time table in the Tihon's Edition (part II).

A1 (III) : Ascensions de la sphère droite (F, f. 56)

Tit : ὀρθὴ σφαῖρα σὺν μεσουρανήματ(ι)[6] πανταχῆ.[7]

♂	Καρκίνου				Λέοντος				Παρθένου			
μº	ἀναφ		ωº χº ξ ξ		ἀναφ		ωº χº ξ ξ		ἀναφ		ωº χº ξ ξ	
α	ρπα	ς	ι	λς	σιγ	ιη	ιε	μβ	σμγ	ζ	ιγ	ιγ
β	ρπβ	ιβ	ι	μθ	σιδ	κ	ιε	μς	σμδ	δ	ιβ	νη
γ	ρπγ	ιη	ια	β	σιε	κβ	ιε	να	σμε	α	ιβ	μδ
δ	ρπδ	κδ	ια	ιε	σις	κδ	ιε	νε	σμε	νη	ιβ	κθ
ε	ρπε	λ	ια	κη	σιζ	κς	ιε	νθ	σμς	νε	ιβ	ιδ
ς	ρπς	λε	ια	μβ	σιη	κη	ις	γ	σμζ	να	ια	νθ
ζ	ρπζ	μ	ια	νε	σιθ	κθ	ις	θ	σμη	μζ	ια	με
η	ρπη	με	ιβ	ζ	σκ	λ	ις	ιε	σμθ	μγ	ια	λ
θ	ρπθ	ν	ιβ	κ	σκα	λα	ις	κα	σν	λθ	ια	ις
ι	ρϟ	νε	ιβ	λβ	σκβ	λβ	ις	ιε	σνα	λε	ια	α
ια	ρϟβ	ō	ιβ	με	σκγ	λβ	ις	θ	σνβ	λα	ι	μς
ιβ	ρϟγ	ε	ιβ	νζ	σκδ	λβ	ις	β	σνγ	κζ	ι	λα
ιγ	ρϟδ	ι	ιγ	η	σκε	λβ	ιε	νς	σνδ	κγ	ι	ιε
ιδ	ρϟε	ιε	ιγ	ιθ	σκς	λβ	ιε	μθ	σνε	ιθ	θ	νη
ιε	ρϟς	κ	ιγ	λ	σκζ	λβ	ιε	μβ	σνς	ιε	θ	μβ
ις	ρϟζ	κε	ιγ	μα	σκη	λβ	ιε	λε	σνζ	ι	θ	κε
ιζ	ρϟη	λ	ιγ	νβ	σκθ	λβ	ιε	κζ	σνη	ε	θ	θ
ιη	ρϟθ	λδ	ιδ	γ	σλ	λβ	ιε	ιθ	σνθ	ō	η	νβ
ιθ	σ	λη	ιδ	ιβ	σλα	λα	ιε	ια	σνθ	νε	η	λς
κ	σα	μβ	ιδ	κ	σλβ	λ	ιε	β	σξ	ν	η	ιθ
κα	σβ	μς	ιδ	κθ	σλγ	κη	ιδ	νδ	σξα	με	η	β
κβ	σγ	ν	ιδ	λζ	σλδ	κς	ιδ	με	σξβ	μ	ζ	με
κγ	σδ	νδ	ιδ	με	σλε	κδ	ιδ	λς	σξγ	λε	ζ	κη
κδ	σε	νη	ιδ	νγ	σλς	κβ	ιδ	κζ	σξδ	λ	ζ	ια
κε	σζ	α	ιε	α	σλζ	κ	ιδ	ιζ	σξε	κε	ς	νε
κς	ση	δ	ιε	η	σλη	ιη	ιδ	ζ	σξς	κ	ς	λθ
κζ	σθ	ζ	ιε	ις	σλθ	ιζ	ιγ	νζ	σξζ	ιε	ς	κγ
κη	σι	ι	ιε	κγ	σμ	ιδ	ιγ	μζ	σξη	ι	ς	ς
κθ	σια	ιγ	ιε	λ	σμα	ιβ	ιγ	λζ	σξθ	ε	ε	ν
λ	σιβ	ις	ιε	λζ	σμβ	ι	ιγ	κζ	σο	ō	ε	λδ
	λβ	ις			κθ	νδ			κζ	ν		

Figure 6: Ptolemy's equation of time table in the Tihon's Edition (part III).

A1 (IV) : Ascensions de la sphère droite (F, f. 56v)

Tit : ὀρθὴ σφαῖρα σὺν μεσουρανήματι•πανταχῇ.[8]

♂	Ζυγοῦ				Σκορπίου				Τοξότου			
μοι	ἀναφ		ωρ χρ ξ ξ		ἀναφ		ωρ χρ ξ ξ		ἀναφ		ωρ χρ ξ ξ	
α	σο	νε	ε	ιη	σϟη	μη	ō	ō	τκη	μζ	δ	λς
β	σοα	ν	ε	β	σϟθ	μς	ō	ō	τκθ	ν	ε	ō
γ	σοβ	με	δ	μζ	τ	μδ	ō	ō	τλ	νγ	ε	κγ
δ	σογ	μ	δ	λα	τα	μβ	ō	α	τλα	νς	ε	μζ
ε	σοδ	λε	δ	ις	τβ	μ	ō	α	τλβ	νθ	ς	ι
ς	σοε	λ	δ	ō	τγ	λη	ō	β	τλδ	β	ς	λδ
ζ	σος	κε	γ	με	τδ	λς	ō	γ	τλε	ς	ς	νθ
η	σοζ	κ	γ	λ	τε	λδ	ō	δ	τλς	ι	ζ	κδ
θ	σοη	ιε	γ	ιε	τς	λβ	ō	ε	τλζ	ιδ	ζ	μθ
ι	σοθ	ι	γ	ō	τζ	λ	ō	ς	τλη	ιη	η	ιε
ια	σπ	ε	β	με	τη	κθ	ō	ιδ	τλθ	κβ	η	μ
ιβ	σπα	ō	β	λα	τθ	κη	ō	κβ	τμ	κς	θ	ς
ιγ	σπα	νε	β	ιθ	τι	κη	ō	λ	τμα	λ	θ	λδ
ιδ	σπβ	ν	β	η	τια	κη	ō	λθ	τμβ	λε	ι	γ
ιε	σπγ	με	α	νη	τιβ	κη	ō	μζ	τμγ	μ	ι	λα
ις	σπδ	μα	α	μη	τιγ	κη	ō	νς	τμδ	με	ια	ō
ιζ	σπε	λζ	α	λη	τιδ	κη	α	δ	τμε	ν	ια	κθ
ιη	σπς	λγ	α	κη	τιε	κη	α	ιγ	τμς	νε	ια	νη
ιθ	σπζ	κθ	α	ιη	τις	κη	α	κς	τμη	ō	ιβ	κη
κ	σπη	κε	α	θ	τιζ	κη	α	μ	τμθ	ε	ιβ	νθ
κα	σπθ	κα	α	α	τιη	κθ	α	νγ	τν	ι	ιγ	κθ
κβ	σϟ	ιζ	ō	νγ	τιθ	λ	β	ζ	τνα	ιε	ιδ	ō
κγ	σϟα	ιγ	ō	με	τκ	λα	β	κα	τνβ	κ	ιδ	λα
κδ	σϟβ	θ	ō	λζ	τκα	λβ	β	λε	τνγ	κε	ιε	β
κε	σϟγ	ε	ō	λα	τκβ	λδ	β	να	τνδ	λ	ιε	λγ
κς	σϟδ	β	ō	κδ	τκγ	λς	γ	ζ	τνε	λς	ις	δ
κζ	σϟδ[9]	νθ	ō	ιη	τκδ	λη	γ	κγ	τνς	μβ	ις	λε
κη	σϟε	νς	ō	ιβ	τκε	μ	γ	μ	τνζ	μη	ιζ	ς
κθ	σϟς	νγ	ō	ς	τκς	μβ	γ	νς	τνη	νδ	ιζ	λζ
λ	σϟζ	ν	ō	ō	τκζ	μδ	δ	ιγ	τξ	ō	ιη	θ
	κζ	ν			κθ	νδ			λβ	ις		

Figure 7: Ptolemy's equation of time table in the Tihon's Edition (part IV).

and the one reproduced by Stahlman in Fig. 8, 9, 10 and 11 [8][pp. 206-209] (Codex Vaticano Graeco 1291, ff. 22r-23v):

(Table of) sphaera recta of simultaneous culmination everywhere (f.22r)

	♑		♒		♓	
λ	α	E	α	E	α	E
1	1;6	18;40	33;18	31;16	63;7	32;13
2	2;12	19;11	34;20	31;28	64;4	32;0
3	3;18	19;42	35;22	31;40	65;1	31;47
4	4;24	20;13	36;24	31;52	65;58	31;33
5	5;30	20;44	37;26	32;4	66;55	31;20
6	6;35	21;15	38;28	32;15	67;51	31;6
7	7;40	21;45	39;29	32;24	68;47	30;50
8	8;45	22;14	40;30	32;32	69;43	30;34
9	9;50	22;44	41;31	32;41	70;39	30;18
10	10;55	23;13	42;32	32;49	71;35	30;2
11	12;0	23;42	43;32	32;58	72;31	29;46
12	13;5	24;11	44;32	33;6	73;27	29;30
13	14;10	24;38	45;32	33;9	74;23	29;11
14	15;15	25;4	46;32	33;12	75;19	28;51
15	16;20	25;31	47;32	33;15	76;15	28;32
16	17;25	25;57	48;32	33;18	77;10	28;12
17	18;30	26;24	49;32	33;20	78;5	27;53
18	19;34	26;50	50;32	33;23	79;0	27;33
19	20;38	27;13	51;31	33;20	79;55	27;11
20	21;42	27;35	52;30	33;17	80;50	26;49
21	22;46	27;58	53;28	33;14	81;45	26;27
22	23;50	28;20	54;26	33;11	82;40	26;5
23	24;54	28;42	55;24	33;8	83;35	25;43
24	25;58	29;4	56;22	33;5	84;30	25;21
25	27;1	29;24	57;20	32;59	85;25	24;58
26	28;4	29;44	58;18	32;52	86;20	24;34
27	29;7	30;4	59;16	32;45	87;15	24;11
28	30;10	30;24	60;14	32;39	88;10	23;47
29	31;13	30;44	61;12	32;33	89;5	23;24
30	32;16	31;4	62;10	32;26	90;0	23;0

Figure 8: Ptolemy's equation of time table in the Stahlman's thesis (part I).

(Table of) sphaera recta of simultaneous culmination everywhere (f.22v)

	♈		♉		♊	
λ	α	E	α	E	α	E
1	90;55	22;36	118;46	10;57	148;47	6;16
2	91;50	22;11	119;46	10;40	149;50	6;19
3	92;45	21;47	120;44	10;24	150;53	6;23
4	93;40	21;22	121;42	10;7	151;56	6;26
5	94;35	20;58	122;40	9;51	152;59	6;30
6	95;30	20;33	123;38	9;35	154;2	6;34
7	96;25	20;9	124;36	9;21	155;6	6;39
8	97;20	19;44	125;34	9;6	156;10	6;44
9	98;15	19;19	126;32	8;51	157;14	6;50
10	99;10	18;54	127;30	8;37	158;18	6;55
11	100;5	18;29	128;29	8;23	159;22	7;1
12	101;0	18;4	129;28	8;10	160;26	7;6
13	101;55	17;40	130;28	7;58	161;30	7;15
14	102;50	17;16	131;28	7;47	162;35	7;24
15	103;45	16;51	132;28	7;37	163;40	7;33
16	104;41	16;27	133;28	7;27	164;45	7;42
17	105;37	16;2	134;28	7;17	165;50	7;51
18	106;33	15;38	135;28	7;7	166;55	8;0
19	107;29	15;16	136;28	7;1	168;0	8;11
20	108;25	14;54	137;28	6;55	169;5	8;22
21	109;21	14;31	138;29	6;50	170;10	8;33
22	110;17	14;9	139;30	6;44	171;15	8;44
23	111;13	13;46	140;31	6;39	172;20	8;56
24	112;9	13;24	141;32	6;34	173;25	9;8
25	113;5	13;2	142;34	6;30	174;30	9;20
26	114;2	12;41	143;36	6;26	175;36	9;32
27	114;59	12;19	144;38	6;23	176;42	9;45
28	115;56	11;57	145;40	6;19	177;48	9;57
29	116;53	11;35	146;42	6;16	178;54	10;10
30	117;50	11;13	147;44	6;12	180;0	10;23

Figure 9: Ptolemy's equation of time table in the Stahlman's thesis (part II).

(Table of) sphaera recta of simultaneous culmination everywhere (f.23r)

	♋		♌		♍	
λ	α	E	α	E	α	E
1	181;6	10;36	213;14	15;42	243;7	13;13
2	182;12	10;49	214;20	15;46	244;4	12;58
3	183;18	11;2	215;22	15;51	245;1	12;44
4	184;24	11;15	216;24	15;55	245;58	12;29
5	185;30	11;29	217;26	15;59	246;55	12;14
6	186;35	11;42	218;28	16;3	247;51	11;59
7	187;40	11;55	219;29	16;9	248;47	11;45
8	188;45	12;7	220;30	16;15	249;43	11;30
9	189;50	12;20	221;31	16;21	250;39	11;16
10	190;55	12;32	222;32	16;15	251;35	11;1
11	192;0	12;45	223;32	16;9	252;31	10;46
12	193;5	12;57	224;32	16;2	253;27	10;31
13	194;10	13;8	225;32	15;56	254;23	10;15
14	195;15	13;19	226;32	15;49	255;19	9;58
15	196;20	13;30	227;32	15;42	256;15	9;42
16	197;25	13;41	228;32	15;35	257;10	9;25
17	198;30	13;52	229;32	15;27	258;5	9;9
18	199;34	14;3	230;32	15;19	259;0	8;52
19	200;38	14;12	231;31	15;11	259;55	8;36
20	201;42	14;20	232;30	15;2	260;50	8;19
21	202;46	14;29	233;28	14;54	261;45	8;2
22	203;50	14;37	234;26	14;45	262;40	7;45
23	204;54	14;45	235;24	14;36	263;35	7;28
24	205;58	14;53	236;22	14;27	264;30	7;11
25	207;1	15;1	237;20	14;17	265;25	6;55
26	208;4	15;8	238;18	14;7	266;20	6;39
27	209;7	15;16	239;16	13;57	267;15	6;23
28	210;10	15;23	240;14	13;47	268;10	6;6
29	211;13	15;30	241;12	13;37	269;5	5;50
30	212;16	15;37	242;10	13;27	270;0	5;34

Figure 10: Ptolemy's equation of time table in the Stahlman's thesis (part III).

(Table of) sphaera recta of simultaneous culmination everywhere (f.23v)

	♎		♏		♐	
λ	α	E	α	E	α	E
1	270;55	5;18	298;48	0;0	328;47	4;36
2	271;50	5;2	299;46	0;0	329;50	5;0
3	272;45	4;47	300;44	0;0	330;53	5;23
4	273;40	4;31	301;42	0;1	331;56	5;47
5	274;35	4;16	302;40	0;1	332;59	6;10
6	275;30	4;0	303;38	0;2	334;2	6;34
7	276;25	3;45	304;36	0;3	335;6	6;59
8	277;20	3;30	305;34	0;4	336;10	7;24
9	278;15	3;15	306;32	0;5	337;14	7;49
10	279;10	3;0	307;30	0;6	338;18	8;15
11	280;5	2;45	308;29	0;14	339;22	8;40
12	281;0	2;31	309;28	0;22	340;26	9;6
13	281;55	2;19	310;28	0;30	341;30	9;34
14	282;50	2;8	311;28	0;39	342;35	10;3
15	283;45	1;58	312;28	0;47	343;40	10;31
16	284;41	1;48	313;28	0;56	344;45	11;0
17	285;37	1;38	314;28	1;4	345;50	11;29
18	286;33	1;28	315;28	1;13	346;55	11;58
19	287;29	1;18	316;28	1;26	348;0	12;28
20	288;25	1;9	317;28	1;40	349;5	12;58
21	289;21	1;1	318;29	1;53	350;10	13;29
22	290;17	0;53	319;30	[illegible];7	351;15	14;0
23	291;13	0;45	320;31	2;21	352;20	14;31
24	292;9	0;37	321;32	2;35	353;25	15;2
25	293;5	0;31	322;34	2;51	354;30	15;33
26	294;2	0;24	323;36	3;7	355;36	16;4
27	294;59	0;18	324;38	3;23	356;42	16;35
28	295;56	0;12	325;40	3;40	357;48	17;6
29	296;53	0;6	326;42	3;56	358;54	17;37
30	297;50	0;0	327;44	4;13	360;0	18;9

Figure 11: Ptolemy's equation of time table in the Stahlman's thesis (part IV).

In Fig. 12 we can see the equation of time in hour units:

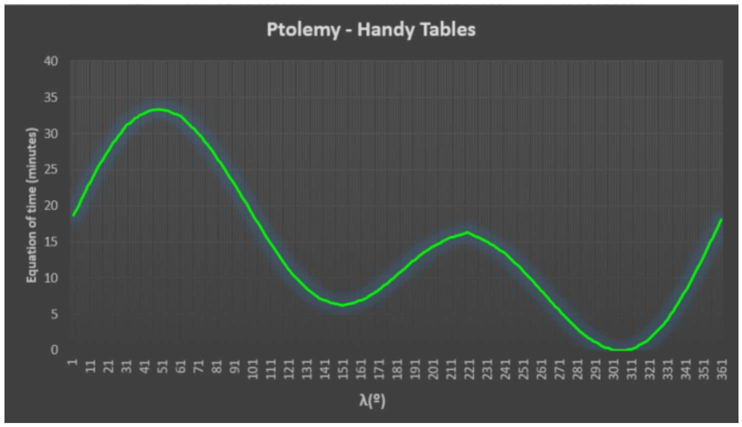


Figure 12: Ptolemy's equation of time in the *Handy Tables* as a function of λ.

The table presents an outlier in the entry 0;16,15h for $\lambda = 220°$ and it is the same in the table presented by Stahlman and by Tihon.

3.3 Analysis of the Table

We recomputed the equation of time table according to Eq.1 (because the argument is the true solar longitude) using the two procedures explained in the previous sections. For the recomputation by the use of modern methods we recompute the solar equation using Eq. 3 and the right ascension using Eq. 4. Results and choice of parameters are displayed in the following subsection.

3.3.1 Results for the recomputation by modern method

Through a least square estimation using Benno van Dalen's software *Zij Manager*, and assuming that the argument used was the true solar longitude λ, we find the 95% confidence intervals of the underlying parameters:

- $\varepsilon = \langle 23;51,50 - 23;52,26\rangle$
- $e = \langle 2;29,51 - 2;30,0\rangle$
- $\lambda_A = \langle 65;57,25 - 66;0,38\rangle$
- $E_0 = \langle 3;34,3 - 3;34,9\rangle$.

For the recomputation we used some historical values found in the *Almagest*, in the Theon of Alexandria's *Commentary on the Almagest*, *Great Commentary on the Handy Tables*, *Small Commentary on the Handy Tables*, some values are recomputed by the least square (LS) estimation together with Benno van Dalen's analyses:

- $\varepsilon = 23;51,20°$ (historical);

- $e = 2;30$ parts (historical);

- $\lambda_A = 66°$ (LS);

- $E_0 = 3;34,6$ (LS);

With this parameters we get the best fit with a mean value for the differences $E_M = 1.4583$ and a standard deviation $\sigma = 6.2806$ (values are given in seconds).

All the computation and graphics are made by Excel and in Fig. 13 we can find a sample of the file:

	A	B	D	E	F	G	H	I	K
1			Manuscript (sexagesimal)			Manuscript	Manuscript	ET (decimal)	ET (decimal)
2		λ (º)	hours	minutes	seconds	(minutes)	(seconds)	(hour units)	(degrees)
3	Ptolemy	(true)							
4	Handy Tables	1	0	18	40	18,66666667	1120	0,31111111	4,66666667
5	Equation of Time	2	0	19	11	19,18333333	1151	0,31972222	4,79583333
6		3	0	19	42	19,7	1182	0,32833333	4,925
7		4	0	20	13	20,21666667	1213	0,33694444	5,05416667
8		5	0	20	44	20,73333333	1244	0,34555556	5,18333333
9		6	0	21	15	21,25	1275	0,35416667	5,3125
10		7	0	21	45	21,75	1305	0,3625	5,4375
11		8	0	22	14	22,23333333	1334	0,37055556	5,55833333
12		9	0	22	44	22,73333333	1364	0,37888889	5,68333333
13		10	0	23	13	23,21666667	1393	0,38694444	5,80416667
14		11	0	23	42	23,7	1422	0,395	5,925
15		12	0	24	11	24,18333333	1451	0,40305556	6,04583333
16		13	0	24	38	24,63333333	1478	0,41055556	6,15833333
17		14	0	25	4	25,06666667	1504	0,41777778	6,26666667
18		15	0	25	31	25,51666667	1531	0,42527778	6,37916667
19		16	0	25	57	25,95	1557	0,4325	6,4875

(a) Manuscript data.

Figure 13: Sample of our Excel file.

I did not work on different ways of rounding: starting from the original table I converted the values of the equation of time in decimal degrees because my recomputation is in decimal degrees. I worked with all the decimal digits in my Excel files to get the result that, at the end, is reconverted in the original units of the table without rounding, then I convert this last one in sexagesimals so there is a sort of truncation. The differences are given from the difference between the original table in sexagesimals and my final result in sexagesimals.

3.3.2 Recomputation by the use of Tables

In this subsection we show our results by recomputation of the equation of time table using the right ascension table and the solar equation table in the *Handy Tables* performing a linear interpolation in the solar equation table to change the argument from $\bar{\kappa}$ to κ.

We know that E_0 is calculated for the minimum of the equation of time (0;0,0h for $\lambda = 210°, 211°, 212°, 213°$) so we recompute it putting Eq. 1 equal to zero for $\lambda = 210°$ and for $\lambda = 211°$ with $\lambda_A = 66°$ and we get $E_0 = 3;34,11°$ and $E_0 = 3;34,15°$ respectively.

We show now in Table (1) the results of our analyses with their relatives standard deviations σ and averages E_M (given in seconds) and we made a recomputation also using the previous value for the initial constant from LS:

$E_0 = 3;34,6°$	$\sigma = 6.5197$ $E_M = 0.9277$
$E_0 = 3;34,11°$	$\sigma = 6.5197$ $E_M = 0.2955$
$E_0 = 3;34,15°$	$\sigma = 6.5197$ $E_M = 0.5535$

Table 1: Standard deviations σ and averages E_M (given in seconds)

We can state that, according to the smallest standard deviation and average value, the best fit is for $E_0 = 3;34,11°$.

In Table 2 and followings we show the data used: in the first column the argument, in column 2 and 3 the entries for the equation of time, in column 4 and 5 the recomputation of the entries and in column 6 the differences given in seconds:

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
1	18	40	18	40	0
2	19	11	19	15	-4
3	19	42	19	47	-5
4	20	13	20	19	-6
5	20	44	20	54	-10
6	21	15	21	22	-7
7	21	45	21	50	-5
8	22	14	22	21	-7
9	22	44	22	49	-5
10	23	13	23	17	-4
11	23	42	23	45	-3
12	24	11	24	13	-2
13	24	38	24	40	-2
14	25	4	25	8	-4
15	25	31	25	36	-5
16	25	57	26	4	-7
17	26	24	26	31	-7
18	26	50	26	55	-5
19	27	13	27	19	-6
20	27	35	27	43	-8
21	27	58	28	6	-8
22	28	20	28	29	-9
23	28	42	28	50	-8
24	29	4	29	14	-10
25	29	24	29	33	-9
26	29	44	29	49	-5
27	30	4	30	9	-5
28	30	24	30	28	-4
29	30	44	30	45	-1
30	31	4	31	4	0
31	31	16	31	20	-4
32	31	28	31	32	-4
33	31	40	31	48	-8
34	31	52	32	0	-8
35	32	4	32	12	-8
36	32	15	32	27	-12
37	32	24	32	36	-12
38	32	32	32	43	-11
39	32	41	32	51	-10
40	32	49	32	59	-10

Table 2: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
41	32	58	33	3	-5
42	33	6	33	7	-1
43	33	9	33	11	-2
44	33	12	33	15	-3
45	33	15	33	19	-4
46	33	18	33	23	-5
47	33	20	33	27	-7
48	33	23	33	28	-5
49	33	20	33	27	-7
50	33	17	33	27	-10
51	33	14	33	20	-6
52	33	11	33	15	-4
53	33	8	33	8	0
54	33	5	33	3	2
55	32	59	32	59	0
56	32	52	32	52	0
57	32	45	32	46	-1
58	32	39	32	40	-1
59	32	33	32	32	1
60	32	26	32	24	2
61	32	13	32	14	-1
62	32	0	32	4	-4
63	31	47	31	52	-5
64	31	33	31	40	-7
65	31	20	31	28	-8
66	31	6	31	12	-6
67	30	50	30	56	-6
68	30	34	30	40	-6
69	30	18	30	24	-6
70	30	2	30	8	-6
71	29	46	29	52	-6
72	29	30	29	36	-6
73	29	11	29	17	-6
74	28	51	29	0	-9
75	28	32	28	44	-12
76	28	12	28	24	-12
77	27	53	28	1	-8
78	27	33	27	40	-7
79	27	11	27	17	-6
80	26	49	26	53	-4

Table 3: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part II.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
81	26	27	26	32	-5
82	26	5	26	9	-4
83	25	43	25	45	-2
84	25	21	25	21	0
85	24	58	24	57	1
86	24	34	24	33	1
87	24	11	24	9	2
88	23	47	23	45	2
89	23	24	23	21	3
90	23	0	22	57	3
91	22	36	22	33	3
92	22	11	22	8	3
93	21	47	21	44	3
94	21	22	21	20	2
95	20	58	20	56	2
96	20	33	20	32	1
97	20	9	20	8	1
98	19	44	19	40	4
99	19	19	19	16	3
100	18	54	18	52	2
101	18	29	18	24	5
102	18	4	18	0	4
103	17	40	17	35	5
104	17	16	17	7	9
105	16	51	16	43	8
106	16	27	16	19	8
107	16	2	15	55	7
108	15	38	15	34	4
109	15	16	15	10	6
110	14	54	14	47	7
111	14	31	14	25	6
112	14	9	14	1	8
113	13	46	13	38	8
114	13	24	13	17	7
115	13	2	12	52	10
116	12	41	12	32	9
117	12	19	12	12	7
118	11	57	11	52	5
119	11	35	11	31	4
120	11	13	11	11	2

Table 4: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part III.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
121	10	57	10	55	2
122	10	40	10	39	1
123	10	24	10	22	2
124	10	7	10	6	1
125	9	51	9	50	1
126	9	35	9	33	2
127	9	21	9	17	4
128	9	6	9	1	5
129	8	51	8	45	6
130	8	37	8	28	9
131	8	23	8	16	7
132	8	10	8	0	10
133	7	58	7	51	7
134	7	47	7	43	4
135	7	37	7	31	6
136	7	27	7	22	5
137	7	17	7	14	3
138	7	7	7	5	2
139	7	1	6	53	8
140	6	55	6	45	10
141	6	50	6	39	11
142	6	44	6	32	12
143	6	39	6	28	11
144	6	34	6	22	12
145	6	30	6	19	11
146	6	26	6	19	7
147	6	23	6	16	7
148	6	19	6	15	4
149	6	16	6	14	2
150	6	12	6	14	-2
151	6	16	6	14	2
152	6	19	6	17	2
153	6	23	6	17	6
154	6	26	6	21	5
155	6	30	6	24	6
156	6	34	6	24	10
157	6	39	6	27	12
158	6	44	6	35	9
159	6	50	6	42	8
160	6	55	6	47	8

Table 5: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part IV.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
161	7	1	6	53	8
162	7	6	6	58	8
163	7	15	7	6	9
164	7	24	7	17	7
165	7	33	7	27	6
166	7	42	7	37	5
167	7	51	7	48	3
168	8	0	7	58	2
169	8	11	8	8	3
170	8	22	8	19	3
171	8	33	8	29	4
172	8	44	8	39	5
173	8	56	8	50	6
174	9	8	8	59	9
175	9	20	9	10	10
176	9	32	9	25	7
177	9	45	9	41	4
178	9	57	9	53	4
179	10	10	10	8	2
180	10	23	10	24	-1
181	10	36	10	36	0
182	10	49	10	51	-2
183	11	2	11	7	-5
184	11	15	11	23	-8
185	11	29	11	39	-10
186	11	42	11	50	-8
187	11	55	12	2	-7
188	12	7	12	14	-7
189	12	20	12	26	-6
190	12	32	12	37	-5
191	12	45	12	49	-4
192	12	57	13	1	-4
193	13	8	13	12	-4
194	13	19	13	24	-5
195	13	30	13	36	-6
196	13	41	13	48	-7
197	13	52	13	59	-7
198	14	3	14	7	-4
199	14	12	14	17	-5
200	14	20	14	27	-7

Table 6: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part V.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
201	14	29	14	34	-5
202	14	37	14	45	-8
203	14	45	14	54	-9
204	14	53	15	2	-9
205	15	1	15	9	-8
206	15	8	15	13	-5
207	15	16	15	17	-1
208	15	23	15	24	-1
209	15	30	15	29	1
210	15	37	15	36	1
211	15	42	15	40	2
212	15	46	15	40	6
213	15	51	15	44	7
214	15	55	15	48	7
215	15	59	15	48	11
216	16	3	15	52	11
217	16	9	15	52	17
218	16	15	15	51	24
219	16	21	15	51	30
220	16	15	15	51	24
221	16	9	15	47	22
222	16	2	15	43	19
223	15	56	15	39	17
224	15	49	15	35	14
225	15	42	15	31	11
226	15	35	15	27	8
227	15	27	15	23	4
228	15	19	15	19	0
229	15	11	15	11	0
230	15	2	15	3	-1
231	14	54	14	52	2
232	14	45	14	43	2
233	14	36	14	31	5
234	14	27	14	20	7
235	14	17	14	10	7
236	14	7	14	0	7
237	13	57	13	52	5
238	13	47	13	44	3
239	13	37	13	34	3
240	13	27	13	24	3

Table 7: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part VI.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
241	13	13	13	12	1
242	12	58	13	0	-2
243	12	44	12	48	-4
244	12	29	12	36	-7
245	12	14	12	24	-10
246	11	59	12	8	-9
247	11	45	11	52	-7
248	11	30	11	36	-6
249	11	16	11	20	-4
250	11	1	11	4	-3
251	10	46	10	49	-3
252	10	31	10	36	-5
253	10	15	10	20	-5
254	9	58	10	4	-6
255	9	42	9	49	-7
256	9	25	9	32	-7
257	9	9	9	13	-4
258	8	52	8	57	-5
259	8	36	8	40	-4
260	8	19	8	21	-2
261	8	2	8	4	-2
262	7	45	7	45	0
263	7	28	7	29	-1
264	7	11	7	12	-1
265	6	55	6	53	2
266	6	39	6	37	2
267	6	23	6	21	2
268	6	6	6	5	1
269	5	50	5	49	1
270	5	34	5	33	1
271	5	18	5	17	1
272	5	2	5	0	2
273	4	47	4	44	3
274	4	31	4	28	3
275	4	16	4	12	4
276	4	0	3	57	3
277	3	45	3	44	1
278	3	30	3	28	2
279	3	15	3	12	3
280	3	0	3	0	0

Table 8: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
281	2	45	2	44	1
282	2	31	2	31	0
283	2	19	2	19	0
284	2	8	2	3	5
285	1	58	1	51	7
286	1	48	1	42	6
287	1	38	1	31	7
288	1	28	1	22	6
289	1	18	1	14	4
290	1	9	1	3	6
291	1	1	0	53	8
292	0	53	0	45	8
293	0	45	0	37	8
294	0	37	0	29	8
295	0	31	0	20	11
296	0	24	0	16	8
297	0	18	0	12	6
298	0	12	0	8	4
299	0	6	0	3	3
300	0	0	59	59	1
301	0	0	59	59	1
302	0	0	59	59	1
303	0	0	59	58	2
304	0	1	59	59	2
305	0	1	0	2	-1
306	0	2	0	1	1
307	0	3	0	2	1
308	0	4	0	5	-1
309	0	5	0	5	0
310	0	6	0	4	2
311	0	14	0	12	2
312	0	22	0	16	6
313	0	30	0	23	7
314	0	39	0	35	4
315	0	47	0	43	4
316	0	56	0	50	6
317	1	4	1	1	3
318	1	13	1	10	3
319	1	26	1	18	8
320	1	40	1	28	12

Table 9: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
321	1	53	1	41	12
322	2	7	1	55	12
323	2	21	2	10	11
324	2	35	2	24	11
325	2	51	2	41	10
326	3	7	3	1	6
327	3	23	3	19	4
328	3	40	3	35	5
329	3	56	3	52	4
330	4	13	4	10	3
331	4	36	4	31	5
332	5	0	4	54	6
333	5	23	5	17	6
334	5	47	5	37	10
335	6	10	6	0	10
336	6	34	6	24	10
337	6	59	6	51	8
338	7	24	7	19	5
339	7	49	7	43	6
340	8	15	8	10	5
341	8	40	8	37	3
342	9	6	9	2	4
343	9	34	9	28	6
344	10	3	9	57	6
345	10	31	10	25	6
346	11	0	10	55	5
347	11	29	11	26	3
348	11	58	11	56	2
349	12	28	12	25	3
350	12	58	12	57	1
351	13	29	13	27	2
352	14	0	13	56	4
353	14	31	14	26	5
354	15	2	14	54	8
355	15	33	15	23	10
356	16	4	15	57	7
357	16	35	16	29	6
358	17	6	17	1	5
359	17	37	17	36	1
360	18	9	18	8	1

Table 10: Ptolemy's Handy Tables: data from manuscript, recomputation and differences, part IX.

In Fig. 14 the pattern of the differences, given in seconds:

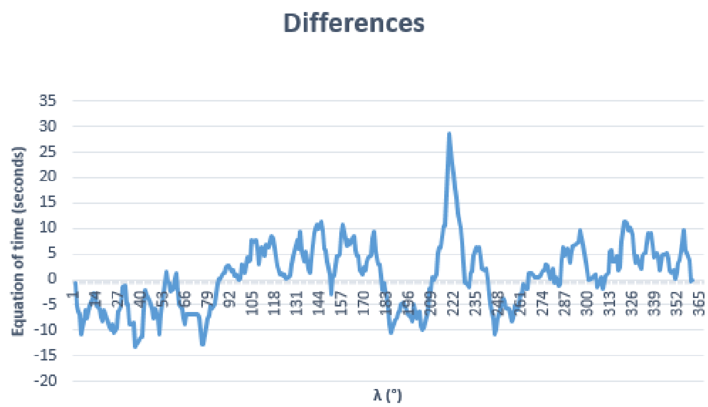


Figure 14: Ptolemy's equation of time differences, given in seconds.

3.4 Conclusions

In our recomputation, after the choice of parameters (done both with LS and historical considerations), we perform other attempts changing the parameters in our Excel file in order to verify if our first choice is the best one. The recomputation by the use of tables is the one indicated in the primary sources and from our results we get the best fit for $\lambda_A = 66°$ and for $E_0 = 3;34,11°$.

We tried also other methods to recompute the solar equation and the right ascension tables but without success. One procedure is shown in the Appendix A and it explains how to extract the solar equation and the right ascension from the equation of time table according to Eq. **??** and Eq. **??**.

Another procedure consists in recomputing the right ascension table using the table from the *Almagest*, given in intervals of 10 degrees, and performing linear interpolation.

For the solar equation we worked on the chord table in the *Almagest*.

In all these three cases we did not get better results than the ones we showed in this report.

3.5 List of Manuscripts

These are the manuscript used by Stahalman, Tihon and Mercy:

- Florence, Biblioteca Medicea Laurenziana gr. 28/48 (Handy Tables): II 133;
- Vatican, Biblioteca Apostolica Vaticana gr. 1291 (Handy Tables): II 111, 133, note 31.

4 Al-Khwārizmī

Al-Khwārizmī (c. 780 – c. 850) was a Persian astronomer, geographer and mathematician who lived in Baghdad during the reigns of the Abbasid caliphs al-Ma'mūn, al-Mu'tasim and al-Wāthiq. His main astronomical work was the *Sindhind Zīj* that was based on Indian methods (taken from Sindhind, an Arabic translation of the Brāhmasputasiddhānta by the Indian astronomer Brahmagupta) instead of the Ptolemy's model in the Almagest.

4.1 Overview

4.1.1 Sindhind Zīj

The *Sindhind Zīj* was based on Indian methods (taken from Sindhind, an Arabic translation of the Brāhmasputasiddhānta by the Indian astronomer Brahmagupta) instead of the Ptolemy's model in the Almagest. We only have a Latin translation by Adelard of Bath of a recension by Maslama al-Majrītī (Cordoba, c. 980) and together with the *Toledan Tables* some Indian methods used by al-Khwārizmī were diffused in the Western Europe. Al-Majrītī converted the planetary tables from the Persian to the Arabic calendar and adapted some tables to the longitude of Cordoba.

The main sources of the *Sindhind Zīj* are:

- the Latin translation by Adelard of Bath of al-Majrītī's recension of the smaller version of al-Khwārizmī's zīj (12th century, available in 9 manuscripts; Neugebauer, 1962, translated the Latin version of the al-Khwārizmī's zīj in English with a new commentary);
- the commentary on the larger version of al-Khwārizmī's zīj by Ibn al-Muthanna, 10th century (the Arabic original is lost and we have a Latin translation by Hugo Sanctallensis plus two Hebrew translations);
- the commentary on al-Khwārizmī's zīj by Ibn Masrūr (10th century, not published);
- The Toledan Tables by al-Zarqālī (or Azarquiel) in the 11th century, the original Arabic is lost but many Latin versions of tables and explanatory texts exist.

The original al-Khwārizmī's zīj is composed by the following tables: chronological, mean motions, solar equation, lunar equation, solar declination, lunar latitude, planetary equations, planetary stations, planetary latitudes, lunar visibility, sine, right ascension, oblique ascension, shadow length (cotangent), true solar and lunar motion, equation of time, mean oppositions and conjunctions, lunar eclipses, parallax, solar eclipses, equation of the houses, projection of the rays and excess of revolution tables.

4.2 Presentation of the table

The argument is the true solar longitude in intervals of one degree. The table starts with Aries and the entries are given in minutes and seconds: the minimum is $0;0,0$ and it occurs at Aquarius $22°$, the maximum $0;34,28^h$ at Scorpio $(8-9)°$.

Pagiia examinationis dierum (O) C 80r, O 137r.

Gradus regulares [1]	Aries		Taurus		Gemini		Cancer		Leo		Virgo	
	Examinatio dierum [2]		Examina-io dierun		Examinatio dierum		Examinatio dierum		Examinatio dierum		Examinatio dierum [2]	
	Min. [1]	Sec. [2]	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
1	8	8	18	48	23	28	19	28 [i]	14	40	17	48
2	8	28	19	4	23	28	19	12	14 [r]	40	18	4
3	8	48	19	20	23	27	19	0	14	40	18	20
4	9	12	19	40	23	26	18	48	14	40	18	40
5	9	32	19	56	23	24	18	32	14	40	19	0
6	9	52	20	12	23	20	18	20	14	40	19	16
7	10	16	20	28	23	16	18	8 [k]	14	40	19	32
8	10	40	20	40	23	12	17	56	14	41 [s]	19	48
9	11	4	20	52	23	6	17	44	14	42	20	4
10	11	28	21	4	23	0	17	32 [l]	14	44	20	24
11	11	48	21	20	22	54	17	20	14	48	20	44
12	12	12	21	32	22	44 [d]	17	8	14	52	21	0
13	12	40	21	44	22	36	17	0	14 [r]	56	21	20
14	13	4	21	52	22	28 [e]	16	48	15	0	21	40
15	13	24	22	4	22	20	16	36	15	4	21	56
16	13	48	22	16	22	12 [f]	16	28	15	10 [t]	22	16 [u]
17	14	8	22	24	22	4	16	16	15	20	22	36
18	14	28	22	32 [b]	21	56	16	8	15	28	22	52
19	14	48	22	40	21	44	16	0	15	36	23	12
20	15	8	22	48	21	32	15	52 [m]	15	44	23	32 [v]
21	15	28	22	56	21	24 [g]	15	44	15	52	23	52
22	15	52	23	0	21	12	15	36	16	0	24	16
23	16	12	23	8	21	0	15	32	16	8	24	36
24	16	32 [a]	23	12	20	48	15	24 [n]	16	20	24	52
25	16	52	23	16	20	36 [h]	15	16 [o]	16	32	25	12
26	17	8	23	20	20	24	15	8	16	48	25	32
27	17	28	23	24	20	14	15	0	17	0	25	52
28	17	48	23	26 [c]	20	4	14	56 [p]	17	12	26	12
29	18	8	23	27	19	54	14	52 [q]	17	24	26	32
30	18	28	23	28	19	40	14	44	17	36	26	52

Figure 15: Al-Khwārizmī's equation of time table, part I.

C 80v, O 137v. **Pagina examinationis dierum** (O)

Gradus regulares 1)	Libra		Scorpius		Sagittarius		Capricornus		Aquarius		Pisces	
	Examinatio dierum 2)		Examinatio dierum		Examinatio dierum		Examinatio dierum		Examinatio dierum		Examinatio dierum	
	Min. 3)	Sec. 4)	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
1	27	8	34	8	31	8	17	16	3	28	0	24 k)
2	27	28	34	12	30	48	16	44	3	8	0	32
3	27	44	34	16	30	28	16	12	2	48	0	40
4	28	4	34	20	30	4	15	40	2	32 i)	0	48
5	28	24	34	22	29	40	15	8	2	16	0	56
6	28	40	34	24	29	20	14	36	2	0	1	4
7	29	0	34	26	28	56	14	4	1	48	1	16 l)
8	29	20	34	27	28	36	13	32	1	36	1	28
9	29	36	34	28	28	12	13	4	1	24	1	40
10	29	52	34	28	27	44	12	32	1	12	1	52
11	30	8	34	27	27	16	12	4	1	3	2	4
12	30	28	34	26	26	52	11	36	0	52	2	20
13	30	44	34	24	26	28	11	4	0	40	2	36
14	31	4	34	20	26	0	10	36	0	32	2	52
15	31	20	34	16	25	32	10	8	0	24	3	4
16	31	32	34	12	25	4	9	36	0	16	3	20
17	31	44	34	4	24	36	9	8	0	10	3	36
18	32 a)	0	33	56	24	8	8	40	0	6	3	52
19	32	12	33	48	23	36	8	12	0	4	4	12
20	32	24	33	36	23	4	7	44	0	2	4	28
21	32	40	33	28	22	36	7	16	0	1	4	48
22	32 b)	52	33	16	22	8	6	52	0	0	5	12
23	33 c)	4	33 e)	4	21	36	6	28	0	1	5	32 m)
24	33	16	32	52	21	8	6	0	0	2	5	48
25	33	24	52	40	20	40	5	32	0	4	6	8
26	33	32	32	28	20	16	5	8	0	6	6	28
27	33	40	32	12	19	40	4	48 h)	0	8	6	48
28	33	48 d)	31	56	19	0	4	28	0	10	7	8
29	33	54	31	40	18	24 g)	4	8	0	14	7	28
30	34	0	31	24	17 f)	52	3	48	0	20	7	48

Figure 16: Al-Khwārizmī's equation of time table, part II.

4.3 Analyses of the Table

This is our set of parameters:

- $\varepsilon = 23;51^\circ$ and $\varepsilon = 24^\circ$ (LS and historical);
- $e = 2;20$ and $e = 2;30$ parts (LS and historical);
- $\lambda_A = 82;39^\circ$ (LS);
- $E_0 = 4,30^\circ$ and $E_0 = 4,20,55^\circ$ (LS and recomputed by hand according to the minimum of the equation of time for $\lambda = 322^\circ$);

4.3.1 Results by the use of modern method

The best fit is for $\varepsilon = 23;51^\circ$, $e = 2;30$ and $E_0 = 4,30^\circ$ with $E_M = 0.2$ and a standard deviation $\sigma = 34.1$ (values are given in seconds).

4.3.2 Results by the use of Tables

We used the right ascension table and the solar equation table in the *Sindhind Zīj* and the best fit is for $E_0 = 4,30°$ with $E_M = 0.1598$ and a standard deviation $\sigma = 43.5944$ (values are given in seconds). This best fit is given by the reconstruction of the solar equation performing a change of variable by linear interpolation.

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
1	8	8	9	27	-79
2	8	28	9	49	-81
3	8	48	10	11	-83
4	9	12	10	33	-81
5	9	32	10	54	-82
6	9	52	11	16	-84
7	10	16	11	38	-82
8	10	40	12	1	-81
9	11	4	12	23	-79
10	11	28	12	45	-77
11	11	48	13	7	-79
12	12	12	13	29	-77
13	12	40	13	50	-70
14	13	4	14	12	-68
15	13	24	14	34	-70
16	13	48	14	55	-67
17	14	8	15	16	-68
18	14	28	15	37	-69
19	14	48	15	58	-70
20	15	8	16	19	-71
21	15	28	16	39	-71
22	15	52	16	59	-67
23	16	12	17	19	-67
24	16	32	17	39	-67
25	16	52	17	58	-66
26	17	8	18	16	-68
27	17	28	18	35	-67
28	17	48	18	52	-64
29	18	8	19	10	-62
30	18	28	19	27	-59
31	18	48	19	44	-56
32	19	4	20	0	-56
33	19	20	20	17	-57
34	19	40	20	31	-51
35	19	56	20	45	-49
36	20	12	21	0	-48
37	20	28	21	13	-45
38	20	40	21	26	-46
39	20	52	21	39	-47
40	21	4	21	49	-45

Table 11: Al-Khwārizmī: data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
41	21	20	22	2	-42
42	21	32	22	13	-41
43	21	44	22	23	-39
44	21	52	22	32	-40
45	22	4	22	41	-37
46	22	16	22	49	-33
47	22	24	22	56	-32
48	22	32	23	3	-31
49	22	40	23	10	-30
50	22	48	23	15	-27
51	22	56	23	20	-24
52	23	0	23	24	-24
53	23	8	23	28	-20
54	23	12	23	31	-19
55	23	16	23	33	-17
56	23	20	23	34	-14
57	23	24	23	35	-11
58	23	26	23	35	-9
59	23	27	23	35	-8
60	23	28	23	34	-6
61	23	28	23	32	-4
62	23	28	23	30	-2
63	23	27	23	26	1
64	23	26	23	23	3
65	23	24	23	18	6
66	23	20	23	14	6
67	23	16	23	8	8
68	23	12	23	3	9
69	23	6	22	56	10
70	23	0	22	49	11
71	22	54	22	41	13
72	22	44	22	33	11
73	22	36	22	25	11
74	22	28	22	16	12
75	22	20	22	7	13
76	22	12	21	57	15
77	22	4	21	47	17
78	21	56	21	36	20
79	21	44	21	24	20
80	21	32	21	13	19

Table 12: Al-Khwārizmī: data from manuscript, recomputation and differences, part II.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
81	21	24	21	1	23
82	21	12	20	49	23
83	21	0	20	43	17
84	20	48	20	25	23
85	20	36	20	13	23
86	20	24	20	0	24
87	20	14	19	47	27
88	20	4	19	36	28
89	19	54	19	22	32
90	19	40	19	9	31
91	19	28	18	57	31
92	19	12	18	42	30
93	19	0	18	30	30
94	18	48	18	17	31
95	18	32	18	4	28
96	18	20	17	52	28
97	18	8	17	40	28
98	17	56	17	27	29
99	17	44	17	15	29
100	17	32	17	3	29
101	17	20	16	51	29
102	17	8	16	39	29
103	17	0	16	27	33
104	16	48	16	17	31
105	16	36	16	7	29
106	16	28	15	57	31
107	16	16	15	47	29
108	16	8	15	38	30
109	16	0	15	29	31
110	15	52	15	21	31
111	15	44	15	13	31
112	15	36	15	5	31
113	15	32	14	58	34
114	15	24	14	52	32
115	15	16	14	47	29
116	15	8	14	41	27
117	15	0	14	37	23
118	14	56	14	32	24
119	14	52	14	29	23
120	14	44	14	26	18

Table 13: Al-Khwārizmī: data from manuscript, recomputation and differences, part III.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
121	14	40	14	23	17
122	14	40	14	22	18
123	14	40	14	21	19
124	14	40	14	20	20
125	14	40	14	20	20
126	14	40	14	21	19
127	14	40	14	22	18
128	14	41	14	24	17
129	14	42	14	27	15
130	14	44	14	30	14
131	14	48	14	34	14
132	14	52	14	39	13
133	14	56	14	45	11
134	15	0	14	50	10
135	15	4	14	56	8
136	15	10	15	3	7
137	15	20	15	11	9
138	15	28	15	19	9
139	15	36	15	28	8
140	15	44	15	37	7
141	15	52	15	47	5
142	16	0	15	58	2
143	16	8	16	9	-1
144	16	20	16	21	-1
145	16	32	16	33	-1
146	16	48	16	45	3
147	17	0	16	58	2
148	17	12	17	12	0
149	17	24	17	26	-2
150	17	36	17	40	-4
151	17	48	17	55	-7
152	18	4	18	11	-7
153	18	20	18	26	-6
154	18	40	18	42	-2
155	19	0	18	58	2
156	19	16	19	15	1
157	19	32	19	32	0
158	19	48	19	49	-1
159	20	4	20	7	-3
160	20	24	20	25	-1

Table 14: Al-Khwārizmī: data from manuscript, recomputation and differences, part IV.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
161	20	44	20	43	1
162	21	0	21	2	-2
163	21	20	21	20	0
164	21	40	21	39	1
165	21	56	21	58	-2
166	22	16	22	17	-1
167	22	36	22	36	0
168	22	52	22	55	-3
169	23	12	23	14	-2
170	23	32	23	34	-2
171	23	52	23	54	-2
172	24	16	24	13	3
173	24	36	24	33	3
174	24	52	24	52	0
175	25	12	25	12	0
176	25	32	25	31	1
177	25	52	25	50	2
178	26	12	26	10	2
179	26	32	26	29	3
180	26	52	26	47	5
181	27	8	27	6	2
182	27	28	27	25	3
183	27	44	27	43	1
184	28	4	28	2	2
185	28	24	28	19	5
186	28	40	28	37	3
187	29	0	28	54	6
188	29	20	29	11	9
189	29	36	29	28	8
190	29	52	29	45	7
191	30	8	30	1	7
192	30	28	30	16	12
193	30	44	30	31	13
194	31	4	30	46	18
195	31	20	31	0	20
196	31	32	31	14	18
197	31	44	31	28	16
198	32	0	31	40	20
199	32	12	31	53	19
200	32	24	32	5	19

Table 15: Al-Khwārizmī: data from manuscript, recomputation and differences, part V.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
201	32	40	32	16	24
202	32	52	32	27	25
203	33	4	32	37	27
204	33	16	32	46	30
205	33	24	32	55	29
206	33	32	33	4	28
207	33	40	33	11	29
208	33	48	33	18	30
209	33	54	33	24	30
210	34	0	33	30	30
211	34	8	33	35	33
212	34	12	33	39	33
213	34	16	33	43	33
214	34	20	33	45	35
215	34	22	33	47	35
216	34	24	33	48	36
217	34	26	33	49	37
218	34	27	33	49	38
219	34	28	33	48	40
220	34	28	33	46	42
221	34	27	33	43	44
222	34	26	33	40	46
223	34	24	33	35	49
224	34	20	33	30	50
225	34	16	33	24	52
226	34	12	33	17	55
227	34	4	33	9	55
228	33	56	33	2	54
229	33	48	32	52	56
230	33	36	32	42	54
231	33	28	32	31	57
232	33	16	32	20	56
233	33	4	32	7	57
234	32	52	31	54	58
235	32	40	31	40	60
236	32	28	31	25	63
237	32	12	31	9	63
238	31	56	30	52	64
239	31	40	30	35	65
240	31	24	30	17	67

Table 16: Al-Khwārizmī: data from manuscript, recomputation and differences, part VI.

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
241	31	8	29	58	70
242	30	48	29	39	69
243	30	28	29	18	70
244	30	4	28	57	67
245	29	40	28	35	65
246	29	20	28	13	67
247	28	56	27	50	66
248	28	36	27	27	69
249	28	12	27	2	70
250	27	44	26	37	67
251	27	16	26	12	64
252	26	52	25	46	66
253	26	28	25	20	68
254	26	0	24	53	67
255	25	32	24	26	66
256	25	4	23	58	66
257	24	36	23	29	67
258	24	8	23	2	66
259	23	36	22	31	65
260	23	4	22	1	63
261	22	36	21	31	65
262	22	8	21	5	63
263	21	36	20	31	65
264	21	8	20	0	68
265	20	40	19	30	70
266	20	16	18	59	77
267	19	40	18	29	71
268	19	0	17	58	62
269	18	24	17	26	58
270	17	52	16	55	57
271	17	16	16	24	52
272	16	44	15	53	51
273	16	12	15	23	49
274	15	40	14	52	48
275	15	8	14	21	47
276	14	36	13	51	45
277	14	4	13	21	43
278	13	32	12	51	41
279	13	4	12	21	43
280	12	32	11	51	41

Table 17: Al-Khwārizmī: data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
281	12	4	11	21	43
282	11	36	10	52	44
283	11	4	10	24	40
284	10	36	9	56	40
285	10	8	9	28	40
286	9	36	9	1	35
287	9	8	8	35	33
288	8	40	8	9	31
289	8	12	7	44	28
290	7	44	7	19	25
291	7	16	6	55	21
292	6	52	6	31	21
293	6	28	6	8	20
294	6	0	5	46	14
295	5	32	5	24	8
296	5	8	5	4	4
297	4	48	4	43	5
298	4	28	4	24	4
299	4	8	4	5	3
300	3	48	3	47	1
301	3	28	3	30	-2
302	3	8	3	14	-6
303	2	48	2	58	-10
304	2	32	2	43	-11
305	2	16	2	29	-13
306	2	0	2	16	-16
307	1	48	2	4	-16
308	1	36	1	53	-17
309	1	24	1	42	-18
310	1	12	1	33	-21
311	1	3	1	24	-21
312	0	52	1	16	-24
313	0	40	1	9	-29
314	0	32	1	3	-31
315	0	24	0	57	-33
316	0	16	0	53	-37
317	0	10	0	49	-39
318	0	6	0	46	-40
319	0	4	0	44	-40
320	0	2	0	43	-41

Table 18: Al-Khwārizmī: data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
321	0	1	0	43	-42
322	0	0	0	43	-43
323	0	1	0	45	-44
324	0	2	0	47	-45
325	0	4	0	50	-46
326	0	6	0	53	-47
327	0	8	0	58	-50
328	0	10	1	3	-53
329	0	14	1	9	-55
330	0	20	1	16	-56
331	0	24	1	23	-59
332	0	32	1	32	-60
333	0	40	1	40	-60
334	0	48	1	50	-62
335	0	56	2	0	-64
336	1	4	2	11	-67
337	1	16	2	22	-66
338	1	28	2	35	-67
339	1	40	2	48	-68
340	1	52	3	1	-69
341	2	4	3	15	-71
342	2	20	3	30	-70
343	2	36	3	45	-69
344	2	52	4	1	-69
345	3	4	4	17	-73
346	3	20	4	33	-73
347	3	36	4	50	-74
348	3	52	5	8	-76
349	4	12	5	25	-73
350	4	28	5	44	-76
351	4	48	6	3	-75
352	5	12	6	22	-70
353	5	32	6	42	-70
354	5	48	7	1	-73
355	6	8	7	21	-73
356	6	28	7	42	-74
357	6	48	8	2	-74
358	7	8	8	23	-75
359	7	28	8	44	-76
360	7	48	9	6	-78

Table 19: Al-Khwārizmī: data from manuscript, recomputation and differences, part IX.

The shape of the differences present a structure (in both procedures), it means that we are far from the right algorithm, so we put aside the recomputation for this astronomer. More details in Appendix B.

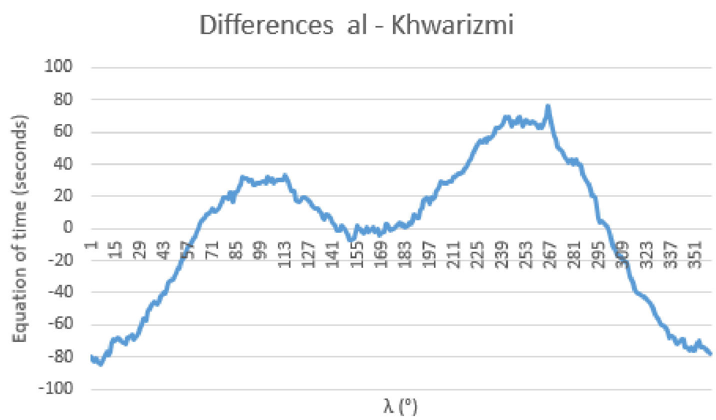


Figure 17: Al-Khwārizmī's equation of time differences, given in seconds.

4.4 Conclusions

We get the best fit for the modern recomputation but we find out the same set of parameters with both methods.

4.5 List of Manuscripts

- Cairo, Dār al-Kutub, Taymūr riaāda 99 (Ibn Masrūr's commentary on al-Khwārizmī's zīj): IV 199;
- Chartes, Bibliothèque publique 214 (al-Khwārizmī's zīj in Latin): IV 199,218;
- Madrid, Biblioteca Nacional, lat. 10016 (al-Khwārizmī's zīj in Latin): IV 199;
- Oxford, Bodleian Library, Auct. F.I. 9 (al-Khwārizmī's zīj in Latin): IV 199, 218, ff. 99v-160r;
- Oxford, Bodleian Library, Corpus Christi College 283 (al-Khwārizmī's zīj in Latin): IV 199;
- Oxford, Bodleian Library, Merton College 259 (astronomical rules close to al-Khwārizmī): IV 199;
- Paris, Bibliothèque Mazarine 3642 (al-Khwārizmī's zīj in Latin): IV 199.

5 Al-Battānī

Al-Battānī (c. 858 – 929) was an Arabic astronomer and mathematician, he was born in Harran, Upper Mesopotamia (now in Turkey) and he lived and worked in al-Raqqa, today in Syria.

He determined the value for the solar year as 365 days, 5 hours, 46 minutes and 24 seconds; he corrected some of Ptolemy's results and he discovered that the direction of the Sun's apogee was changing. He introduced the use of sines and tangents in calculation, he calculated the values for the precession of the equinoxes ($1°$ in 66 years) and for the obliquity of the ecliptic ($23°35'$). He was quoted by Copernicus in the *De Revolutionibus Orbium Coelestium* and also by Tycho Brahe. Kepler and Galileo showed interest in some of his observations.

5.1 Overview

See Appendix C for primary and secondary sources.

5.1.1 Sābi'Zīj

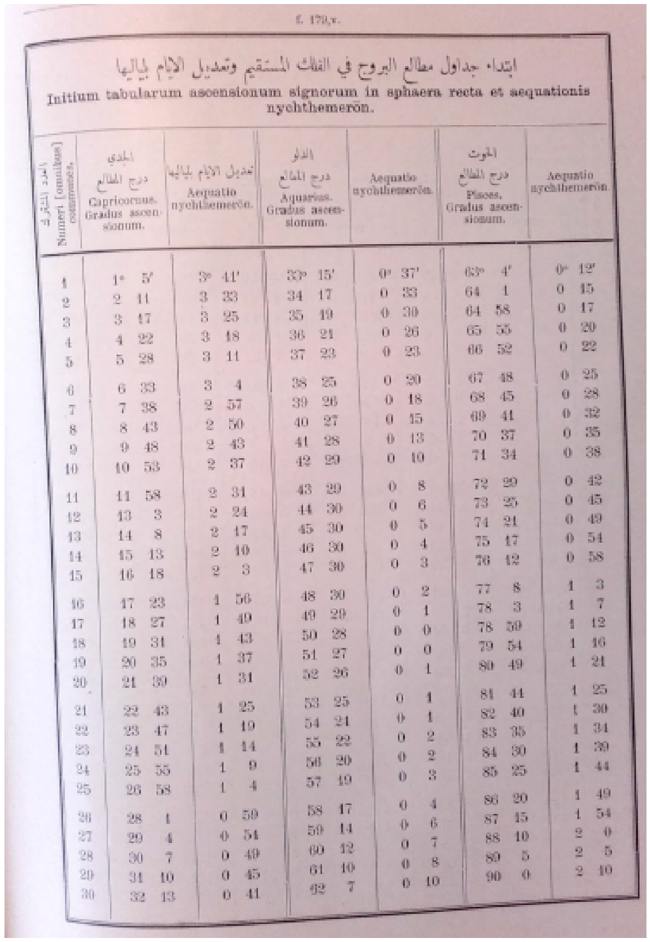
His most important astronomical work with tables is often indicated as the *Sābi'Zīj* purely based on Ptolemaic astronomy and it was influential in the Islamic East (because many astronomers adopted its parameters) and in the West (because many of its tables were distributed as a part of the *Toledan Tables*).

5.2 Presentation of the table

In *Tabule Astronomice illustrissimi Alfontij regis castelle* [10] by Ratdolt we can find an equation of time table. The heading is *Tabula elevationu Signoru in circulo directo*, the first column is *Gradus equales* for the solar longitude in intervals of one degree, then we find other two columns for each sign (four zodiacal signs per table) starting with Capricornus: the *Ascensiones* and the *Aequatio dierum* both given in grades and minutes.

In Nallino the equation of time table is called *nychthemeron* [11], on the heading we find *Initium tabularum ascensionum signorum in sphaera recta et aequationis nychthemeron.* The argument, the solar longitude, is in the first column and it is given in intervals of one degree; the table starts with Capricorn in which we find, in the second column, the ascension given in degrees and minutes; in the third column we find the *aequatio nychthemeron* in grades and minutes. We find three zodiacal signs per each table.

In Pedersen the table [12] is cited in all the canons and it is called *tabula circula directi*, for the ascension values we find *ascensiones* or *elevationes*, for the equation of time we find *aequatio dierum cum noctibus suis.* The tables normally starts with Capricorn. The table is the same as al-Battānī in Nallino II p. 61-64, and it shows the values for single degrees. The value used for the obliquity is $\varepsilon = 23°35'$, there are systematic errors present in the al-Battānī's tradition. To recompute the table Pedersen used this value for the ecliptic and the Nallino's value for the solar eccentricity $e = 2;4,45$ parts, for the solar apogee $\lambda_A = 82;15°$, and $E_0 = 4;6,30°$ for the epoch constant chosen to make the tabular values positive.



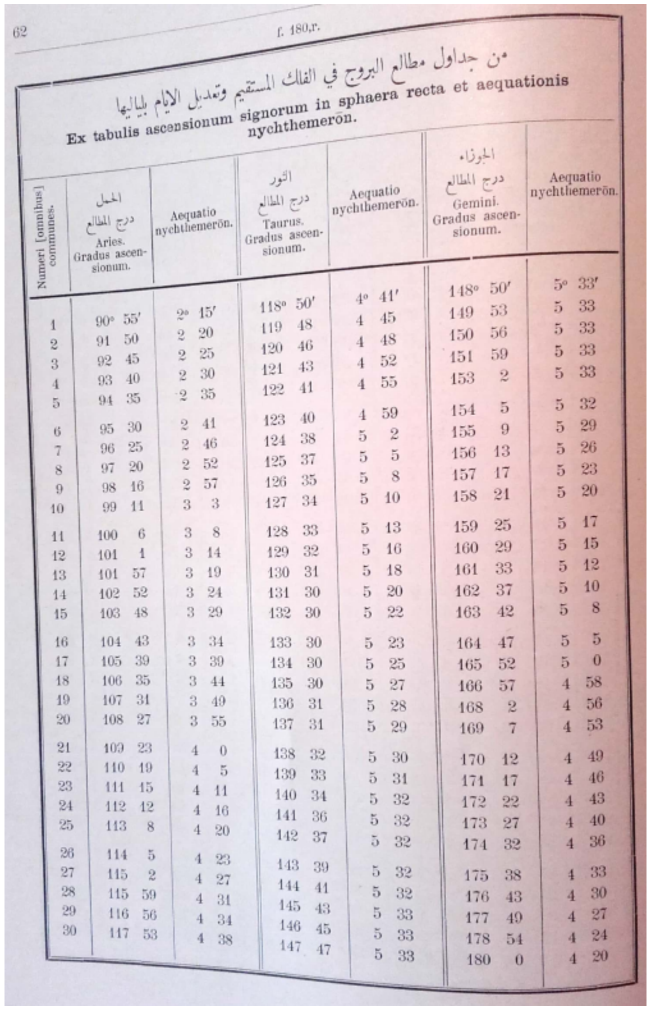
f. 179,v.

ابتداء جداول مطالع البروج في الفلك المستقيم وتعديل الايام بلياليها

Initium tabularum ascensionum signorum in sphaera recta et aequationis nychthemerōn.

العدد المشترك Numeri [omnibus] communes.	الجدي درج المطالع Capricornus. Gradus ascensionum.	تعديل الايام بلياليها Aequatio nychthemerōn.	الدلو درج المطالع Aquarius. Gradus ascensionum.	Aequatio nychthemerōn.	الحوت درج المطالع Pisces. Gradus ascensionum.	Aequatio nychthemerōn.
1	1° 5'	3° 41'	33° 15'	0° 37'	63° 4'	0° 12'
2	2 11	3 33	34 17	0 33	64 1	0 15
3	3 17	3 25	35 19	0 30	64 58	0 17
4	4 22	3 18	36 21	0 26	65 55	0 20
5	5 28	3 11	37 23	0 23	66 52	0 22
6	6 33	3 4	38 25	0 20	67 48	0 25
7	7 38	2 57	39 26	0 18	68 45	0 28
8	8 43	2 50	40 27	0 15	69 41	0 32
9	9 48	2 43	41 28	0 13	70 37	0 35
10	10 53	2 37	42 29	0 10	71 34	0 38
11	11 58	2 31	43 29	0 8	72 29	0 42
12	13 3	2 24	44 30	0 6	73 25	0 45
13	14 8	2 17	45 30	0 5	74 21	0 49
14	15 13	2 10	46 30	0 4	75 17	0 54
15	16 18	2 3	47 30	0 3	76 12	0 58
16	17 23	1 56	48 30	0 2	77 8	1 3
17	18 27	1 49	49 29	0 1	78 3	1 7
18	19 31	1 43	50 28	0 0	78 59	1 12
19	20 35	1 37	51 27	0 0	79 54	1 16
20	21 39	1 31	52 26	0 1	80 49	1 21
21	22 43	1 25	53 25	0 1	81 44	1 25
22	23 47	1 19	54 24	0 1	82 40	1 30
23	24 51	1 14	55 22	0 2	83 35	1 34
24	25 55	1 9	56 20	0 2	84 30	1 39
25	26 58	1 4	57 19	0 3	85 25	1 44
26	28 1	0 59	58 17	0 4	86 20	1 49
27	29 4	0 54	59 14	0 6	87 15	1 54
28	30 7	0 49	60 12	0 7	88 10	2 0
29	31 10	0 45	61 10	0 8	89 5	2 5
30	32 13	0 41	62 7	0 10	90 0	2 10

Figure 18: Al-Battānī's equation of time table part I in Nallino.



62 f. 180,r.

من جداول مطالع البروج في الفلك المستقيم وتعديل الايام بلياليها

Ex tabulis ascensionum signorum in sphaera recta et aequationis nychthemerōn.

Numeri [omnibus] communes.	الحمل درج المطالع Aries. Gradus ascensionum.	Aequatio nychthemerōn.	الثور درج المطالع Taurus. Gradus ascensionum.	Aequatio nychthemerōn.	الجوزاء درج المطالع Gemini. Gradus ascensionum.	Aequatio nychthemerōn.
1	90° 55′	2° 15′	118° 50′	4° 41′	148° 50′	5° 33′
2	91 50	2 20	119 48	4 45	149 53	5 33
3	92 45	2 25	120 46	4 48	150 56	5 33
4	93 40	2 30	121 43	4 52	151 59	5 33
5	94 35	2 35	122 41	4 55	153 2	5 33
6	95 30	2 41	123 40	4 59	154 5	5 32
7	96 25	2 46	124 38	5 2	155 9	5 29
8	97 20	2 52	125 37	5 5	156 13	5 26
9	98 16	2 57	126 35	5 8	157 17	5 23
10	99 11	3 3	127 34	5 10	158 21	5 20
11	100 6	3 8	128 33	5 13	159 25	5 17
12	101 1	3 14	129 32	5 16	160 29	5 15
13	101 57	3 19	130 31	5 18	161 33	5 12
14	102 52	3 24	131 30	5 20	162 37	5 10
15	103 48	3 29	132 30	5 22	163 42	5 8
16	104 43	3 34	133 30	5 23	164 47	5 5
17	105 39	3 39	134 30	5 25	165 52	5 0
18	106 35	3 44	135 30	5 27	166 57	4 58
19	107 31	3 49	136 31	5 28	168 2	4 56
20	108 27	3 55	137 31	5 29	169 7	4 53
21	109 23	4 0	138 32	5 30	170 12	4 49
22	110 19	4 5	139 33	5 31	171 17	4 46
23	111 15	4 11	140 34	5 32	172 22	4 43
24	112 12	4 16	141 36	5 32	173 27	4 40
25	113 8	4 20	142 37	5 32	174 32	4 36
26	114 5	4 23	143 39	5 32	175 38	4 33
27	115 2	4 27	144 41	5 32	176 43	4 30
28	115 59	4 31	145 43	5 33	177 49	4 27
29	116 56	4 34	146 45	5 33	178 54	4 24
30	117 53	4 38	147 47	5 33	180 0	4 20

Figure 19: Al-Battānī's equation of time table part II in Nallino.

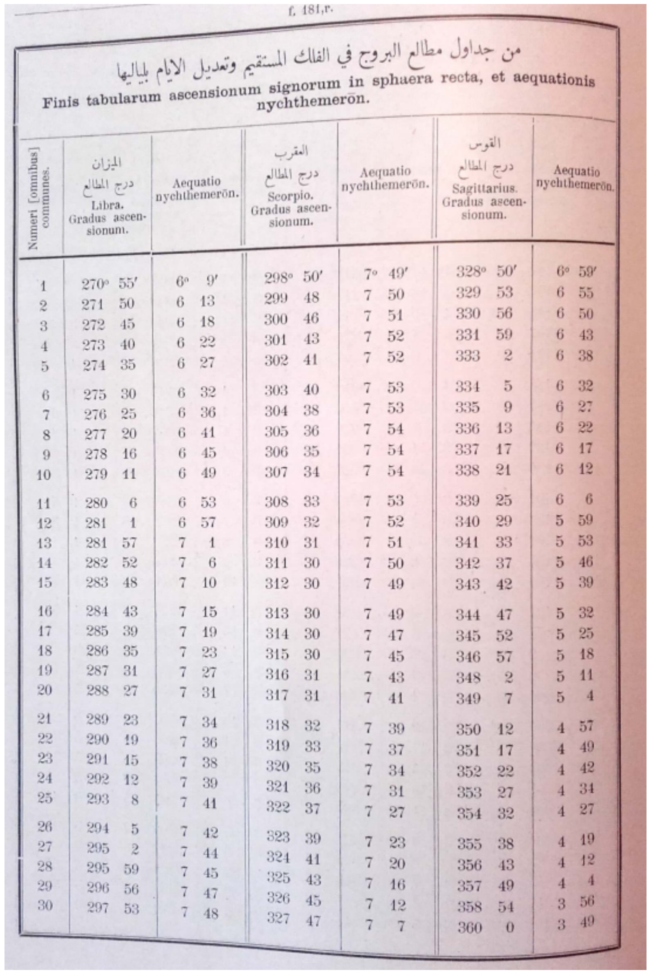
f. 180,v.

من جداول مطالع البروج في الفلك المستقيم وتعديل الايام بلياليها

Ex tabulis ascensionum signorum in sphaera recta, et aequationis nychthemerōn.

Numeri [omnibus] communes.	السرطان درج المطالع Cancer. Gradus ascensionum.	Aequatio nychthemerōn.	الاسد درج المطالع Leo. Gradus ascensionum.	Aequatio nychthemerōn.	السنبلة درج المطالع Virgo. Gradus ascensionum.	Aequatio nychthemerōn.
1	181° 5′	4° 17′	213° 15′	3° 4′	243° 4′	3° 48′
2	182 10	4 14	214 17	3 4	244 1	3 51
3	183 16	4 10	215 19	3 4	244 58	3 54
4	184 22	4 7	216 21	3 4	245 55	3 57
5	185 28	4 3	217 23	3 4	246 52	4 1
6	186 33	4 0	218 25	3 4	247 48	4 6
7	187 38	3 57	219 26	3 4	248 45	4 11
8	188 43	3 55	220 27	3 4	249 41	4 16
9	189 48	3 51	221 28	3 4	250 37	4 21
10	190 53	3 48	222 29	3 5	251 33	4 26
11	191 58	3 45	223 29	3 6	252 29	4 31
12	193 3	3 42	224 30	3 7	253 25	4 36
13	194 8	3 39	225 30	3 8	254 21	4 41
14	195 13	3 36	226 30	3 9	255 17	4 46
15	196 18	3 33	227 30	3 10	256 12	4 51
16	197 23	3 30	228 30	3 11	257 8	4 56
17	198 27	3 27	229 29	3 13	258 3	5 1
18	199 31	3 25	230 28	3 14	258 59	5 6
19	200 35	3 23	231 27	3 16	259 54	5 11
20	201 39	3 21	232 26	3 17	260 49	5 16
21	202 43	3 19	233 25	3 19	261 44	5 21
22	203 47	3 17	234 24	3 21	262 40	5 26
23	204 51	3 15	235 22	3 24	263 35	5 31
24	205 55	3 13	236 20	3 27	264 30	5 36
25	206 58	3 11	237 19	3 30	265 25	5 41
26	208 1	3 9	238 17	3 33	266 20	5 45
27	209 4	3 8	239 14	3 36	267 15	5 50
28	210 7	3 7	240 12	3 39	268 10	5 55
29	211 10	3 6	241 10	3 42	269 5	5 59
30	212 13	3 5	242 7	3 45	270 0	6 4

Figure 20: Al-Battānī's equation of time table part III in Nallino.



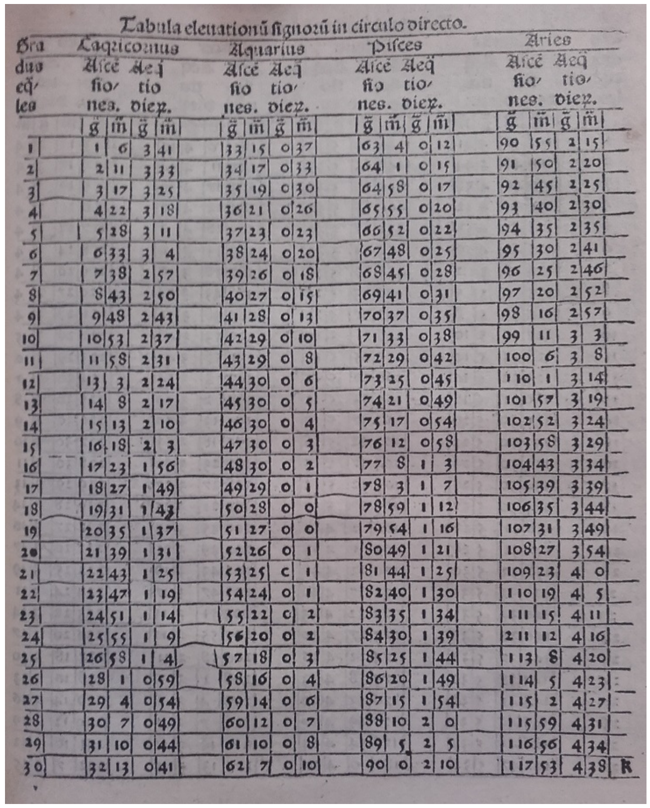
f. 181,r.

من جداول مطالع البروج في الفلك المستقيم وتعديل الايام بلياليها

Finis tabularum ascensionum signorum in sphaera recta, et aequationis nychthemerōn.

Numeri [omnibus] communes.	الميزان درج المطالع Libra. Gradus ascensionum.	Aequatio nychthemerōn.	العقرب درج المطالع Scorpio. Gradus ascensionum.	Aequatio nychthemerōn.	القوس درج المطالع Sagittarius. Gradus ascensionum.	Aequatio nychthemerōn.
1	270° 55′	6° 9′	298° 50′	7° 49′	328° 50′	6° 59′
2	271 50	6 13	299 48	7 50	329 53	6 55
3	272 45	6 18	300 46	7 51	330 56	6 50
4	273 40	6 22	301 43	7 52	331 59	6 43
5	274 35	6 27	302 41	7 52	333 2	6 38
6	275 30	6 32	303 40	7 53	334 5	6 32
7	276 25	6 36	304 38	7 53	335 9	6 27
8	277 20	6 41	305 36	7 54	336 13	6 22
9	278 16	6 45	306 35	7 54	337 17	6 17
10	279 11	6 49	307 34	7 54	338 21	6 12
11	280 6	6 53	308 33	7 53	339 25	6 6
12	281 1	6 57	309 32	7 52	340 29	5 59
13	281 57	7 1	310 31	7 51	341 33	5 53
14	282 52	7 6	311 30	7 50	342 37	5 46
15	283 48	7 10	312 30	7 49	343 42	5 39
16	284 43	7 15	313 30	7 49	344 47	5 32
17	285 39	7 19	314 30	7 47	345 52	5 25
18	286 35	7 23	315 30	7 45	346 57	5 18
19	287 31	7 27	316 31	7 43	348 2	5 11
20	288 27	7 31	317 31	7 41	349 7	5 4
21	289 23	7 34	318 32	7 39	350 12	4 57
22	290 19	7 36	319 33	7 37	351 17	4 49
23	291 15	7 38	320 35	7 34	352 22	4 42
24	292 12	7 39	321 36	7 31	353 27	4 34
25	293 8	7 41	322 37	7 27	354 32	4 27
26	294 5	7 42	323 39	7 23	355 38	4 19
27	295 2	7 44	324 41	7 20	356 43	4 12
28	295 59	7 45	325 43	7 16	357 49	4 4
29	296 56	7 47	326 45	7 12	358 54	3 56
30	297 53	7 48	327 47	7 7	360 0	3 49

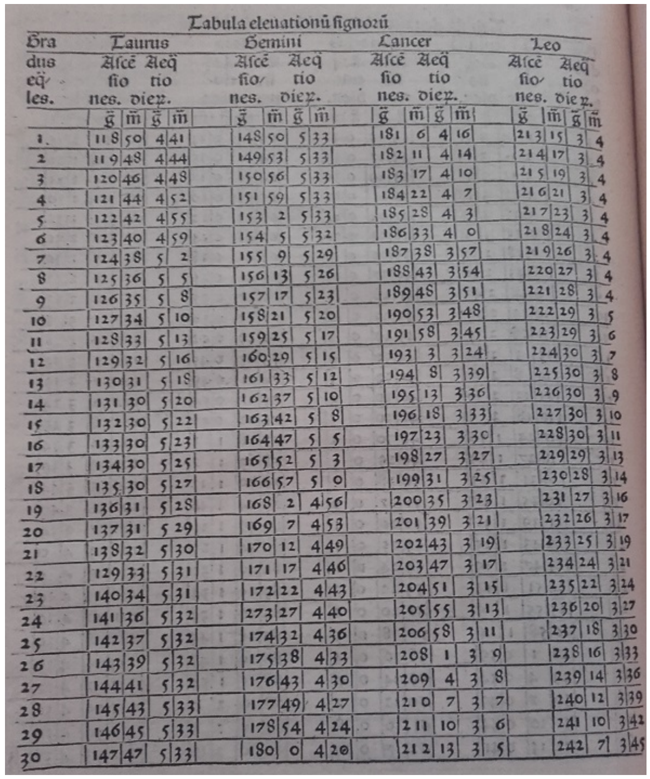
Figure 21: Al-Battānī's equation of time table part IV in Nallino.



Tabula eleuationū signorū in circulo directo.

Gradus eq̄les	Capricornus Ascēsiones g	Capricornus Ascēsiones m	Capricornus Aeq̄tio dierū g	Capricornus Aeq̄tio dierū m	Aquarius Ascēsiones g	Aquarius Ascēsiones m	Aquarius Aeq̄tio dierū g	Aquarius Aeq̄tio dierū m	Pisces Ascēsiones g	Pisces Ascēsiones m	Pisces Aeq̄tio dierū g	Pisces Aeq̄tio dierū m	Aries Ascēsiones g	Aries Ascēsiones m	Aries Aeq̄tio dierū g	Aries Aeq̄tio dierū m
1	1	6	3	41	33	15	0	37	63	4	0	12	90	55	2	15
2	2	11	3	33	34	17	0	33	64	1	0	15	91	50	2	20
3	3	17	3	25	35	19	0	30	64	58	0	17	92	45	2	25
4	4	22	3	18	36	21	0	26	65	55	0	20	93	40	2	30
5	5	28	3	11	37	23	0	23	66	52	0	22	94	35	2	35
6	6	33	3	4	38	24	0	20	67	48	0	25	95	30	2	41
7	7	38	2	57	39	26	0	18	68	45	0	28	96	25	2	46
8	8	43	2	50	40	27	0	15	69	41	0	31	97	20	2	52
9	9	48	2	43	41	28	0	13	70	37	0	35	98	16	2	57
10	10	53	2	37	42	29	0	10	71	33	0	38	99	11	3	3
11	11	58	2	31	43	29	0	8	72	29	0	42	100	6	3	8
12	13	3	2	24	44	30	0	6	73	25	0	45	110	1	3	14
13	14	8	2	17	45	30	0	5	74	21	0	49	101	57	3	19
14	15	13	2	10	46	30	0	4	75	17	0	54	102	52	3	24
15	16	18	2	3	47	30	0	3	76	12	0	58	103	58	3	29
16	17	23	1	56	48	30	0	2	77	8	1	3	104	43	3	34
17	18	27	1	49	49	29	0	1	78	3	1	7	105	39	3	39
18	19	31	1	43	50	28	0	0	78	59	1	12	106	35	3	44
19	20	35	1	37	51	27	0	0	79	54	1	16	107	31	3	49
20	21	39	1	31	52	26	0	1	80	49	1	21	108	27	3	54
21	22	43	1	25	53	25	c	1	81	44	1	25	109	23	4	0
22	23	47	1	19	54	24	0	1	82	40	1	30	110	19	4	5
23	24	51	1	14	55	22	c	2	83	35	1	34	111	15	4	11
24	25	55	1	9	56	20	0	2	84	30	1	39	211	12	4	16
25	26	58	1	4	57	18	0	3	85	25	1	44	113	8	4	20
26	28	1	0	59	58	16	0	4	86	20	1	49	114	5	4	23
27	29	4	0	54	59	14	0	6	87	15	1	54	115	2	4	27
28	30	7	0	49	60	12	0	7	88	10	2	0	115	59	4	31
29	31	10	0	44	61	10	0	8	89	5	2	5	116	56	4	34
30	32	13	0	41	62	7	0	10	90	0	2	10	117	53	4	38 R

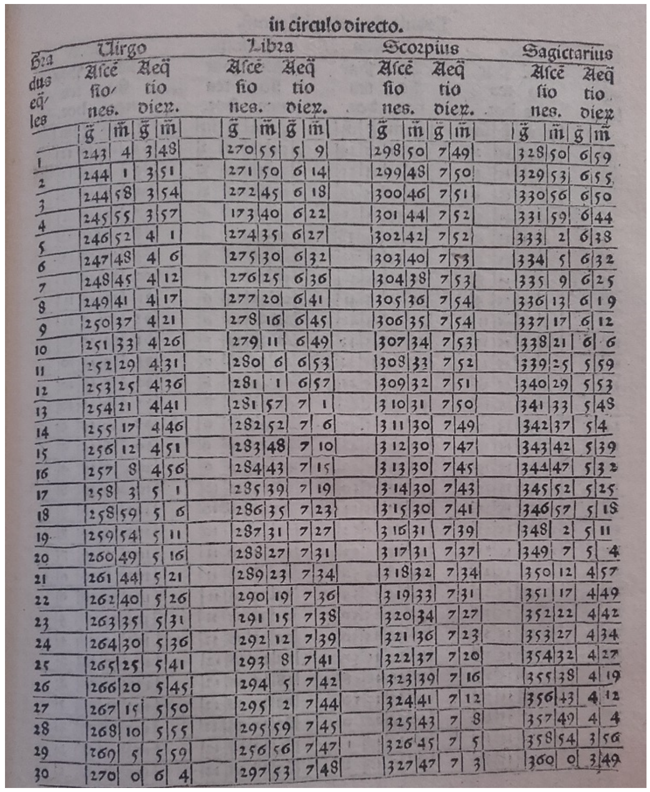
Figure 22: Al-Battānī's equation of time table part I in Ratdolt.



Tabula eleuationũ signorũ

Gradus eq̃les.	Taurus Ascẽsiones g	Taurus Ascẽsiones m	Taurus Aeq̃tio dierũ g	Taurus Aeq̃tio dierũ m	Gemini Ascẽsiones g	Gemini Ascẽsiones m	Gemini Aeq̃tio dierũ g	Gemini Aeq̃tio dierũ m	Cancer Ascẽsiones g	Cancer Ascẽsiones m	Cancer Aeq̃tio dierũ g	Cancer Aeq̃tio dierũ m	Leo Ascẽsiones g	Leo Ascẽsiones m	Leo Aeq̃tio dierũ g	Leo Aeq̃tio dierũ m
1.	118	50	4	41	148	50	5	33	181	6	4	16	213	15	3	4
2	119	48	4	44	149	53	5	33	182	11	4	14	214	17	3	4
3	120	46	4	48	150	56	5	33	183	17	4	10	215	19	3	4
4	121	44	4	52	151	59	5	33	184	22	4	7	216	21	3	4
5	122	42	4	55	153	2	5	33	185	28	4	3	217	23	3	4
6	123	40	4	59	154	5	5	32	186	33	4	0	218	24	3	4
7	124	38	5	2	155	9	5	29	187	38	3	57	219	26	3	4
8	125	36	5	5	156	13	5	26	188	43	3	54	220	27	3	4
9	126	35	5	8	157	17	5	23	189	48	3	51	221	28	3	4
10	127	34	5	10	158	21	5	20	190	53	3	48	222	29	3	5
11	128	33	5	13	159	25	5	17	191	58	3	45	223	29	3	6
12	129	32	5	16	160	29	5	15	193	3	3	24	224	30	3	7
13	130	31	5	18	161	33	5	12	194	8	3	39	225	30	3	8
14	131	30	5	20	162	37	5	10	195	13	3	36	226	30	3	9
15	132	30	5	22	163	42	5	8	196	18	3	33	227	30	3	10
16	133	30	5	23	164	47	5	5	197	23	3	30	228	30	3	11
17	134	30	5	25	165	52	5	3	198	27	3	27	229	29	3	13
18	135	30	5	27	166	57	5	0	199	31	3	25	230	28	3	14
19	136	31	5	28	168	2	4	56	200	35	3	23	231	27	3	16
20	137	31	5	29	169	7	4	53	201	39	3	21	232	26	3	17
21	138	32	5	30	170	12	4	49	202	43	3	19	233	25	3	19
22	129	33	5	31	171	17	4	46	203	47	3	17	234	24	3	21
23	140	34	5	31	172	22	4	43	204	51	3	15	235	22	3	24
24	141	36	5	32	273	27	4	40	205	55	3	13	236	20	3	27
25	142	37	5	32	174	32	4	36	206	58	3	11	237	18	3	30
26	143	39	5	32	175	38	4	33	208	1	3	9	238	16	3	33
27	144	41	5	32	176	43	4	30	209	4	3	8	239	14	3	36
28	145	43	5	33	177	49	4	27	210	7	3	7	240	12	3	39
29	146	45	5	33	178	54	4	24	211	10	3	6	241	10	3	42
30	147	47	5	33	180	0	4	20	212	13	3	5	242	7	3	45

Figure 23: Al-Battānī's equation of time table part II in Ratdolt.



in circulo directo.

Gradus equales	Virgo Ascensiones g	m	Aequatio dierum g	m	Libra Ascensiones g	m	Aequatio dierum g	m	Scorpius Ascensiones g	m	Aequatio dierum g	m	Sagictarius Ascensiones g	m	Aequatio dierum g	m
1	243	4	3	48	270	55	5	9	298	50	7	49	328	50	6	59
2	244	1	3	51	271	50	6	14	299	48	7	50	329	53	6	55
3	244	58	3	54	272	45	6	18	300	46	7	51	330	56	6	50
4	245	55	3	57	173	40	6	22	301	44	7	52	331	59	6	44
5	246	52	4	1	274	35	6	27	302	42	7	52	333	2	6	38
6	247	48	4	6	275	30	6	32	303	40	7	53	334	5	6	32
7	248	45	4	12	276	25	6	36	304	38	7	53	335	9	6	25
8	249	41	4	17	277	20	6	41	305	36	7	54	336	13	6	19
9	250	37	4	21	278	16	6	45	306	35	7	54	337	17	6	12
10	251	33	4	26	279	11	6	49	307	34	7	53	338	21	6	6
11	252	29	4	31	280	6	6	53	308	33	7	52	339	25	5	59
12	253	25	4	36	281	1	6	57	309	32	7	51	340	29	5	53
13	254	21	4	41	281	57	7	1	310	31	7	50	341	33	5	48
14	255	17	4	46	282	52	7	6	311	30	7	49	342	37	5	4
15	256	12	4	51	283	48	7	10	312	30	7	47	343	42	5	39
16	257	8	4	56	284	43	7	15	313	30	7	45	344	47	5	32
17	258	3	5	1	285	39	7	19	314	30	7	43	345	52	5	25
18	258	59	5	6	286	35	7	23	315	30	7	41	346	57	5	18
19	259	54	5	11	287	31	7	27	316	31	7	39	348	2	5	11
20	260	49	5	16	288	27	7	31	317	31	7	37	349	7	5	4
21	261	44	5	21	289	23	7	34	318	32	7	34	350	12	4	57
22	262	40	5	26	290	19	7	36	319	33	7	31	351	17	4	49
23	263	35	5	31	291	15	7	38	320	34	7	27	352	22	4	42
24	264	30	5	36	292	12	7	39	321	36	7	23	353	27	4	34
25	265	25	5	41	293	8	7	41	322	37	7	20	354	32	4	27
26	266	20	5	45	294	5	7	42	323	39	7	16	355	38	4	19
27	267	15	5	50	295	2	7	44	324	41	7	12	356	43	4	12
28	268	10	5	55	295	59	7	45	325	43	7	8	357	49	4	4
29	269	5	5	59	256	56	7	47	326	45	7	5	358	54	3	56
30	270	0	6	4	297	53	7	48	327	47	7	3	360	0	3	49

Figure 24: Al-Battānī's equation of time table part III in Ratdolt.

5.3 Analyses of the table

Our set of parameters is:

- $\varepsilon = 23;35°$ (historical);

- $e = 2;4,45$ parts (historical);

- $\lambda_A = 82;15°$ (historical);

- $E_0 = 4;6,30°$ (historical);

We used the solar equation table and right ascension tables from the *Toledan Tables*.

5.3.1 Recomputation by modern method

With this set of parameters we get the best fit with a mean value for the differences $E_M = 0.05$ and a standard deviation $\sigma = 2.088$ (values are given in minutes of degrees).

5.3.2 Recomputation by the use of tables

With this set of parameters we get the best fit with a mean value for the differences $E_M = 0.45$ and a standard deviation $\sigma = 2.12$ (values are given in minutes of degree) with the double interpolation to change the argument of the solar equation.

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
1	3	41	3	43	-2
2	3	33	3	35	-2
3	3	25	3	27	-2
4	3	18	3	20	-2
5	3	11	3	12	-1
6	3	4	3	5	-1
7	2	57	2	58	-1
8	2	50	2	51	-1
9	2	43	2	44	-1
10	2	37	2	37	0
11	2	31	2	30	1
12	2	24	2	23	1
13	2	17	2	16	1
14	2	10	2	9	1
15	2	3	2	2	1
16	1	56	1	55	1
17	1	49	1	49	0
18	1	43	1	43	0
19	1	37	1	38	-1
20	1	31	1	32	-1
21	1	25	1	26	-1
22	1	19	1	20	-1
23	1	14	1	14	0
24	1	9	1	8	1
25	1	4	1	4	0
26	0	59	0	59	0
27	0	54	0	54	0
28	0	49	0	49	0
29	0	45	0	45	0
30	0	41	0	40	1
31	0	37	0	36	1
32	0	33	0	33	0
33	0	30	0	29	1
34	0	26	0	26	0
35	0	23	0	22	1
36	0	20	0	20	0
37	0	18	0	16	2
38	0	15	0	14	1
39	0	13	0	11	2
40	0	10	0	9	1

Table 20: Al-Battānī: data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
41	0	8	0	8	0
42	0	6	0	5	1
43	0	5	0	4	1
44	0	4	0	3	1
45	0	3	0	1	2
46	0	2	0	0	2
47	0	1	0	0	1
48	0	0	-1	59	1
49	0	0	-1	59	1
50	0	1	-1	59	2
51	0	1	-1	59	2
52	0	1	-1	59	2
53	0	2	0	0	2
54	0	2	0	1	1
55	0	3	0	1	2
56	0	4	0	2	2
57	0	6	0	4	2
58	0	7	0	5	2
59	0	8	0	7	1
60	0	10	0	9	1
61	0	12	0	11	1
62	0	15	0	13	2
63	0	17	0	16	1
64	0	20	0	18	2
65	0	22	0	20	2
66	0	25	0	24	1
67	0	28	0	26	2
68	0	32	0	29	3
69	0	35	0	33	2
70	0	38	0	35	3
71	0	42	0	40	2
72	0	45	0	44	1
73	0	49	0	47	2
74	0	54	0	51	3
75	0	58	0	56	2
76	1	3	1	0	3
77	1	7	1	4	3
78	1	12	1	8	4
79	1	16	1	13	3
80	1	21	1	18	3

Table 21: Al-Battānī: data from manuscript, recomputation and differences, part II.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
81	1	25	1	23	2
82	1	30	1	27	3
83	1	34	1	32	2
84	1	39	1	37	2
85	1	44	1	42	2
86	1	49	1	47	2
87	1	54	1	52	2
88	2	0	1	57	3
89	2	5	2	3	2
90	2	10	2	8	2
91	2	15	2	13	2
92	2	20	2	19	1
93	2	25	2	24	1
94	2	30	2	29	1
95	2	35	2	35	0
96	2	41	2	40	1
97	2	46	2	46	0
98	2	52	2	51	1
99	2	57	2	56	1
100	3	3	3	2	1
101	3	8	3	7	1
102	3	14	3	13	1
103	3	19	3	18	1
104	3	24	3	23	1
105	3	29	3	28	1
106	3	34	3	34	0
107	3	39	3	39	0
108	3	44	3	44	0
109	3	49	3	49	0
110	3	55	3	54	1
111	4	0	3	59	1
112	4	5	4	4	1
113	4	11	4	9	2
114	4	16	4	13	3
115	4	20	4	18	2
116	4	23	4	22	1
117	4	27	4	26	1
118	4	31	4	30	1
119	4	34	4	35	-1
120	4	38	4	39	-1

Table 22: Al-Battānī: data from manuscript, recomputation and differences, part III.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
121	4	41	4	43	-2
122	4	45	4	46	-1
123	4	48	4	50	-2
124	4	52	4	54	-2
125	4	55	4	58	-3
126	4	59	5	0	-1
127	5	5	5	3	2
128	5	5	5	6	-1
129	5	8	5	9	-1
130	5	10	5	12	-2
131	5	13	5	14	-1
132	5	16	5	17	-1
133	5	18	5	20	-2
134	5	20	5	22	-2
135	5	22	5	24	-2
136	5	23	5	26	-3
137	5	25	5	27	-2
138	5	27	5	29	-2
139	5	28	5	30	-2
140	5	29	5	31	-2
141	5	30	5	32	-2
142	5	31	5	33	-2
143	5	32	5	34	-2
144	5	32	5	34	-2
145	5	32	5	34	-2
146	5	32	5	34	-2
147	5	32	5	34	-2
148	5	33	5	34	-1
149	5	33	5	34	-1
150	5	33	5	34	-1
151	5	33	5	33	0
152	5	33	5	32	1
153	5	33	5	31	2
154	5	33	5	30	3
155	5	33	5	29	4
156	5	32	5	28	4
157	5	29	5	26	3
158	5	26	5	24	2
159	5	23	5	22	1
160	5	20	5	20	0

Table 23: Al-Battānī: data from manuscript, recomputation and differences, part IV.

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
161	5	17	5	18	-1
162	5	15	5	16	-1
163	5	12	5	14	-2
164	5	10	5	12	-2
165	5	8	5	9	-1
166	5	5	5	6	-1
167	5	0	5	3	-3
168	4	58	5	0	-2
169	4	56	4	57	-1
170	4	53	4	54	-1
171	4	49	4	51	-2
172	4	46	4	48	-2
173	4	43	4	46	-3
174	4	40	4	43	-3
175	4	36	4	40	-4
176	4	33	4	36	-3
177	4	30	4	33	-3
178	4	27	4	29	-2
179	4	24	4	26	-2
180	4	20	4	22	-2
181	4	17	4	19	-2
182	4	14	4	16	-2
183	4	10	4	12	-2
184	4	7	4	8	-1
185	4	3	4	4	-1
186	4	0	4	1	-1
187	3	57	3	58	-1
188	3	55	3	55	0
189	3	51	3	52	-1
190	3	48	3	49	-1
191	3	45	3	46	-1
192	3	42	3	43	-1
193	3	39	3	40	-1
194	3	36	3	37	-1
195	3	33	3	34	-1
196	3	30	3	31	-1
197	3	27	3	29	-2
198	3	25	3	27	-2
199	3	23	3	25	-2
200	3	21	3	22	-1

Table 24: Al-Battānī: data from manuscript, recomputation and differences, part V.

λ(°)	**Manuscript** degrees	**(sexagesimal)** minutes	**Recomputation** degrees	**(sexagesimal)** minutes	**Differences (min of deg)**
201	3	19	3	20	-1
202	3	17	3	18	-1
203	3	15	3	16	-1
204	3	13	3	14	-1
205	3	11	3	12	-1
206	3	9	3	11	-2
207	3	8	3	10	-2
208	3	7	3	9	-2
209	3	6	3	7	-1
210	3	5	3	6	-1
211	3	4	3	6	-2
212	3	4	3	5	-1
213	3	4	3	5	-1
214	3	4	3	4	0
215	3	4	3	4	0
216	3	4	3	3	1
217	3	4	3	4	0
218	3	4	3	4	0
219	3	4	3	5	-1
220	3	5	3	5	0
221	3	6	3	7	-1
222	3	7	3	7	0
223	3	8	3	8	0
224	3	9	3	10	-1
225	3	10	3	11	-1
226	3	11	3	12	-1
227	3	13	3	14	-1
228	3	14	3	16	-2
229	3	16	3	19	-3
230	3	17	3	21	-4
231	3	19	3	23	-4
232	3	21	3	25	-4
233	3	24	3	28	-4
234	3	27	3	31	-4
235	3	30	3	33	-3
236	3	33	3	36	-3
237	3	36	3	40	-4
238	3	39	3	43	-4
239	3	42	3	45	-3
240	3	45	3	49	-4

Table 25: Al-Battānī: data from manuscript, recomputation and differences, part VI.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
241	3	48	3	53	-5
242	3	51	3	57	-6
243	3	54	4	0	-6
244	3	57	4	4	-7
245	4	1	4	8	-7
246	4	6	4	12	-6
247	4	11	4	16	-5
248	4	16	4	20	-4
249	4	21	4	25	-4
250	4	26	4	29	-3
251	4	31	4	34	-3
252	4	36	4	38	-2
253	4	41	4	43	-2
254	4	46	4	47	-1
255	4	51	4	52	-1
256	4	56	4	56	0
257	5	1	5	2	-1
258	5	6	5	6	0
259	5	11	5	11	0
260	5	16	5	16	0
261	5	21	5	21	0
262	5	26	5	25	1
263	5	31	5	30	1
264	5	36	5	35	1
265	5	41	5	40	1
266	5	45	5	45	0
267	5	50	5	50	0
268	5	55	5	55	0
269	5	59	5	59	0
270	6	4	6	4	0
271	6	9	6	9	0
272	6	13	6	13	0
273	6	18	6	18	0
274	6	22	6	23	-1
275	6	27	6	27	0
276	6	32	6	32	0
277	6	36	6	36	0
278	6	41	6	41	0
279	6	45	6	44	1
280	6	49	6	48	1

Table 26: Al-Battānī: data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
281	6	53	6	53	0
282	6	57	6	57	0
283	7	1	7	0	1
284	7	6	7	5	1
285	7	10	7	8	2
286	7	15	7	12	3
287	7	19	7	15	4
288	7	23	7	18	5
289	7	27	7	21	6
290	7	31	7	24	7
291	7	34	7	27	7
292	7	36	7	30	6
293	7	38	7	33	5
294	7	39	7	35	4
295	7	41	7	38	3
296	7	42	7	40	2
297	7	44	7	42	2
298	7	45	7	44	1
299	7	47	7	45	2
300	7	48	7	47	1
301	7	49	7	49	0
302	7	50	7	50	0
303	7	51	7	50	1
304	7	52	7	52	0
305	7	52	7	52	0
306	7	53	7	52	1
307	7	53	7	53	0
308	7	54	7	53	1
309	7	54	7	53	1
310	7	54	7	52	2
311	7	53	7	51	2
312	7	52	7	51	1
313	7	51	7	50	1
314	7	50	7	50	0
315	7	49	7	48	1
316	7	49	7	46	3
317	7	47	7	45	2
318	7	45	7	43	2
319	7	43	7	40	3
320	7	41	7	39	2

Table 27: Al-Battānī: data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript** degrees	**(sexagesimal)** minutes	**Recomputation** degrees	**(sexagesimal)** minutes	**Differences (min of deg)**
321	7	39	7	36	3
322	7	37	7	33	4
323	7	34	7	29	5
324	7	31	7	26	5
325	7	27	7	23	4
326	7	23	7	19	4
327	7	20	7	16	4
328	7	16	7	12	4
329	7	12	7	8	4
330	7	7	7	4	3
331	6	59	6	59	0
332	6	55	6	54	1
333	6	50	6	49	1
334	6	43	6	44	-1
335	6	38	6	39	-1
336	6	32	6	34	-2
337	6	27	6	28	-1
338	6	22	6	22	0
339	6	17	6	16	1
340	6	12	6	10	2
341	6	6	6	4	2
342	5	59	5	58	1
343	5	53	5	52	1
344	5	46	5	46	0
345	5	39	5	39	0
346	5	32	5	32	0
347	5	25	5	25	0
348	5	18	5	18	0
349	5	11	5	11	0
350	5	4	5	4	0
351	4	57	4	57	0
352	4	49	4	50	-1
353	4	42	4	42	0
354	4	34	4	35	-1
355	4	27	4	28	-1
356	4	19	4	20	-1
357	4	12	4	13	-1
358	4	4	4	5	-1
359	3	56	3	58	-2
360	3	49	3	50	-1

Table 28: Al-Battānī: data from manuscript, recomputation and differences, part IX.

And here are the differences in minutes of degrees:

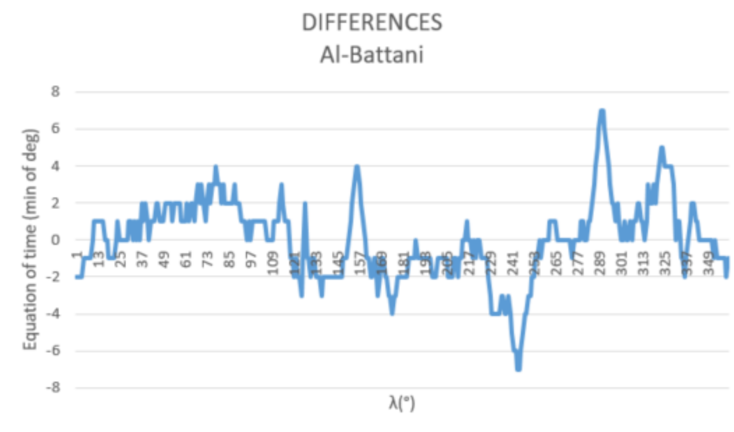


Figure 25: Al-Battānī's equation of time differences, given in minutes of degrees. Table shifted by 90 degrees for the recomputation.

5.4 Conclusions

We get similar results for both methods, the error is small if we exclude some outliers that we did not investigate. In general we need further investigation but we left this astronomer aside to give space to Alfonsine astronomy.

5.5 List of Manuscripts

- Escorial, Real Biblioteca de San Lorenzo de El Escorial, árabe 908 (*Sābi's Zīj* by al-Battānī): IV 200; VI 4; VII 11, note 6; VIII 407 et passim;
- Paris, Bibliothèque de l'Arsenal, 8322 (*Sābi's Zīj* in Castillian): VIII 407 etc;
- Oxford, Bodleian Library, ms. Savile 22 in Latin (*Toledan Tables*;
- Cambridge University Library, Kk.I.1 (*Toledan Tables*): VIII 407.

6 John of Murs

John of Murs (1290 - 1355) was a French astronomer active in Paris in the first half of the 14th century. He was one of the first transmitters of the Alfonsine Tables written in Toledo and he was one of the compilers, together with John of Lignères and John of Saxony (astronomers of King Alfonso of Castile), of the Parisian Alfonsine Tables diffused throughout Europe from the 14th to the 16th century. The Alfonsine tables of Castile are not extant but the material produced in Paris in 1320s is based on a Castilian tradition.

6.1 Overview

His main works are

1. the *Expositio intentionis regis Alfonsii circa tabulas eius*, a text with no tables were he explained the features of the tables of King Alfonso X and he mentioned only William of Saint-Cloud as a predecessor; the data mentioned in the *Expositio* (such as the daily mean motion of the Sun, the maximum solar equation, the radix for the solar motion for the epoch of Alfonso) come from the Parisian Alfonsine Tables [13];

2. the *Tables of 1321* that are tables on the planets, the Sun and the Moon;

3. the *Patefit*, a work on conjunctions and oppositions, including mean and true syzygies;

4. the *Tabulae permanentes* a short text of one table for finding the time interval from mean to true syzygy;

We find some parameters of John of Murs in the tables of John of Vimond of 1320 but probably they knew each other because they came both from Normandy.

See Appendix D for detailed description of primary and secondary sources.

6.2 Presentation of the table

John of Murs' *Kalendarium Solis et Lune* (Brussels, Bibliothèque Royale) consists in a calendar with the daily positions of the Sun for 1321, with canons and tables. For the Sun we have four tables. In the first table (*tabula solis*) we find a cycle of four years, the argument is the month and the day within the month, the year begins in January, in the last column we find the equation of time in minutes of time and this table is calculated for Paris.

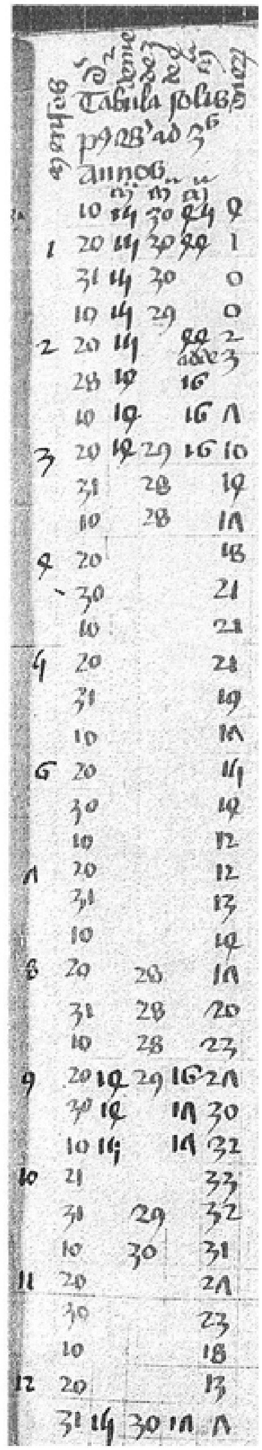


Figure 26: John of Murs' equation of time table in Kalendarium.

John of Murs's tables of 1321 are tables on the planets, the Sun and the Moon. In the table of true position of the Sun, the radices where computed by King Alfonso X for Toledo, the entries are given in physical signs, degrees, minutes and seconds, the table is valid for 1321. We have a *tabula principalis* and a *contratabula* (in which there is no double argument). The argument is the day of the year in intervals of 6 days (beginning in January), in the second column we find the correction to be subtracted from the true position of the Sun for 1321 to find its true position for years after 1321; in the third column we find the hourly velocity of the Sun in minutes and seconds per hours, in the fourth column we find the equation of time given in minutes and seconds of time. The external values of the equation of time are: minimum at $0;0°$ between the 24 jan-12 Feb, maximum at $5;14,30°$ the 6th of May, the other minimum is $3;0°$ between 18-24 July and the other maximum is $8;9,30°$ the 24th October (the entries in the table are in hours). All the parameters for the Sun are found in Parisian Alfonsine Tables.

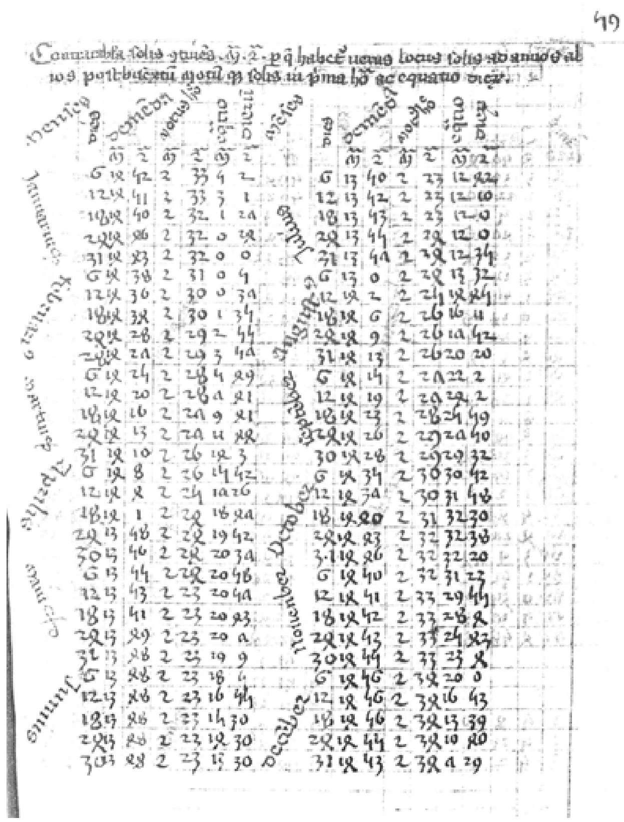


Figure 27: John of Murs' equation of time, in Tables of 1321, with the argument given in days of the year.

We have also another equation of time table, this time with λ as argument:

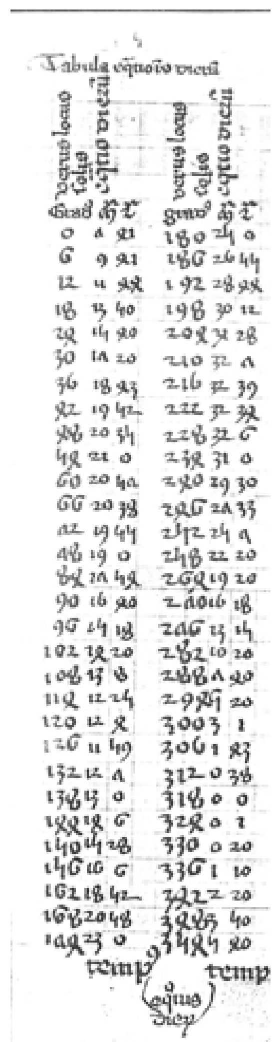


Figure 28: John of Murs' equation of time, in Tables of 1321, with λ as argument.

6.3 Analyses of the table

In the case of John of Murs, after a preliminary analyses by the use of modern formulas we decided to focus on the recomputation by the use of tables that gave us better results.

John of Murs's tables are really difficult to recompute because the argument is given in irregular intervals.

We used the *Toledan Tables* for the right ascension, for the solar equation we used both *Toledan Tables* and *Parisian Alfonsine Tables* (PAT) in order to find out the right algorithm.

For the initial constant we also tried other values recomputed by hand according to the minimum of the equation of time.

In the tables with the days as argument we make a conversion to the true solar longitude according to the *verus locus* of the sun from 12 tables (one for each month) in MS Vat. Lat. 3116 that I personally consulted in the Vatican Library in May:

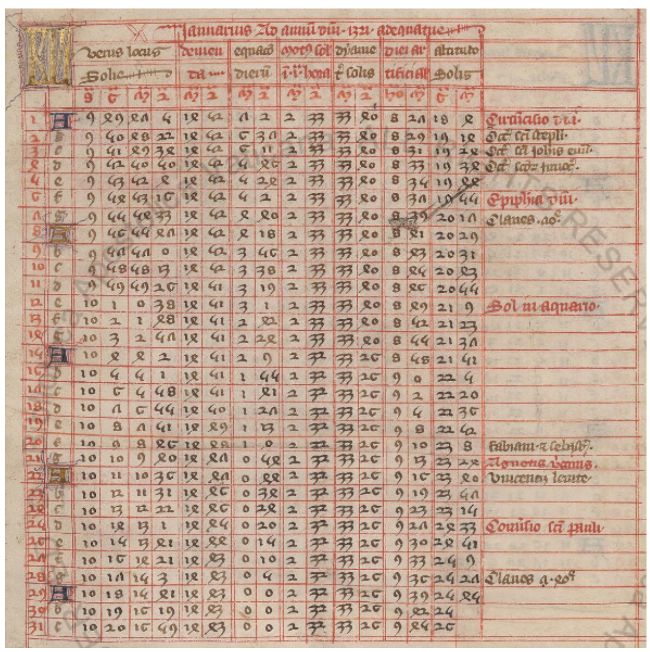


Figure 29: Verus locus, for the month of January, from MS Vat. Lat. 3116.

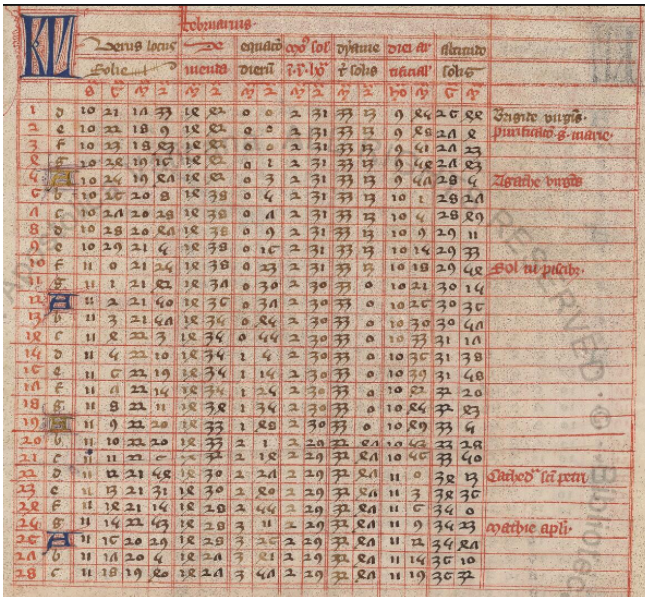


Figure 30: Verus locus, for the month of February, from MS Vat. Lat. 3116.

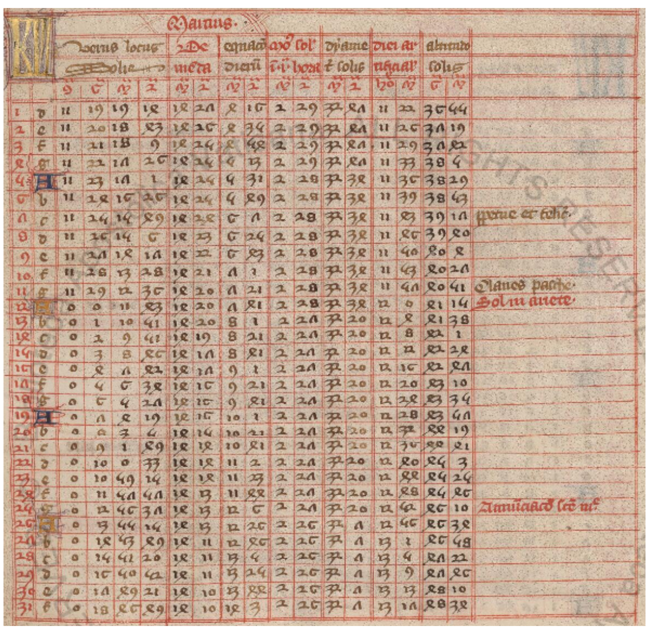


Figure 31: Verus locus, for the month of March, from MS Vat. Lat. 3116.

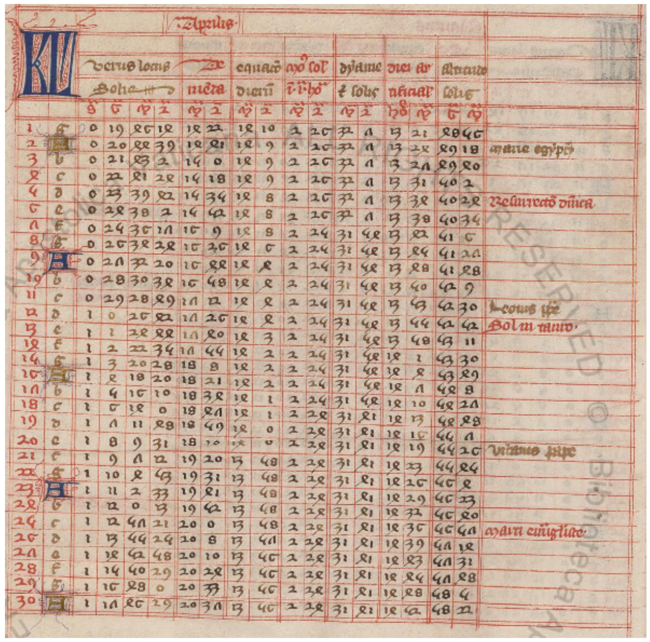


Figure 32: Verus locus, for the month of April, from MS Vat. Lat. 3116.

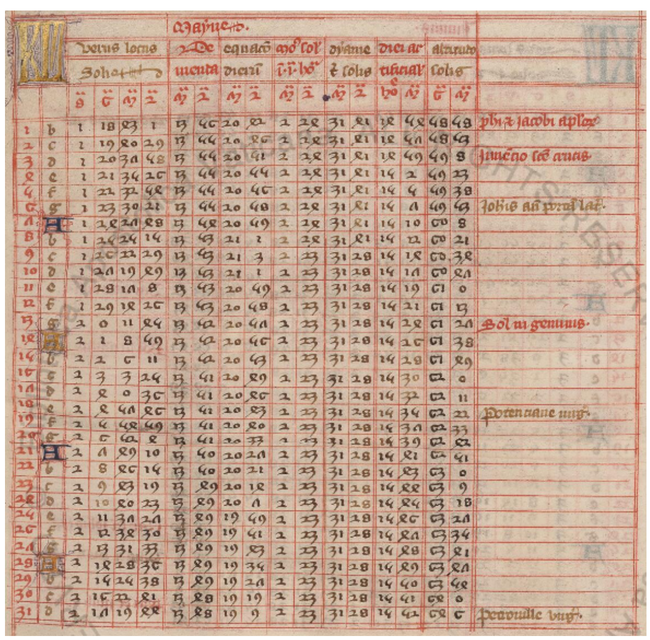


Figure 33: Verus locus, for the month of May, from MS Vat. Lat. 3116.

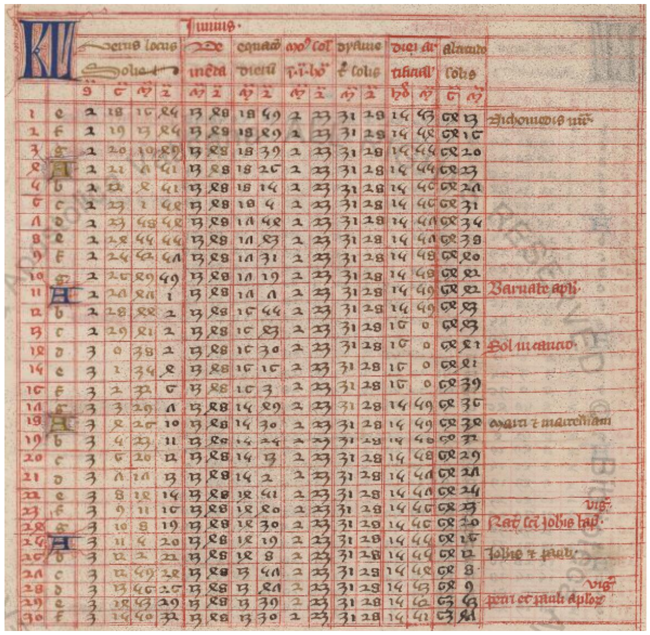


Figure 34: Verus locus, for the month of June, from MS Vat. Lat. 3116.

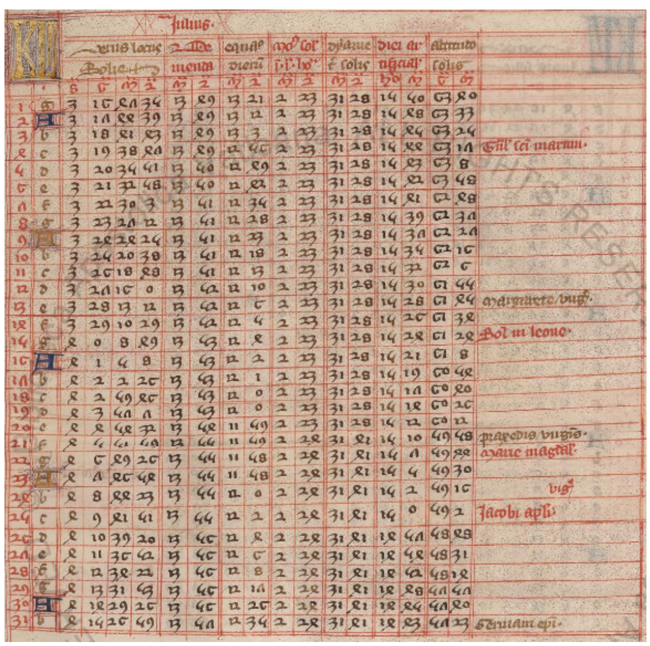


Figure 35: Verus locus, for the month of July, from MS Vat. Lat. 3116.

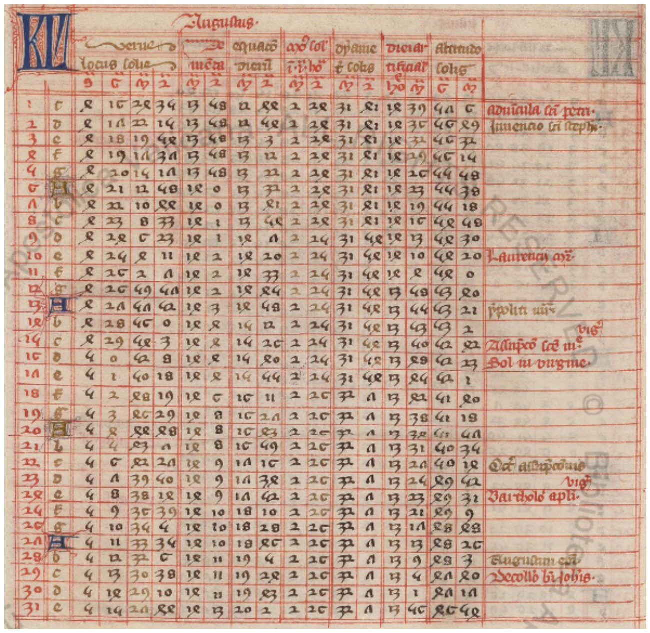


Figure 36: Verus locus, for the month of August, from MS Vat. Lat. 3116.

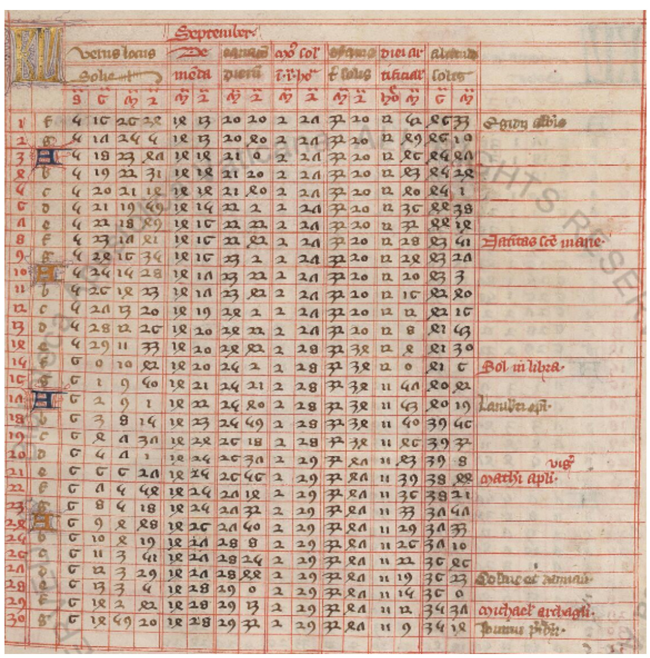


Figure 37: Verus locus, for the month of September, from MS Vat. Lat. 3116.

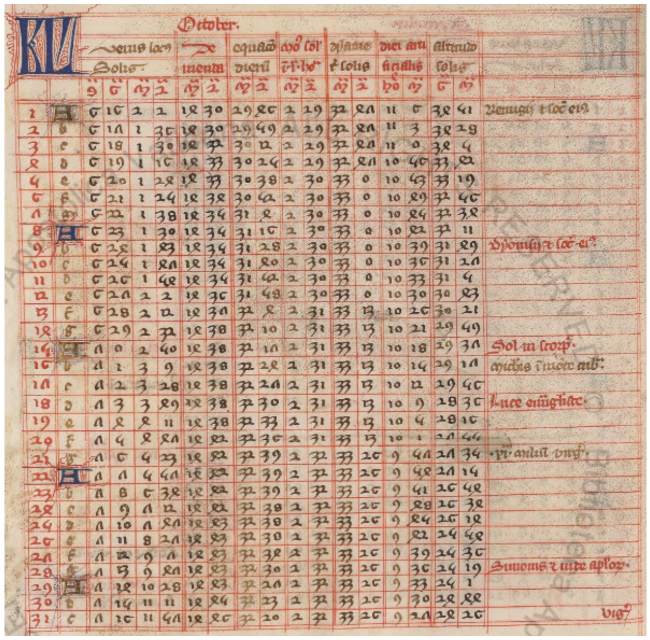


Figure 38: Verus locus, for the month of October, from MS Vat. Lat. 3116.

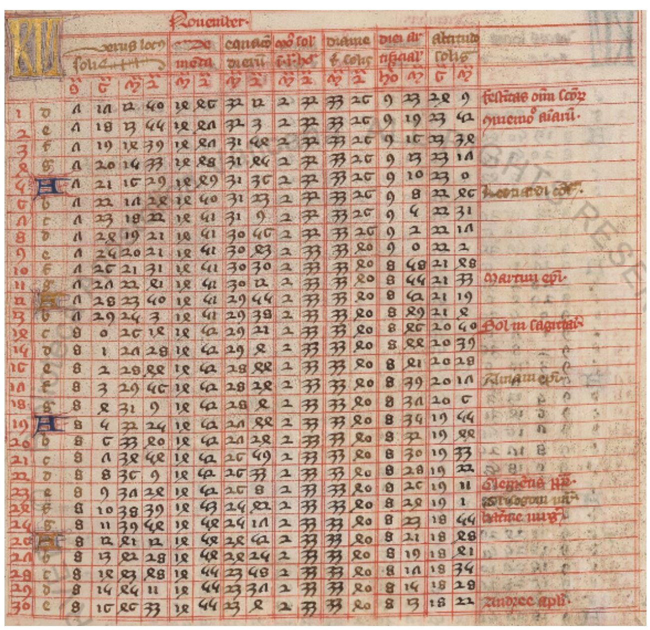


Figure 39: Verus locus, for the month of November, from MS Vat. Lat. 3116.

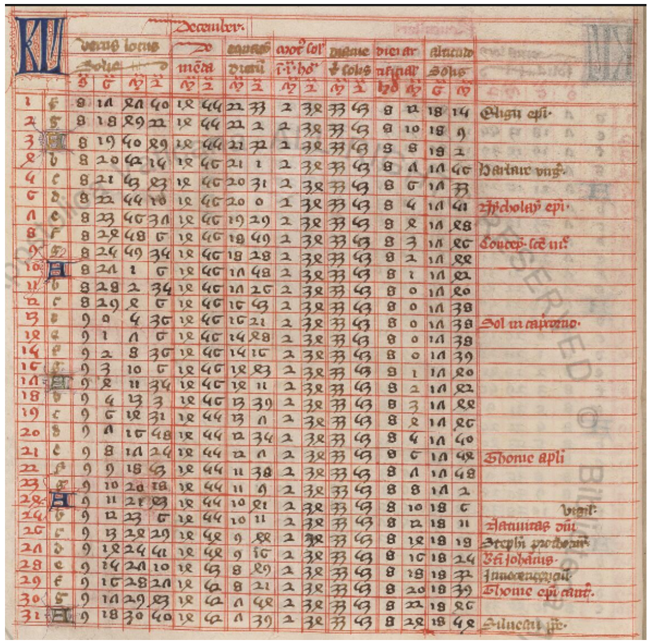


Figure 40: Verus locus, for the month of December, from MS Vat. Lat. 3116.

In the same manuscript, in f. 35r, there is a table with no name that corresponds to the right ascension table that I used with few different values that I corrected.

The manuscript is also available online at `https://digi.vatlib.it/?ling=en`.

6.3.1 Recomputation by the use of tables

We use $\lambda_A = 88, 28, 26°$ from PAT and the best estimation for the initial constant is $E_0 = 4;5,40°$. We used also $E_0 = 4;8,23°$ (calculated for $\lambda = 320°$) and $E_0 = 4;8,23°$ (calculated for $\lambda = 330°$) for the *Kalendarium*, $E_0 = 4;9,27$ (by equating the equation of time to zero for $\lambda = 318°$ for the table in λ) and $E_0 = 3;59,52$ (by equating the equation of time to zero for $\lambda = 320°$ for the table in days) but the best fit is for $E_0 = 4;5,40$ in all the works.

The best fit is for the algorithm with the solar equation table (interpolated in order to change the argument from the mean to the true solar anomaly) from the Parisian Alfonsine Tables and the results are:

- *Tables of 1321* in λ: $E_M = 0.532$ and $\sigma = 9.088$ seconds;
- *Tables of 1321* in days: $E_M = 3.055$ and $\sigma = 10.045$ seconds;
- *Kalendarium*: $E_M = 0.002$ and $\sigma = 0.4334$ minutes of time.

In the following table we can see the original data from manuscript, the recomputation and the differences for *Tables of 1321* in λ:

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
0	7	41	7	43	-2
6	9	41	9	49	-8
12	11	44	11	52	-8
18	13	50	13	56	-6
24	15	40	15	48	-8
30	17	20	17	30	-10
36	18	43	18	54	-11
42	19	52	20	2	-10
48	20	35	20	45	-10
54	21	0	21	6	-6
60	20	57	21	3	-6
66	20	38	20	38	0
72	19	55	19	57	-2
78	19	0	18	56	4
84	17	54	17	43	11
90	16	40	16	31	9
96	15	18	15	10	8
102	14	20	13	58	22
108	13	8	12	57	11
114	12	25	12	15	10
120	12	4	11	50	14
126	11	59	11	42	17
132	12	7	12	6	1
138	13	0	12	49	11
144	14	6	13	57	9
150	15	28	15	19	9
156	17	6	17	1	5
162	18	52	18	50	2
168	20	58	20	54	4
174	23	0	22	57	3

Table 29: John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in λ, part I.

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
180	25	0	25	2	-2
186	26	55	27	1	-6
192	28	44	28	46	-2
198	30	12	30	22	-10
204	31	28	31	36	-8
210	32	7	32	27	-20
216	32	39	32	50	-11
222	32	34	32	47	-13
228	32	6	32	13	-7
234	31	0	31	10	-10
240	29	30	29	34	-4
246	27	33	27	31	2
252	25	7	25	8	-1
258	22	20	22	23	-3
264	19	20	19	22	-2
270	16	18	16	17	1
276	13	14	13	12	2
282	10	20	10	12	8
288	7	40	7	27	13
294	5	20	5	4	16
300	3	1	3	1	0
306	1	43	1	27	16
312	0	38	0	24	14
318	0	0	0	8	-8
324	0	1	0	10	-9
330	0	20	0	12	8
336	1	10	1	5	5
342	2	20	2	20	0
348	3	50	3	57	-7
354	5	40	5	43	-3

Table 30: John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in λ, part II.

for *Tables of 1321* in days λ is taken from MS Vat. Lat. 3116:

$\lambda(°)$	**Day of the Year**	**MS** minutes	**(sexagesimal)** seconds	**Rec.** minutes	**(sexagesimal)** seconds	**Differences (sec)**
295	January 6	5	2	4	40	22
301	12	3	1	2	41	20
307	18	1	27	1	16	11
313	24	0	24	0	18	6
320	31	0	0	0	12	-12
326	February 6	0	5	0	4	1
332	12	0	37	0	28	9
338	18	1	35	1	26	9
344	24	2	55	2	52	3
348	28	3	57	3	57	0
354	March 6	5	49	5	43	6
360	12	7	41	7	43	-2
6	18	9	41	9	49	-8
11	24	11	44	11	34	10
18	31	14	3	13	56	7
24	April 6	15	52	15	47	5
30	12	17	26	17	30	-4
36	18	18	47	18	54	-7
42	24	19	52	20	2	-10
47	30	20	37	20	38	-1
53	May 6	20	58	21	3	-5
59	12	20	57	21	7	-10
64	18	20	43	20	50	-7
70	24	20	7	20	12	-5
77	31	19	9	19	7	2
83	June 6	18	5	17	59	6
88	12	16	55	16	53	2
94	18	15	30	15	33	-3
100	24	14	30	14	21	9
105	30	13	30	13	23	7

Table 31: John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in days with λ from MS Vat. Lat. 3116, part I.

$\lambda(^\circ)$	**Day of the Year**	**MS** minutes	**(sexagesimal)** seconds	**Rec.** minutes	**(sexagesimal)** seconds	**Differences (sec)**
111	July 6	12	42	12	34	8
117	12	12	10	11	58	12
122	18	12	0	11	45	15
128	24	12	0	11	47	13
135	31	12	35	12	26	9
141	August 6	13	32	13	19	13
146	12	14	45	14	23	22
152	18	16	11	15	52	19
158	24	17	52	17	35	17
165	31	20	20	19	51	29
171	September 6	22	2	21	56	6
177	12	24	2	24	0	2
183	18	25	59	26	2	-3
189	24	27	50	27	54	-4
194	30	29	32	29	22	10
201	October 6	30	52	31	2	-10
207	12	31	58	32	4	-6
213	18	32	30	32	41	-11
219	24	32	38	32	53	-15
226	31	32	20	32	26	-6
232	November 6	31	23	31	32	-9
238	12	29	55	30	11	-16
244	18	28	4	28	16	-12
250	24	25	43	25	57	-14
256	30	23	4	23	21	-17
262	December 6	20	0	20	25	-25
269	12	16	53	16	47	6
275	18	13	39	13	41	-2
281	24	10	40	10	41	-1
288	31	7	29	7	27	2

Table 32: John of Murs data from manuscript, recomputation and differences for *Tables of 1321* in days λ from MS Vat. Lat. 3116, part II.

and for *Kalendarium* (also in this case λ is taken from MS Vat. Lat. 3116):

$\lambda(°)$	**Day of the Year**	**Manuscript** minutes	**Recomputation** minutes	**Differences** minutes
299	January 10	4	3	1
309	20	1	0	1
320	31	0	0	0
330	February 10	0	0	0
340	20	2	1	1
348	28	3	3	0
358	March 10	7	7	0
8	20	10	10	0
18	31	14	13	1
28	April 10	17	16	1
38	20	18	19	-1
47	30	21	20	1
57	May 10	21	21	0
66	20	21	20	1
77	31	19	19	0
86	June 10	17	17	0
96	20	15	15	0
105	30	14	13	1
115	July 10	12	12	0
124	20	12	11	1
135	31	13	12	1
145	August 10	14	14	0
154	20	17	16	1
165	31	20	19	1
175	September 10	23	23	0
185	20	27	26	1
195	30	30	29	1
205	October 10	32	31	1
215	20	33	32	1
226	31	32	32	0
236	November 10	31	30	1
246	20	27	27	0
256	30	23	23	0
267	December 10	18	17	1
277	20	13	12	1
288	31	7	7	0

Table 33: John of Murs data from manuscript, recomputation and differences for Kalendarium with λ from MS Vat. Lat. 3116.

Here the shape of the differences in the three works:

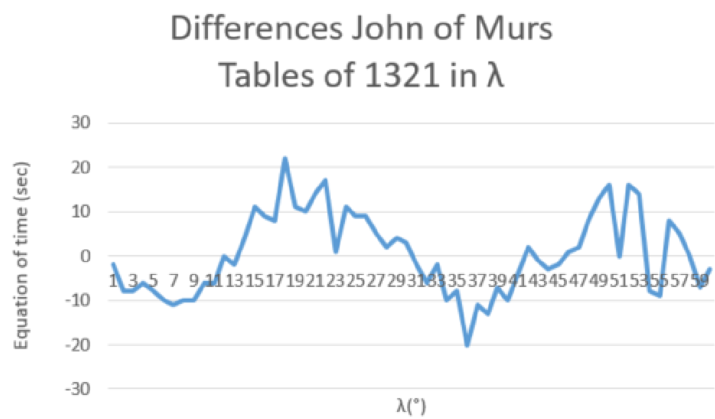


Figure 41: John of Murs' equation of time differences, given in seconds, in *Tables of 1321 in* λ.

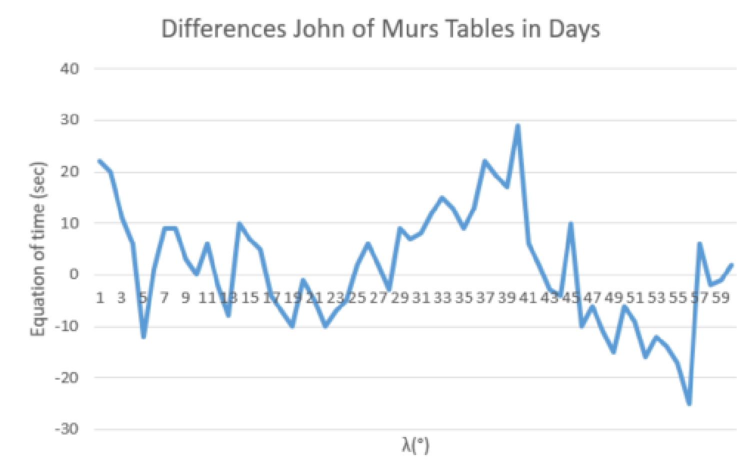


Figure 42: John of Murs' equation of time differences, given in seconds, in *Tables of 1321 in days.*

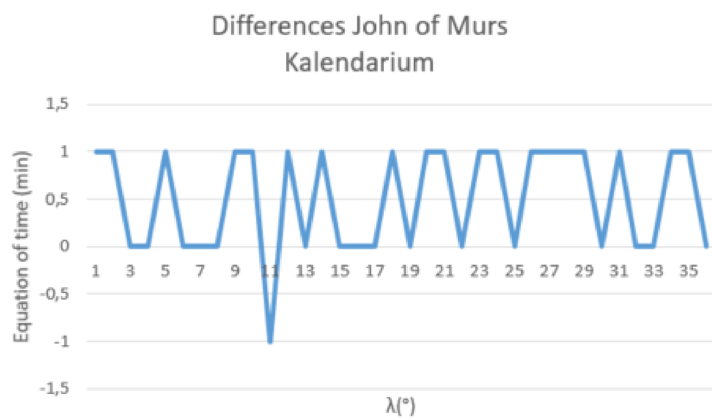


Figure 43: John of Murs' equation of time differences, given in minutes of time, in *Kalendarium*.

6.4 Conclusions

This is the hardest case of recomputation because the tables of John of Murs are given in irregular intervals for the argument so we have to recompute each single value for the equation of time, one by one. Moreover we do not know the underlying parameters.

From our recomputation we can state that the solar apogee and the solar equation table from the Parisian Alfonsine Tables are the most plausible values for John of Murs' equation of time.

6.5 List of Manuscripts

- Lisbon, MS Ajuda 52-XII-34 (57v-58v);
- Oxford, Bodleian Library, MS Can.Misc.501;
- London, British Library, Royal MS 12.C.XVII;
- Erfurt, Biblioteca Amploniana, MS 4°360 (*Patefit*);
- Erfurt, Biblioteca Amploniana, MS 4°371 (*Patefit*);
- Vatican, Biblioteca Apostolica, MS Lat. 3116 (*Patefit*);
- Brussels, Bibliothèque royale, MS 1086-1115 (24r-25v, 26r-v).

7 Peter of Saint Omer & John of Lignères

7.1 Overview

In the *Toledan Tables* we find an equation of time table (*Tabula aequationis dierum cum noctibus suis*) that should belong to Peter of Saint Omer and could have been a source for John of Lignères, so we put these two astronomers in the same section.

7.1.1 Peter of Saint Omer

Peter of Saint Omer was active in Paris in the 1290s. He wrote a treatise, *Tractatus de semissis* describing an equatorium for the calculation of planetary longitudes. In the *Toledan Tables* we find an equation of time table (*Tabula aequationis dierum cum noctibus suis*) that should belong to Peter of Saint Omer and could have been a source for John of Lignères.

He was also known also as Petrus de Sancto Ademaro [14] [13], see Appendix E.

7.1.2 John of Lignères

John of Lignères was born in 1290 [15] and we do not know his date of death; he lived in Paris from 1320 to 1335 [14] and he published astronomical tables and canons (often confused among the other astronomers of the period like John of Murs, John of Saxony), theory of planets, treatises on instruments and mathematical works which contributed to the diffusion of Alfonsine astronomy in Latin West.

He wrote a treatise on the theory of the planets, *Spera concentrica vel circulus concentricus dicitur* especially on the motion of the eighth sphere and other treatises on instruments (the saphea, the equatorium, the directorium). On the equatorium in particular he wrote *Quia nobilissima scientia astronomie non potest* and *Primo linea recta que est in medio regule.*

We find equation of time tables in *Tractatus diversi de scientiis* of 1401-1500 (Paris, Bibliothèque nationale de France, MS 7282, 110v), in *Canones Joannis de Ligneriis, aliàs, de Lineriis* of 1401-1500 (Paris, Bibliothèque nationale de France, MS 7295A, 163v) and in *Johannes de Lineriis: Tabulae aequationum planetarum* (Vatican, Biblioteca Apostolica, MS Pal. lat. 1412, 101v).

See Appendix E.

7.2 Presentation of the table

This table is described (but not present) in *Tractatus de semissis* with a minimun value of $0;0°$ in Aquarius $18° - 25°$ and a maximum value of $7;57°$ in Scorpio $8° - 9°$ so we can state that belongs to Peter of Saint Omer. The argument is the solar longitude in degrees (*Linee numeri Gradus aequales*) in intervals of one degree, tha table starts with Capricorn and the equation of time (*Aequatio dierum*) is given in degrees and minutes.

In Cracow, Jagiellonian Library, we can find a manuscript containing a copy of the *Tabulae Resolutae* in which the maximum value for the equation of time is $7;57°$ in Scorpio $8° - 9°$ as in the table of Peter of Saint Omer [6].

We find equation of time tables in *Tractatus diversi de scientiis* of 1401-1500 (Paris, Bibliothèque nationale de France, MS 7282, 110v) where the solar longitude is the argument in intervals of one degree, the table starts with Capricorn, the minimum value of the equation of time in degrees and minutes is $0;0°$ in Aquarius $18° - 25°$ and the maximum value is $7;57°$ in Scorpio $8° - 9°$.

In *Canones Joannis de Ligneriis, aliàs, de Lineriis* of 1401-1500 (Paris, Bibliothèque nationale de France, MS 7295A, 163v) the solar longitude is the argument in intervals of one degree, the table starts with Capricorn, the minimum value of the equation of time in degrees and minutes is $0;0°$ in Aquarius $18° - 25°$ and the maximum value is $7;57°$ in Scorpio $8° - 9°$.

In *Johannes de Lineriis: Tabulae aequationum planetarum* (Vatican, Biblioteca Apostolica, MS Pal. lat. 1412, 101v) the solar longitude is the argument in intervals of one degree, the table starts with Capricorn, the minimum value of the equation of time in degrees and minutes is $0;0°$ in Aquarius $18° - 25°$ and the maximum value is $7;57°$ in Scorpio $8° - 9°$.

In May 2018 I visited the Vatican Library to consult the manuscript Pal. Lat. 1412 in which we find the equation of time in f. 101v:

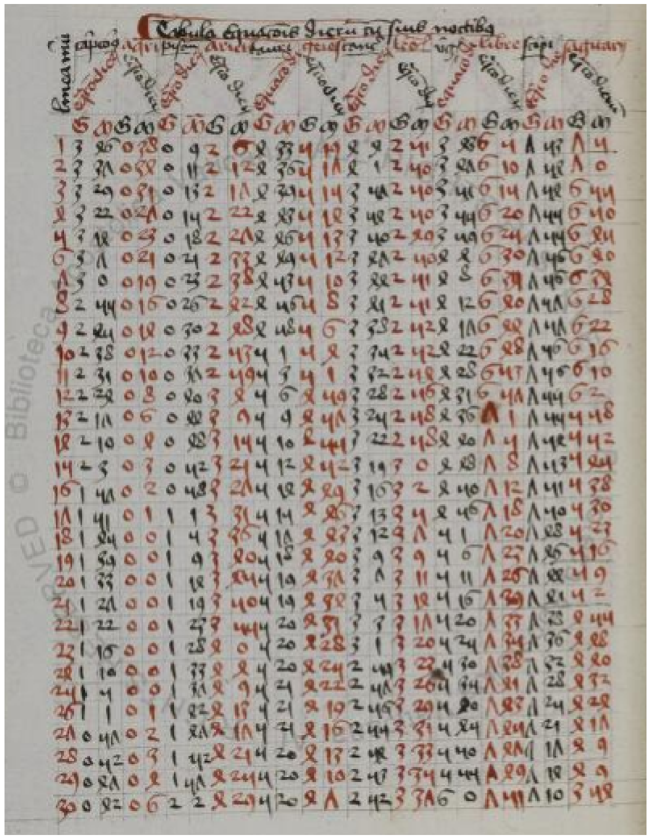


Figure 44: Equation of time table in Vatican, Biblioteca Apostolica, MS Pal. Lat. 1412, 101v .

In f. 72r we find a table with the following values for the sun motion and the solar apogee:

- motus solis: $4\ 38°; 17, 13, 48, 33, 40, 57$
- augis solis: $1\ 11°, 25, 22, 59, 43, 20, 43$

In f. 102v we find:

- motus solis: $4\ 48°; 27, 21, 48, 49, 20$

The solar apogee is different from the value I used in my recomputation.

We find the *Tabula equationis solis prima* in ff. 85r-v and it is the same table (with only some different entries to be taken into account) that I used in my recomputation.

In ff. 96v and 97r we find the right ascension table *Motus signorum in circulo rect* that it is the same that I used for my recomputation.

This manuscript is also available online at `https://digi.vatlib.it/?ling=en`.

7.3 Analyses of the table

For our recomputation we used the right ascension table and the solar equation table available in the *Toledan Tables*.

We also made an attempt using solar equation from PAT but the best fit is for TT.

7.3.1 Recomputation by modern method

Our set of parameters is:

- $\varepsilon = 23;33,30°$ (historical);
- $e = 2;4,45$ parts (historical);
- $\lambda_A = 87;13°$ (LS);
- $E_0 = 4;0,55°$ (LS).

For the initial constant we also try other values recomputed by hand according to the minimum of the equation of time and we get different values by varying the solar longitude: for $\lambda = 318°$ we get $E_0 = 3;56,38$, for $\lambda = 325°$ we get $E_0 = 3;54,30°$ but the best fit is with the value from LS and we get: $\sigma = 1.6352$, $E_M = 0,4916$ minutes of degree.

7.3.2 Recomputation by the use of tables

For the solar equation from TT we get the best fit from the original table without performing linear interpolation.

In Table 34 there is a synthesis of our analyses giving the values for the solar apogee and the epoch constant with the standard deviation σ and the average E_M of the differences for our recomputation of the equation of time:

Peter of Saint Omer
$\lambda_A = 87;12°$
$E_0 = 4;0,53°$
$\sigma = 1.2198$
$E_M = 0,0598$

Table 34: Standard deviations σ and averages E_M (given in minutes of degrees) for our recomputation with the estimation of λ_A and E_0.

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
1	3	46	3	47	-1
2	3	37	3	39	-2
3	3	29	3	32	-3
4	3	22	3	24	-2
5	3	14	3	17	-3
6	3	7	3	10	-3
7	3	0	3	1	-1
8	2	52	2	54	-2
9	2	45	2	47	-2
10	2	38	2	40	-2
11	2	31	2	33	-2
12	2	24	2	26	-2
13	2	17	2	19	-2
14	2	10	2	12	-2
15	2	3	2	5	-2
16	1	57	1	59	-2
17	1	51	1	52	-1
18	1	45	1	46	-1
19	1	39	1	40	-1
20	1	33	1	34	-1
21	1	27	1	28	-1
22	1	22	1	22	0
23	1	16	1	16	0
24	1	10	1	11	-1
25	1	5	1	5	0
26	1	1	1	0	1
27	0	57	0	55	2
28	0	52	0	51	1
29	0	47	0	46	1
30	0	42	0	42	0
31	0	38	0	37	1
32	0	34	0	33	1
33	0	31	0	30	1
34	0	27	0	26	1
35	0	23	0	22	1
36	0	21	0	20	1
37	0	19	0	17	2
38	0	16	0	14	2
39	0	14	0	11	3
40	0	12	0	8	4

Table 35: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
41	0	10	0	7	3
42	0	8	0	5	3
43	0	6	0	4	2
44	0	4	0	3	1
45	0	3	0	1	2
46	0	2	0	0	2
47	0	1	-1	58	3
48	0	0	-1	58	2
49	0	0	-1	58	2
50	0	0	-1	57	3
51	0	0	-1	57	3
52	0	0	-1	58	2
53	0	0	-1	58	2
54	0	0	-1	59	1
55	0	0	0	0	0
56	0	1	0	1	0
57	0	2	0	2	0
58	0	3	0	3	0
59	0	4	0	5	-1
60	0	6	0	6	0
61	0	9	0	8	1
62	0	11	0	10	1
63	0	13	0	12	1
64	0	15	0	15	0
65	0	18	0	18	0
66	0	21	0	20	1
67	0	23	0	23	0
68	0	26	0	26	0
69	0	30	0	29	1
70	0	33	0	33	0
71	0	37	0	36	1
72	0	40	0	39	1
73	0	44	0	43	1
74	0	48	0	47	1
75	0	52	0	51	1
76	0	56	0	55	1
77	1	1	1	0	1
78	1	5	1	4	1
79	1	9	1	8	1
80	1	14	1	13	1

Table 36: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part II.

$\lambda(°)$	**Manuscript** degrees	**(sexagesimal)** minutes	**Recomputation** degrees	**(sexagesimal)** minutes	**Differences (min of deg)**
81	1	19	1	18	1
82	1	23	1	21	2
83	1	28	1	26	2
84	1	33	1	31	2
85	1	37	1	36	1
86	1	42	1	41	1
87	1	47	1	46	1
88	1	52	1	51	1
89	1	57	1	57	0
90	2	2	2	2	0
91	2	7	2	7	0
92	2	12	2	12	0
93	2	17	2	17	0
94	2	22	2	23	-1
95	2	27	2	28	-1
96	2	33	2	33	0
97	2	38	2	39	-1
98	2	43	2	44	-1
99	2	48	2	49	-1
100	2	53	2	54	-1
101	2	59	3	0	-1
102	3	4	3	4	0
103	3	9	3	10	-1
104	3	15	3	16	-1
105	3	21	3	20	1
106	3	27	3	25	2
107	3	31	3	31	0
108	3	36	3	35	1
109	3	40	3	40	0
110	3	45	3	45	0
111	3	50	3	50	0
112	3	55	3	55	0
113	4	0	3	59	1
114	4	4	4	4	0
115	4	9	4	8	1
116	4	13	4	13	0
117	4	17	4	17	0
118	4	21	4	21	0
119	4	25	4	25	0
120	4	29	4	29	0

Table 37: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part III.

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
121	4	33	4	32	1
122	4	36	4	36	0
123	4	39	4	40	-1
124	4	43	4	43	0
125	4	46	4	46	0
126	4	49	4	50	-1
127	4	53	4	53	0
128	4	56	4	55	1
129	4	58	4	59	-1
130	5	1	5	1	0
131	5	3	5	3	0
132	5	6	5	6	0
133	5	9	5	8	1
134	5	10	5	10	0
135	5	12	5	11	1
136	5	14	5	13	1
137	5	15	5	14	1
138	5	17	5	16	1
139	5	18	5	18	0
140	5	19	5	18	1
141	5	19	5	19	0
142	5	20	5	20	0
143	5	20	5	20	0
144	5	20	5	21	-1
145	5	21	5	22	-1
146	5	21	5	21	0
147	5	21	5	21	0
148	5	20	5	21	-1
149	5	20	5	21	-1
150	5	20	5	20	0
151	5	19	5	19	0
152	5	17	5	18	-1
153	5	15	5	17	-2
154	5	14	5	16	-2
155	5	13	5	15	-2
156	5	12	5	13	-1
157	5	10	5	12	-2
158	5	8	5	9	-1
159	5	6	5	7	-1
160	5	4	5	5	-1

Table 38: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part IV.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
161	5	1	5	3	-2
162	4	59	5	1	-2
163	4	57	4	59	-2
164	4	55	4	56	-1
165	4	52	4	54	-2
166	4	49	4	51	-2
167	4	46	4	48	-2
168	4	43	4	45	-2
169	4	40	4	42	-2
170	4	37	4	39	-2
171	4	34	4	36	-2
172	4	31	4	33	-2
173	4	28	4	30	-2
174	4	25	4	26	-1
175	4	22	4	23	-1
176	4	19	4	20	-1
177	4	16	4	16	0
178	4	13	4	13	0
179	4	10	4	9	1
180	4	7	4	6	1
181	4	4	4	3	1
182	4	1	3	59	2
183	3	57	3	56	1
184	3	54	3	52	2
185	3	50	3	49	1
186	3	47	3	46	1
187	3	44	3	42	2
188	3	41	3	39	2
189	3	38	3	36	2
190	3	35	3	33	2
191	3	32	3	30	2
192	3	28	3	27	1
193	3	25	3	24	1
194	3	22	3	21	1
195	3	19	3	18	1
196	3	16	3	16	0
197	3	13	3	12	1
198	3	12	3	10	2
199	3	9	3	8	1
200	3	7	3	6	1

Table 39: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part V.

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
201	3	5	3	4	1
202	3	3	3	2	1
203	3	1	3	0	1
204	2	59	2	58	1
205	2	57	2	56	1
206	2	56	2	55	1
207	2	55	2	54	1
208	2	54	2	53	1
209	2	53	2	51	2
210	2	52	2	51	1
211	2	51	2	50	1
212	2	50	2	50	0
213	2	50	2	49	1
214	2	50	2	49	1
215	2	49	2	48	1
216	2	50	2	48	2
217	2	51	2	49	2
218	2	51	2	49	2
219	2	52	2	50	2
220	2	52	2	50	2
221	2	53	2	52	1
222	2	54	2	53	1
223	2	56	2	55	1
224	2	58	2	56	2
225	2	59	2	58	1
226	3	0	2	59	1
227	3	2	3	0	2
228	3	5	3	3	2
229	3	7	3	5	2
230	3	9	3	7	2
231	3	11	3	10	1
232	3	14	3	13	1
233	3	17	3	15	2
234	3	20	3	18	2
235	3	23	3	21	2
236	3	26	3	24	2
237	3	29	3	27	2
238	3	32	3	30	2
239	3	35	3	35	0
240	3	39	3	38	1

Table 40: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part VI.

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
241	3	43	3	42	1
242	3	47	3	46	1
243	3	51	3	49	2
244	3	55	3	53	2
245	3	59	3	58	1
246	4	4	4	2	2
247	4	8	4	7	1
248	4	12	4	11	1
249	4	17	4	15	2
250	4	22	4	20	2
251	4	26	4	25	1
252	4	31	4	29	2
253	4	36	4	34	2
254	4	40	4	39	1
255	4	46	4	44	2
256	4	50	4	48	2
257	4	56	4	54	2
258	5	1	4	59	2
259	5	6	5	4	2
260	5	11	5	9	2
261	5	16	5	14	2
262	5	20	5	19	1
263	5	25	5	24	1
264	5	30	5	29	1
265	5	35	5	34	1
266	5	40	5	39	1
267	5	45	5	44	1
268	5	50	5	50	0
269	5	55	5	55	0
270	6	0	6	0	0
271	6	5	6	4	1
272	6	10	6	9	1
273	6	15	6	14	1
274	6	20	6	19	1
275	6	25	6	24	1
276	6	30	6	29	1
277	6	35	6	33	2
278	6	40	6	38	2
279	6	44	6	42	2
280	6	48	6	46	2

Table 41: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
281	6	53	6	51	2
282	6	57	6	55	2
283	7	1	6	59	2
284	7	5	7	3	2
285	7	8	7	7	1
286	7	12	7	10	2
287	7	16	7	15	1
288	7	20	7	18	2
289	7	23	7	21	2
290	7	26	7	25	1
291	7	29	7	28	1
292	7	32	7	31	1
293	7	35	7	33	2
294	7	38	7	36	2
295	7	41	7	38	3
296	7	43	7	41	2
297	7	45	7	43	2
298	7	47	7	45	2
299	7	49	7	47	2
300	7	51	7	49	2
301	7	53	7	50	3
302	7	54	7	52	2
303	7	54	7	53	1
304	7	55	7	54	1
305	7	55	7	55	0
306	7	56	7	55	1
307	7	56	7	56	0
308	7	57	7	56	1
309	7	57	7	56	1
310	7	56	7	56	0
311	7	56	7	55	1
312	7	55	7	55	0
313	7	55	7	54	1
314	7	54	7	53	1
315	7	53	7	51	2
316	7	51	7	50	1
317	7	50	7	48	2
318	7	48	7	47	1
319	7	46	7	45	1
320	7	44	7	42	2

Table 42: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part VIII.

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
321	7	41	7	40	1
322	7	38	7	37	1
323	7	36	7	34	2
324	7	32	7	32	0
325	7	28	7	29	-1
326	7	25	7	25	0
327	7	21	7	21	0
328	7	17	7	17	0
329	7	14	7	13	1
330	7	10	7	8	2
331	7	5	7	4	1
332	7	0	7	0	0
333	6	55	6	55	0
334	6	50	6	50	0
335	6	45	6	45	0
336	6	40	6	39	1
337	6	34	6	34	0
338	6	28	6	28	0
339	6	22	6	22	0
340	6	16	6	16	0
341	6	10	6	10	0
342	6	2	6	4	-2
343	5	58	5	58	0
344	5	52	5	51	1
345	5	45	5	44	1
346	5	38	5	37	1
347	5	30	5	30	0
348	5	23	5	23	0
349	5	16	5	16	0
350	5	9	5	9	0
351	5	2	5	2	0
352	4	55	4	55	0
353	4	48	4	47	1
354	4	40	4	39	1
355	4	32	4	32	0
356	4	24	4	25	-1
357	4	17	4	17	0
358	4	9	4	10	-1
359	4	2	4	2	0
360	3	54	3	54	0

Table 43: Peter of St. Omer/John of Lignères data from manuscript, recomputation and differences, part IX.

Here the shape of the differences:

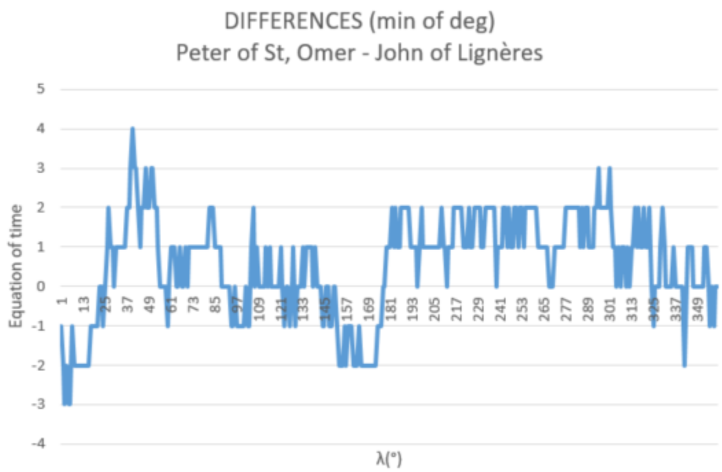


Figure 45: Peter of St. Omer/John of Lignères' equation of time differences, given in minutes of degree.

7.4 Conclusions

We can state that the best fit is given by the approximation of the solar equation (from the *Toledan Tables*) as a function of the true solar anomaly.

We want to underline, by the way, that the difference between the two methods is really small.

We also tried to reconstruct the extracted solar equation underlying the equation of time table according to Benno van Dalen's method with Eq. **??** and we can state that Peter of Saint Omer did not use the solar equation table in the Toledan Tables. Further investigation is needed by the use of the chords table.

From the reconstruction of the extracted right ascension underlying the equation of time table according to Benno van Dalen's method with Eq. **??** we can state that Peter of Saint Omer did not use the right ascension table in the Toledan Tables. Further investigation is needed by the use of linear interpolation between some corrected values (given for multiples of 10 degrees) calculated in the *Almagest*.

The procedure which considers the equation of time as a function of the mean solar longitude as argument has not given yet good results due to an inadequate reconstruction of the right ascension through interpolation used to pass from $\alpha(\lambda)$ to $\alpha(\bar{\lambda})$.

7.5 List of Manuscripts

- Cologne, Historischer Archiv der Stadt, MS W* 178, 7v;

- Cracow, Biblioteka Jagiellońska, MS 551, 78v;
- Erfurt, Bibliotheca Amploniana, MS F 377, 46v;
- Florence, Biblioteca Medicea Laurenziana, MS San Marco 185, 107v;
- Bernkastel-Kues, Cusanusstift, MS 212, 83r;
- London, British Library, MS Egerton 889, 38v;
- Madrid, Biblioteca Nacional, MS 10002, 25r;
- Oxford, Bodleian Library, MS Can. Misc. 27, 79r;
- Paris, Bibliothèque Nationale de France, MS 7282, 110v;
- Paris, Bibliothèque Nationale de France, MS 7286C, 54r;
- Paris, Bibliothèque Nationale de France, MS 7295A, 163v;
- Vatican, Biblioteca Apostolica, MS Pal. lat. 1412, 101v.

8 Levi ben Gerson

Levi ben Gerson (known also as Ralbag) was born in Bagnols, France, in 1288 and died in 1344. He was an astronomer, mathematician, physicist and a philosopher [14]. He lived in Orange and Avignon and he was probably the first astronomer to write in Hebrew indeed in the 14th century a Hebrew tradition flourished in the southern France and he was the main actor. His astronomical writings were known also to later Hebrew writers and he is mentioned also by Abraham Zacut [4].

He relied mainly on al-Battānī and Ptolemy but he refused the lunar model of this last one stating that is was correct only at syzygies and quadrature but not at the octant points. Ptolemy's lunar model also leads to see both sides of the Moon and Levi, according to the eccentric hypothesis, eliminates the epicycles and says that the Moon has no motion on its own, so we can see only one side. He decided this order for the planets: Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn and the fixed stars so with Mercury and Venus above the Sun (even if Ibn Rushd observed their transits over the Sun).

8.1 Overview

He wrote a trigonometrical work (*De sinibus, chordis et arcubus*) in 1343 where he calculated the sin tables basing on Ptolemaic methods. His main work is on philosophy: *Milhamot Adonai (The Wars of the Lords)* divided into six books and the fifth is on astronomy. The fifth book is divided into three treatises, the first is *Sefer Tekunah (Book of Astronomy)*: it deals with trigonometry, description of the Jacob's staff (an astronomical instrument), arrangements on the heavenly motions, the lunar model, solar and planetary motion, order of planets with sizes and distances.

See Appendix F for primary and secondary sources and for the algorithm.

8.2 Presentation of the table

The entries are in time degrees and the parameters are

- $\lambda_A =$ Cancer $3°$;
- $e = 2;14$ parts;
- $\varepsilon = 23;33°$;
- E_0 at Aquarius $20°$ is $4;2°$.

The table is presented with the solar longitude (there are no indications if it is the mean or the true but probably is the true solar longitude) as argument in intervals of one degree, it starts with Aries, the maximum is $8;13°$ in Scorpio $8° - 9°$ and the minimum is $0;0°$ in Aquarius $20° - 25°$.

The equation of time table is in Paris, hebr. 724, 117b and Paris, hebr. 725, 91a:

Table 9

P 724, 125b.
P 725, 97b.

Table for the difference of the days together with their nights
[Equation of time in time degrees][1]

Degrees	Aries	Taurus	Gemini	Cancer	Leo	Virgo	Libra	Scorpio	Sagittarius	Capricorn	Aquarius	Pisces
1	1;59°	4;20°	5; 6°	3;53°	2;45°	3;45°	6;15°	8; 6°	7;22°	4; 2°	0;45°	0; 8°
2	2; 4	;24	; 5	;49	;45	;49	;20	; 8	;17	3;54	;41	; 9
3	; 9	;27	; 4	;46	;45	;53	;25	; 9	;12	;46	;37	;11
4	;14	;30	; 3	;42	;45	3;57	;30	;10	; 7	;38	;33	;13
5	;19	;33	; 2	;39	;45	4; 2	;35	;10	7; 2	;30	;29	;15
6	;25°	;37°	5; 0°	;36°	;46°	; 6°	;40°	;11°	6;57°	;23°	;26°	;18°
7	;30	;40	4;59	;33	;47	;11	;45	;12	;51	;15	;23	;21
8	;35	;43	;57	;30	;47	;16	;50	;12	;45	; 8	;20	;24
9	;40	;46	;55	;27	;48	;20	;54	;13	;39	3; 0	;17	;27
10	;45	;48	;53	;24	;49	;25	6;59	;13	;33	2;53	;14	;30
11	;50°	;50°	;51°	;21°	;50°	;30°	7; 3°	;12°	;26°	;46°	;12°	;33°
12	2;55	;53	;49	;18	;51	;35	; 8	;12	;20	;38	;10	;36
13	3; 0	;55	;47	;15	;53	;40	;12	;11	;14	;31	; 9	;39
14	; 5	;57	;44	;13	;55	;45	;16	;10	; 8	;24	; 7	;43
15	;10	4;58	;42	;10	;57	;50	;20	; 8	6; 1	;17	; 5	;47
16	;15°	5; 0°	;39°	; 7°	2;58°	4;55°	;23°	; 7°	5;54°	;10°	; 3°	;51°
17	;20	; 1	;36	; 5	3; 0	5; 1	;27	; 5	;46	2; 3	; 2	;55
18	;25	; 3	;33	; 3	; 2	; 6	;31	; 4	;39	1;58	; 1	0;59
19	;29	; 4	;30	3; 1	; 4	;11	;35	; 2	;32	;51	0; 1	1; 3
20	;34	; 5	;27	2;59	; 7	;16	;39	8; 0	;25	;44	; 0	; 7
21	;39°	; 6°	;24°	;57°	;10°	;22°	;42°	7;59°	;17°	;38°	; 0°	;12°
22	;43	; 6	;21	;55	;13	;27	;45	;54	;10	;32	; 0	;16
23	;48	; 7	;18	;53	;16	;32	;48	;51	5; 3	;26	; 0	;20
24	;52	; 8	;15	;52	;19	;37	;51	;49	4;55	;20	; 0	;25
25	3;57	; 8	;12	;50	;23	;43	;53	;46	;47	;15	0; 1	;30
26	4; 1°	; 8°	; 9°	;49°	;26°	;48°	;56°	;42°	;40°	;10°	; 2°	;35°
27	; 5	; 8	; 5	;48	;29	;54	7;58	;38	;32	1; 5	; 2	;39
28	; 9	; 8	4; 2	;47	;33	5;59	8; 1	;34	;24	1; 0	; 3	;44
29	;13	; 7	3;59	;46	;36	6; 4	; 3	;30	;16	0;55	; 5	;49
30	4;17	5; 6	3;56	2;46	3;40	6; 9	8; 5	7;26	4; 9	0;50	0; 6	1;54

[1] The column subheadings in P 725 indicate that the entries are in degrees and minutes. In P 724, the subheadings indicate, incorrectly, minutes and seconds.

Taurus 28. With P 724: 8; P 725 reads 7.

Libra 14. With P 724: 16; P 725 reads 17.

Figure 46: Levi ben Gerson's equation of time in Paris, hebr. 724, 117b and Paris, hebr. 725, 91a.

8.3 Vatican Library

In May 2018 I visited the Vatican Library to consult the following manuscripts about Levi ben Gerson: Vat. Lat. 3380 and Vat. Lat. 3098.

8.3.1 Vatican, Latin, 3380

This manuscript was available only in microfilm, quite difficult to read, because the original was under digitization process.

In this manuscript we find *Tabula equationis solis ultimata* (f. 152r).

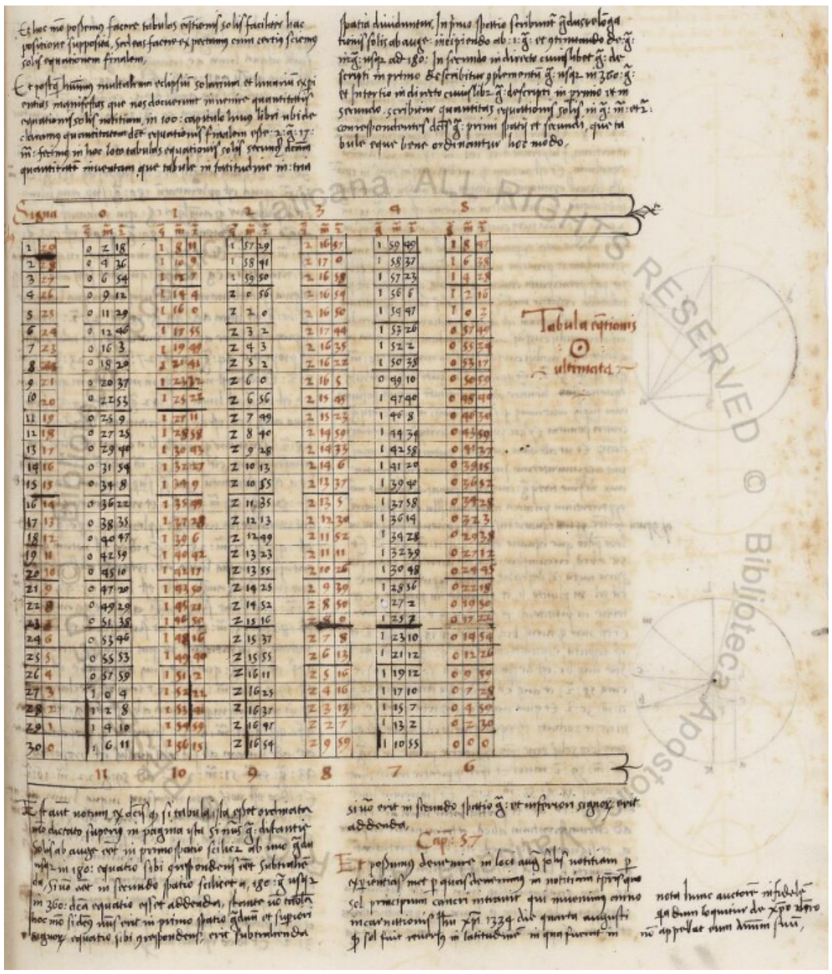


Figure 47: Levi ben Gerson's Tabula equationis solis ultimata, Vat. Lat. 3380, 152r.

8.3.2 Vatican Latin, 3098

Also this manuscript was available only in microfilm because the original was under digitization process.

In the manuscript there are no tables but only canons.

These manuscripts are now available on the website of the Vatican Library `https://digi.vatlib.it/?ling=en`.

8.4 Analyses of the table

In this case we follow Goldstein's method [4] using both solar equation tables that we have (Solar Equation from Paris, hebr. 725, 80b-81a and Tabula equationis solis ultimata in Vat. Lat. 3380) and the right ascension table from Paris, hebr. 724, 116b.

8.4.1 Recomputation by the use of tables

We get that the equation of time table was computed with the solar equation table from Paris hebr. 725 and the results are: $\sigma = 0.4342$, $E_M = 0,0416$ minutes of degrees.

See Appendix F for the algorithm from [4].

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(^\circ)$	**Manuscript** degrees	**(sexagesimal)** minutes	**Recomputation** degrees	**(sexagesimal)** minutes	**Differences (min of deg)**
1	1	59	1	58	1
2	2	4	2	4	0
3	2	9	2	9	0
4	2	14	2	14	0
5	2	19	2	19	0
6	2	25	2	24	1
7	2	30	2	29	1
8	2	35	2	34	1
9	2	40	2	39	1
10	2	45	2	44	1
11	2	50	2	49	1
12	2	55	2	54	1
13	3	0	2	59	1
14	3	5	3	5	0
15	3	10	3	9	1
16	3	15	3	14	1
17	3	20	3	19	1
18	3	25	3	24	1
19	3	29	3	29	0
20	3	34	3	33	1
21	3	39	3	38	1
22	3	43	3	43	0
23	3	48	3	47	1
24	3	52	3	52	0
25	3	57	3	55	2
26	4	1	4	0	1
27	4	5	4	4	1
28	4	9	4	8	1
29	4	13	4	12	1
30	4	17	4	16	1
31	4	20	4	20	0
32	4	24	4	24	0
33	4	27	4	27	0
34	4	30	4	30	0
35	4	33	4	33	0
36	4	37	4	36	1
37	4	40	4	39	1
38	4	43	4	42	1
39	4	46	4	45	1
40	4	48	4	47	1

Table 44: Levi ben Gerson data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
41	4	50	4	50	0
42	4	53	4	52	1
43	4	55	4	55	0
44	4	57	4	56	1
45	4	58	4	58	0
46	5	0	4	59	1
47	5	1	5	1	0
48	5	3	5	2	1
49	5	4	5	4	0
50	5	5	5	4	1
51	5	6	5	5	1
52	5	6	5	6	0
53	5	7	5	6	1
54	5	8	5	7	1
55	5	8	5	8	0
56	5	8	5	7	1
57	5	8	5	7	1
58	5	8	5	7	1
59	5	7	5	7	0
60	5	6	5	6	0
61	5	6	5	6	0
62	5	5	5	5	0
63	5	4	5	4	0
64	5	3	5	2	1
65	5	2	5	1	1
66	5	0	4	59	1
67	4	59	4	58	1
68	4	57	4	56	1
69	4	55	4	54	1
70	4	53	4	52	1
71	4	51	4	50	1
72	4	49	4	48	1
73	4	47	4	46	1
74	4	44	4	43	1
75	4	42	4	41	1
76	4	39	4	38	1
77	4	36	4	35	1
78	4	33	4	33	0
79	4	30	4	30	0
80	4	27	4	27	0

Table 45: Levi ben Gerson data from manuscript, recomputation and differences, part II.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
81	4	24	4	24	0
82	4	21	4	21	0
83	4	18	4	18	0
84	4	15	4	14	1
85	4	12	4	11	1
86	4	9	4	8	1
87	4	5	4	5	0
88	4	2	4	2	0
89	3	59	3	58	1
90	3	56	3	55	1
91	3	53	3	52	1
92	3	49	3	48	1
93	3	46	3	45	1
94	3	42	3	42	0
95	3	39	3	39	0
96	3	36	3	36	0
97	3	33	3	32	1
98	3	30	3	29	1
99	3	27	3	26	1
100	3	24	3	24	0
101	3	21	3	21	0
102	3	18	3	18	0
103	3	15	3	15	0
104	3	13	3	12	1
105	3	10	3	9	1
106	3	7	3	7	0
107	3	5	3	4	1
108	3	3	3	2	1
109	3	1	3	1	0
110	2	59	2	59	0
111	2	57	2	57	0
112	2	55	2	55	0
113	2	53	2	53	0
114	2	52	2	52	0
115	2	50	2	50	0
116	2	49	2	49	0
117	2	48	2	48	0
118	2	47	2	47	0
119	2	46	2	46	0
120	2	46	2	46	0

Table 46: Levi ben Gerson data from manuscript, recomputation and differences, part III.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
121	2	45	2	45	0
122	2	45	2	45	0
123	2	45	2	44	1
124	2	45	2	44	1
125	2	45	2	44	1
126	2	46	2	44	2
127	2	47	2	45	2
128	2	47	2	46	1
129	2	48	2	47	1
130	2	49	2	47	2
131	2	50	2	49	1
132	2	51	2	51	0
133	2	53	2	53	0
134	2	55	2	54	1
135	2	57	2	56	1
136	2	58	2	57	1
137	3	0	2	59	1
138	3	2	3	2	0
139	3	4	3	4	0
140	3	7	3	7	0
141	3	10	3	9	1
142	3	13	3	13	0
143	3	16	3	15	1
144	3	19	3	19	0
145	3	23	3	22	1
146	3	26	3	26	0
147	3	29	3	29	0
148	3	33	3	32	1
149	3	36	3	37	-1
150	3	40	3	40	0
151	3	45	3	44	1
152	3	49	3	48	1
153	3	53	3	52	1
154	3	57	3	56	1
155	4	2	4	1	1
156	4	6	4	6	0
157	4	11	4	11	0
158	4	16	4	15	1
159	4	20	4	20	0
160	4	25	4	25	0

Table 47: Levi ben Gerson data from manuscript, recomputation and differences, part IV.

$\lambda(^\circ)$	**Manuscript** degrees	**(sexagesimal)** minutes	**Recomputation** degrees	**(sexagesimal)** minutes	**Differences (min of deg)**
161	4	30	4	30	0
162	4	35	4	34	1
163	4	40	4	39	1
164	4	45	4	45	0
165	4	50	4	50	0
166	4	55	4	55	0
167	5	1	5	0	1
168	5	6	5	6	0
169	5	11	5	11	0
170	5	16	5	16	0
171	5	22	5	22	0
172	5	27	5	26	1
173	5	32	5	32	0
174	5	37	5	37	0
175	5	43	5	43	0
176	5	48	5	48	0
177	5	54	5	53	1
178	5	59	5	59	0
179	6	4	6	4	0
180	6	9	6	9	0
181	6	15	6	14	1
182	6	20	6	19	1
183	6	25	6	24	1
184	6	30	6	29	1
185	6	35	6	34	1
186	6	40	6	39	1
187	6	45	6	44	1
188	6	50	6	49	1
189	6	54	6	53	1
190	6	59	6	58	1
191	7	3	7	3	0
192	7	8	7	6	2
193	7	12	7	11	1
194	7	16	7	16	0
195	7	20	7	19	1
196	7	23	7	23	0
197	7	27	7	28	-1
198	7	31	7	31	0
199	7	35	7	35	0
200	7	39	7	38	1

Table 48: Levi ben Gerson data from manuscript, recomputation and differences, part V.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
201	7	42	7	41	1
202	7	45	7	45	0
203	7	48	7	47	1
204	7	51	7	50	1
205	7	53	7	53	0
206	7	56	7	56	0
207	7	58	7	58	0
208	8	1	8	0	1
209	8	3	8	2	1
210	8	5	8	4	1
211	8	6	8	5	1
212	8	8	8	7	1
213	8	9	8	8	1
214	8	10	8	9	1
215	8	10	8	10	0
216	8	11	8	11	0
217	8	12	8	12	0
218	8	12	8	11	1
219	8	13	8	12	1
220	8	13	8	12	1
221	8	12	8	11	1
222	8	12	8	11	1
223	8	11	8	11	0
224	8	10	8	9	1
225	8	8	8	8	0
226	8	7	8	6	1
227	8	5	8	5	0
228	8	4	8	3	1
229	8	2	8	2	0
230	8	0	7	59	1
231	7	59	7	56	3
232	7	54	7	54	0
233	7	51	7	51	0
234	7	49	7	48	1
235	7	46	7	45	1
236	7	42	7	41	1
237	7	38	7	38	0
238	7	34	7	34	0
239	7	30	7	30	0
240	7	26	7	25	1

Table 49: Levi ben Gerson data from manuscript, recomputation and differences, part VI.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
241	7	22	7	21	1
242	7	17	7	16	1
243	7	12	7	11	1
244	7	7	7	6	1
245	7	2	7	1	1
246	6	57	6	55	2
247	6	51	6	50	1
248	6	45	6	44	1
249	6	39	6	38	1
250	6	33	6	32	1
251	6	26	6	26	0
252	6	20	6	18	2
253	6	14	6	13	1
254	6	8	6	6	2
255	6	1	6	0	1
256	5	54	5	53	1
257	5	46	5	46	0
258	5	39	5	39	0
259	5	32	5	31	1
260	5	25	5	24	1
261	5	17	5	17	0
262	5	10	5	10	0
263	5	3	5	2	1
264	4	55	4	54	1
265	4	47	4	47	0
266	4	40	4	40	0
267	4	32	4	31	1
268	4	24	4	24	0
269	4	16	4	16	0
270	4	9	4	8	1
271	4	2	4	1	1
272	3	54	3	53	1
273	3	46	3	45	1
274	3	38	3	37	1
275	3	30	3	30	0
276	3	23	3	23	0
277	3	15	3	14	1
278	3	8	3	7	1
279	3	0	3	0	0
280	2	53	2	52	1

Table 50: Levi ben Gerson data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
281	2	46	2	45	1
282	2	38	2	38	0
283	2	31	2	31	0
284	2	24	2	23	1
285	2	17	2	16	1
286	2	10	2	10	0
287	2	3	2	3	0
288	1	58	1	56	2
289	1	51	1	50	1
290	1	44	1	44	0
291	1	38	1	38	0
292	1	32	1	32	0
293	1	26	1	26	0
294	1	20	1	22	-2
295	1	15	1	14	1
296	1	10	1	9	1
297	1	5	1	4	1
298	1	0	0	59	1
299	0	55	0	54	1
300	0	50	0	50	0
301	0	45	0	45	0
302	0	41	0	41	0
303	0	37	0	37	0
304	0	33	0	33	0
305	0	29	0	29	0
306	0	26	0	26	0
307	0	23	0	23	0
308	0	20	0	19	1
309	0	17	0	16	1
310	0	14	0	13	1
311	0	12	0	12	0
312	0	10	0	10	0
313	0	9	0	8	1
314	0	7	0	6	1
315	0	5	0	5	0
316	0	3	0	3	0
317	0	2	0	1	1
318	0	1	0	1	0
319	0	1	0	0	1
320	0	0	0	0	0

Table 51: Levi ben Gerson data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
321	0	0	0	-1	1
322	0	0	0	0	0
323	0	0	0	-1	1
324	0	0	0	0	0
325	0	1	0	1	0
326	0	2	0	1	1
327	0	2	0	2	0
328	0	3	0	3	0
329	0	5	0	4	1
330	0	6	0	5	1
331	0	8	0	7	1
332	0	9	0	9	0
333	0	11	0	11	0
334	0	13	0	13	0
335	0	15	0	16	-1
336	0	18	0	18	0
337	0	21	0	21	0
338	0	24	0	23	1
339	0	27	0	26	1
340	0	30	0	29	1
341	0	33	0	32	1
342	0	36	0	36	0
343	0	39	0	39	0
344	0	43	0	43	0
345	0	47	0	47	0
346	0	51	0	50	1
347	0	55	0	54	1
348	0	59	0	59	0
349	1	3	1	2	1
350	1	7	1	7	0
351	1	12	1	12	0
352	1	16	1	15	1
353	1	20	1	20	0
354	1	25	1	25	0
355	1	30	1	29	1
356	1	35	1	34	1
357	1	39	1	39	0
358	1	44	1	44	0
359	1	49	1	49	0
360	1	54	1	54	0

Table 52: Levi ben Gerson data from manuscript, recomputation and differences, part IX.

Here is the pattern of differences:

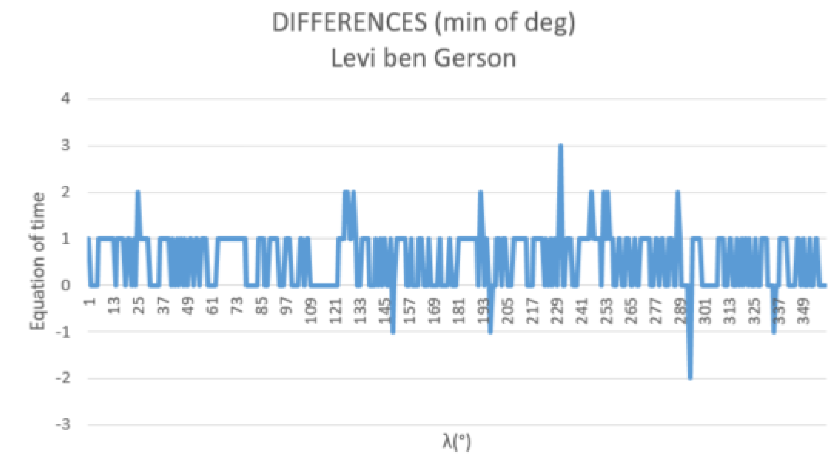


Figure 48: Levi ben Gerson's equation of time differences given in minutes of degree.

From the graphic of the differences we can see the presence of an outlier at $\lambda = 294°$.

8.5 Conclusions

We reproduced exactly the algorithm indicated by Goldstein in his survey [4] and we get an excellent result. In our recomputation we also try to recompute the equation of time using the second solar equation table available (Tabula equationis solis ultimata in Vat. Lat. 3380) and we find out that it is not the one used by Levi. We also try to recompute the equation of time with the mean solar longitude as argument, achieving a result that lead us to conclude that this was not the right argument.

8.6 List of Manuscripts

- Paris, hebr. 724: 12, 14, 32 ff., 74 f.,78 f., 84-86, 93 f., 96, 108, 131, 133, 141, 145 f; (*eq time);
- Paris, hebr. 725: 12, 14, 32 ff., 75 f.,78 f., 93, 108, 117, 133; (*eq time);
- British Museum, hebr., Add. 26,921;
- British Museum, hebr., Or. 10,725;
- Munich, hebr., 314;
- Vatican, Latin, 3098;

- Vatican, Latin, 3380;
- Milan, Ambrosiana, Latin, D 327.

9 John Holbroke

Very few is known about John Holbroke, he was a mathematician and an astronomer, master of Peterhouse in Cambridge from 1418 to 1431 and died in 1437.

See Appendix G for primary and secondary sources.

9.1 Overview

The manuscript in London, British Library, Egerton 889 (written between 1420 and 1437) on astronomy and astrology with various astronomical tables with canons (including by John of Lignères) was probably partly copied by him [16].

9.1.1 Egerton 889

In this manuscript in the British Library of London we find several astronomical tables from many astronomers (among them Simon Bredon, John Maudith, John of Ligneres, John Walter, William Rede and John Holbroke). Folios 150v-151r are on equation on time, a brief description of the table follows.

9.2 Presentation of the table

In the *Egerton 889* in folios 150v-151r there is the equation on time table. The heading of the table is *Tabula equationis dierum cum noctibus suis* and starts with Aquarius. The argument (*Gradus equales*) is the solar longitude in intervals of one degree, the entries are in degrees and minutes, the minimum is $0;0°$ in Aquarius $14° - 21°$ and the maximum is $8;28°$ in Scorpio. $9° - 10°$.

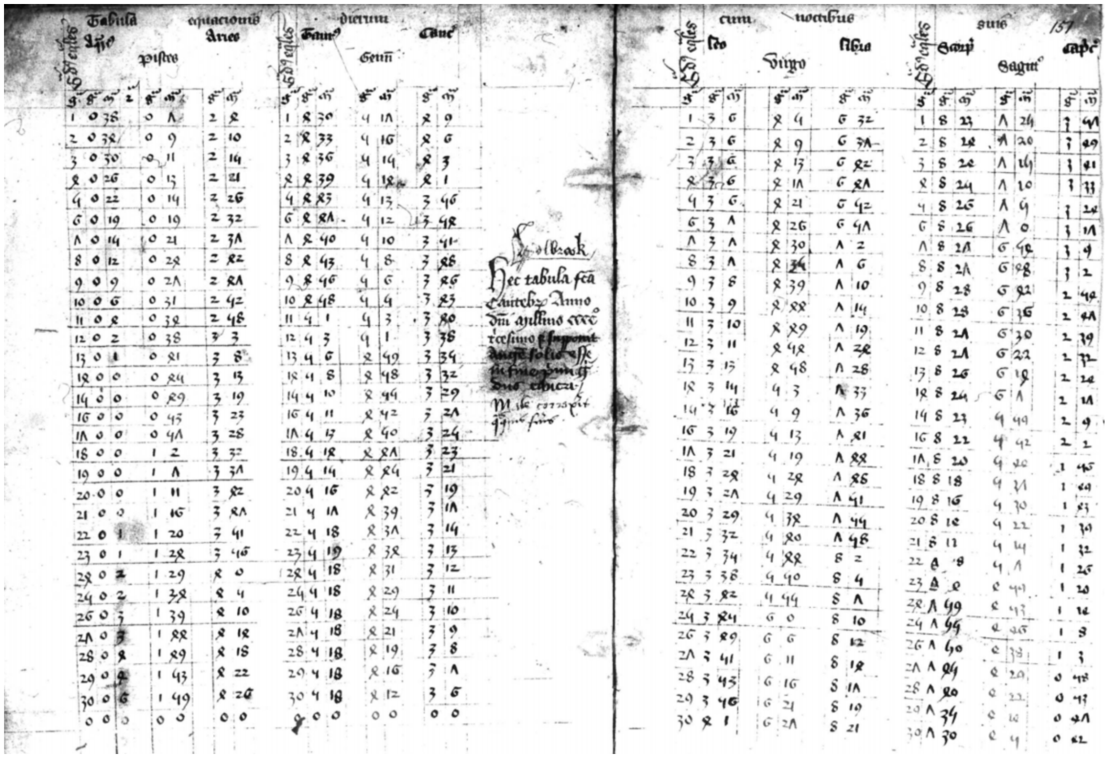


Figure 49: John Holbroke's equation of time in Egerton 889.

9.3 Analyses of the table

For the case of John Holbroke we just performed a recomputation according to the modern formula using a set of parameters from LS and considering λ as argument.

Our set of parameters is:

- $\varepsilon = 23;50,6°$ (LS);
- $e = 2;21,59$ parts (LS);
- $\lambda_A = 88;52,42°$ (LS);
- $E_0 = 4;11,21°$ (LS).

9.3.1 Recomputation by modern method

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
1	0	38	0	41	-3
2	0	34	0	36	-2
3	0	30	0	32	-2
4	0	26	0	29	-3
5	0	22	0	25	-3
6	0	19	0	21	-2
7	0	15	0	18	-3
8	0	12	0	15	-3
9	0	9	0	12	-3
10	0	6	0	10	-4
11	0	4	0	7	-3
12	0	2	0	5	-3
13	0	1	0	3	-2
14	0	0	0	2	-2
15	0	0	0	0	0
16	0	0	0	0	0
17	0	0	0	1	-1
18	0	0	0	2	-2
19	0	0	0	3	-3
20	0	0	0	3	-3
21	0	0	0	4	-4
22	0	1	0	4	-3
23	0	1	0	3	-2
24	0	2	0	3	-1
25	0	2	0	2	0
26	0	3	0	2	1
27	0	3	0	1	2
28	0	4	0	0	4
29	0	4	0	1	3
30	0	6	0	2	4
31	0	7	0	4	3
32	0	9	0	6	3
33	0	11	0	8	3
34	0	13	0	10	3
35	0	15	0	13	2
36	0	19	0	15	4
37	0	21	0	18	3
38	0	24	0	21	3
39	0	27	0	24	3
40	0	31	0	27	4

Table 53: John Holbroke data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
41	0	34	0	31	3
42	0	38	0	34	4
43	0	41	0	38	3
44	0	45	0	42	3
45	0	49	0	45	4
46	0	53	0	49	4
47	0	57	0	54	3
48	1	2	0	58	4
49	1	7	1	2	5
50	1	11	1	7	4
51	1	16	1	11	5
52	1	20	1	16	4
53	1	24	1	20	4
54	1	29	1	25	4
55	1	34	1	30	4
56	1	39	1	35	4
57	1	44	1	40	4
58	1	49	1	45	4
59	1	53	1	50	3
60	1	59	1	55	4
61	2	4	2	0	4
62	2	10	2	6	4
63	2	15	2	11	4
64	2	21	2	16	5
65	2	26	2	21	5
66	2	32	2	27	5
67	2	37	2	32	5
68	2	42	2	37	5
69	2	47	2	43	4
70	2	52	2	48	4
71	2	58	2	53	5
72	3	3	2	59	4
73	3	8	3	4	4
74	3	13	3	9	4
75	3	19	3	14	5
76	3	23	3	19	4
77	3	28	3	25	3
78	3	32	3	30	2
79	3	37	3	35	2
80	3	42	3	40	2

Table 54: John Holbroke data from manuscript, recomputation and differences, part II.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
81	3	47	3	44	3
82	3	51	3	49	2
83	3	56	3	54	2
84	4	0	3	59	1
85	4	5	4	3	2
86	4	10	4	8	2
87	4	14	4	12	2
88	4	18	4	16	2
89	4	22	4	20	2
90	4	26	4	24	2
91	4	30	4	28	2
92	4	33	4	32	1
93	4	36	4	36	0
94	4	39	4	40	-1
95	4	43	4	43	0
96	4	47	4	46	1
97	4	50	4	49	1
98	4	53	4	53	0
99	4	56	4	55	1
100	4	58	4	58	0
101	5	1	5	1	0
102	5	3	5	3	0
103	5	6	5	6	0
104	5	8	5	8	0
105	5	10	5	10	0
106	5	11	5	12	-1
107	5	13	5	13	0
108	5	14	5	15	-1
109	5	15	5	16	-1
110	5	16	5	18	-2
111	5	17	5	19	-2
112	5	18	5	20	-2
113	5	19	5	20	-1
114	5	18	5	21	-3
115	5	18	5	21	-3
116	5	18	5	22	-4
117	5	18	5	22	-4
118	5	18	5	22	-4
119	5	18	5	21	-3
120	5	18	5	21	-3

Table 55: John Holbroke data from manuscript, recomputation and differences, part III.

$\lambda(°)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
121	5	17	5	20	-3
122	5	16	5	20	-4
123	5	15	5	19	-4
124	5	14	5	18	-4
125	5	13	5	17	-4
126	5	12	5	15	-3
127	5	10	5	14	-4
128	5	8	5	12	-4
129	5	6	5	11	-5
130	5	5	5	9	-4
131	5	3	5	7	-4
132	5	1	5	5	-4
133	4	59	5	3	-4
134	4	58	5	0	-2
135	4	55	4	58	-3
136	4	52	4	55	-3
137	4	50	4	53	-3
138	4	47	4	50	-3
139	4	45	4	47	-2
140	4	42	4	45	-3
141	4	39	4	42	-3
142	4	37	4	39	-2
143	4	34	4	36	-2
144	4	31	4	33	-2
145	4	29	4	29	0
146	4	25	4	26	-1
147	4	21	4	23	-2
148	4	19	4	20	-1
149	4	16	4	17	-1
150	4	12	4	14	-2
151	4	9	4	10	-1
152	4	6	4	7	-1
153	4	3	4	4	-1
154	4	1	4	1	0
155	3	56	3	57	-1
156	3	54	3	54	0
157	3	51	3	51	0
158	3	48	3	48	0
159	3	46	3	45	1
160	3	43	3	42	1

Table 56: John Holbroke data from manuscript, recomputation and differences, part IV.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
161	3	40	3	39	1
162	3	38	3	37	1
163	3	35	3	34	1
164	3	32	3	31	1
165	3	29	3	29	0
166	3	27	3	26	1
167	3	25	3	24	1
168	3	23	3	22	1
169	3	21	3	20	1
170	3	19	3	18	1
171	3	17	3	16	1
172	3	15	3	14	1
173	3	13	3	13	0
174	3	12	3	11	1
175	3	11	3	10	1
176	3	10	3	9	1
177	3	9	3	8	1
178	3	8	3	7	1
179	3	7	3	6	1
180	3	6	3	5	1
181	3	6	3	5	1
182	3	6	3	5	1
183	3	6	3	5	1
184	3	6	3	5	1
185	3	6	3	5	1
186	3	7	3	5	2
187	3	7	3	6	1
188	3	7	3	6	1
189	3	8	3	7	1
190	3	9	3	8	1
191	3	10	3	9	1
192	3	11	3	11	0
193	3	13	3	12	1
194	3	15	3	14	1
195	3	16	3	16	0
196	3	19	3	18	1
197	3	21	3	20	1
198	3	24	3	22	2
199	3	27	3	24	3
200	3	29	3	27	2

Table 57: John Holbroke data from manuscript, recomputation and differences, part V.

λ(°)	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
201	3	32	3	30	2
202	3	35	3	32	3
203	3	38	3	35	3
204	3	42	3	39	3
205	3	45	3	42	3
206	3	49	3	45	4
207	3	51	3	49	2
208	3	53	3	52	1
209	3	56	3	56	0
210	4	1	4	0	1
211	4	5	4	4	1
212	4	9	4	8	1
213	4	13	4	12	1
214	4	17	4	16	1
215	4	21	4	21	0
216	4	26	4	25	1
217	4	30	4	30	0
218	4	35	4	35	0
219	4	39	4	39	0
220	4	44	4	44	0
221	4	49	4	49	0
222	4	54	4	54	0
223	4	58	4	59	-1
224	5	3	5	4	-1
225	5	9	5	9	0
226	5	13	5	14	-1
227	5	19	5	19	0
228	5	24	5	24	0
229	5	29	5	29	0
230	5	34	5	35	-1
231	5	40	5	40	0
232	5	44	5	45	-1
233	5	50	5	50	0
234	5	55	5	55	0
235	6	0	6	1	-1
236	6	6	6	6	0
237	6	11	6	11	0
238	6	16	6	16	0
239	6	21	6	21	0
240	6	27	6	26	1

Table 58: John Holbroke data from manuscript, recomputation and differences, part VI.

λ(°)	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
241	6	32	6	31	1
242	6	37	6	36	1
243	6	42	6	41	1
244	6	47	6	46	1
245	6	52	6	51	1
246	6	57	6	56	1
247	7	2	7	1	1
248	7	6	7	5	1
249	7	10	7	10	0
250	7	15	7	14	1
251	7	19	7	19	0
252	7	24	7	23	1
253	7	28	7	27	1
254	7	33	7	31	2
255	7	36	7	35	1
256	7	41	7	39	2
257	7	44	7	42	2
258	7	48	7	46	2
259	7	51	7	49	2
260	7	55	7	53	2
261	7	58	7	56	2
262	8	2	7	59	3
263	8	5	8	1	4
264	8	7	8	4	3
265	8	10	8	7	3
266	8	12	8	9	3
267	8	14	8	11	3
268	8	17	8	13	4
269	8	19	8	15	4
270	8	21	8	17	4
271	8	23	8	18	5
272	8	24	8	19	5
273	8	24	8	20	4
274	8	25	8	21	4
275	8	26	8	22	4
276	8	26	8	23	3
277	8	27	8	23	4
278	8	27	8	23	4
279	8	28	8	23	5
280	8	28	8	23	5

Table 59: John Holbroke data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
281	8	27	8	22	5
282	8	27	8	21	6
283	8	26	8	20	6
284	8	25	8	19	6
285	8	23	8	18	5
286	8	22	8	16	6
287	8	20	8	14	6
288	8	18	8	12	6
289	8	16	8	10	6
290	8	14	8	8	6
291	8	12	8	5	7
292	8	8	8	2	6
293	8	4	7	59	5
294	7	59	7	56	3
295	7	55	7	52	3
296	7	50	7	49	1
297	7	45	7	45	0
298	7	40	7	41	-1
299	7	35	7	36	-1
300	7	30	7	32	-2
301	7	25	7	27	-2
302	7	20	7	22	-2
303	7	15	7	17	-2
304	7	10	7	12	-2
305	7	5	7	6	-1
306	7	0	7	1	-1
307	6	54	6	55	-1
308	6	48	6	49	-1
309	6	42	6	43	-1
310	6	36	6	37	-1
311	6	30	6	30	0
312	6	22	6	24	-2
313	6	14	6	17	-3
314	6	7	6	10	-3
315	5	59	6	3	-4
316	5	52	5	56	-4
317	5	44	5	49	-5
318	5	37	5	41	-4
319	5	30	5	34	-4
320	5	22	5	26	-4

Table 60: John Holbroke data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript**	**(sexagesimal)**	**Recomputation**	**(sexagesimal)**	**Differences (min of deg)**
	degrees	minutes	degrees	minutes	
321	5	15	5	19	-4
322	5	7	5	11	-4
323	4	59	5	3	-4
324	4	53	4	56	-3
325	4	46	4	48	-2
326	4	38	4	40	-2
327	4	29	4	32	-3
328	4	22	4	24	-2
329	4	10	4	16	-6
330	4	5	4	8	-3
331	3	57	4	0	-3
332	3	49	3	52	-3
333	3	41	3	44	-3
334	3	33	3	36	-3
335	3	24	3	29	-5
336	3	17	3	21	-4
337	3	9	3	13	-4
338	3	2	3	5	-3
339	2	54	2	58	-4
340	2	47	2	50	-3
341	2	39	2	43	-4
342	2	32	2	35	-3
343	2	24	2	28	-4
344	2	17	2	21	-4
345	2	9	2	14	-5
346	2	2	2	7	-5
347	1	56	2	0	-4
348	1	49	1	53	-4
349	1	43	1	47	-4
350	1	39	1	40	-1
351	1	32	1	34	-2
352	1	26	1	28	-2
353	1	20	1	22	-2
354	1	14	1	16	-2
355	1	8	1	10	-2
356	1	3	1	5	-2
357	0	58	1	0	-2
358	0	53	0	55	-2
359	0	47	0	50	-3
360	0	42	0	45	-3

Table 61: John Holbroke data from manuscript, recomputation and differences, part IX.

From the recomputation we get this shape for the differences:

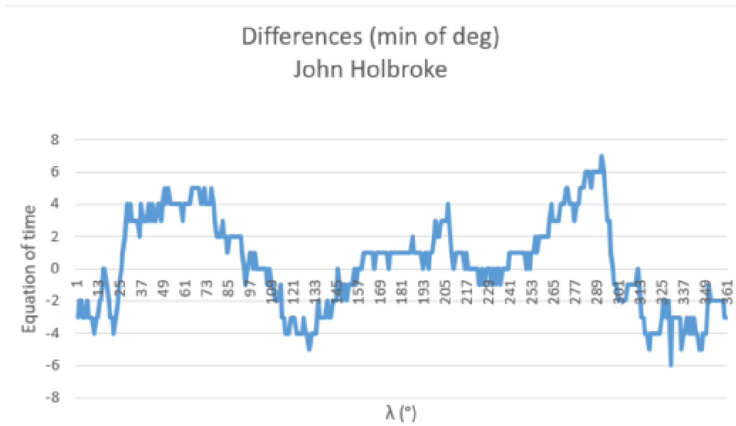


Figure 50: John Holbroke's equation of time differences, given in minutes of degrees.

with standard deviation $\sigma = 2.87$ and average $E_M \approx 0$ minutes of degree.

9.4 Conclusions

We do not get a good fit using the parameters from LS and by considering λ as argument.

We diceded to put aside Holbroke to focus on other astronomers.

9.5 List of Manuscripts

- London, Bodleian Library, Egerton 889, 150v-151r;
- Cambridge, University Library, Ee.3.61, fol. 56r.

10 Giovanni Bianchini

Giovanni Bianchini (1410 - c. 1469) was an Italian astronomer, he was a professor of astronomy at the University of Ferrara. He wrote some astronomical tables published in 1495. He was a merchant in Venice and then he worked for Nicolo' d'Este (Marquis d'Este). He had contacts with Georg Peurbach and Regiomontanus.

10.1 Overview

His main work is *Tabulae astronomiae* in 1495 compiled in Ferrara and addressed to Leonello d'Este. In addition to this work we have two editions of the canons and the tables:

- *Tabule Joa[nni] Blanchini Bononiensis*, Venice: Luca Antonio Giunta, 1526;

- *Luminarium atque planetarum motuum tabulae octoginta quinque*, Basel: Joannes Hervagium, 1553. In this work we have Peurbach's *Tabulae eclipsium* and the Tables of Bianchini edited by Prugner. Tables are also in the followings manuscripts:

  - Naples, Biblioteca Nazionale, MSS VIII.C.34;
  - Nuremberg, Stadtbibliothek, Cent V 57;
  - Rome, Biblioteca Casanatense, MS 1673;
  - Vatican, Biblioteca Apostolica, MS Pal. lat. 1375.

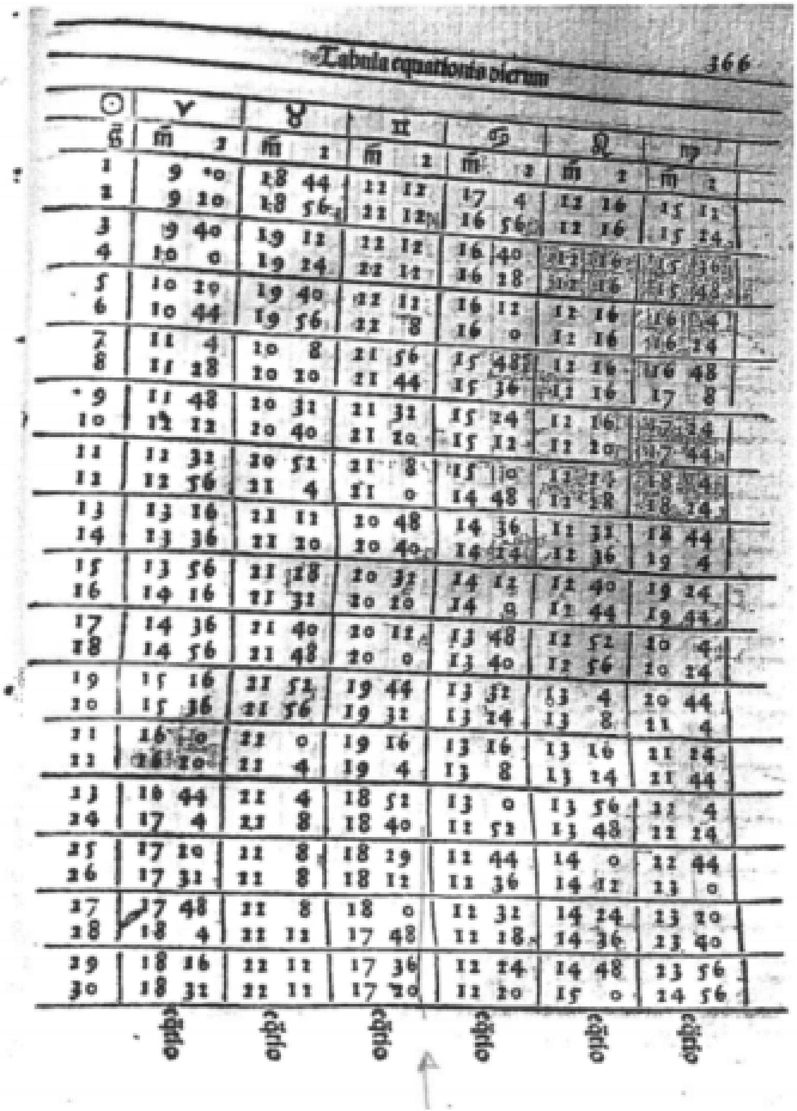
  He also wrote, in 1442, *Compositio instrumenti* on an instrument (*biffa*) used to determine the altitude of stars, *Canones tabularum super primo mobile* dedicated to spherical trigonometry in which we find the value $40;45,4°$ for the latitude of Ferrara and $23;30,30°$ for the obliquity of the ecliptic, *Flores Almagesti* (composed between 1440 and 1455) on arithmetic, algebra and astronomy, *Canones tabularum de eclipsibus luminarium* around 1456-1460 on observations on lunar eclipses, *Tabulas magistrales* on tables where he left the sexagesimal notation for the decimal one.

He did not use a single epoch in his tables but intermediate epochs from which the quantities are counted and this feature distinguishes his tables to the others [17].

See Appendix H for primary and secondary sources.

10.2 Presentation of the table

The entries are given in minutes and seconds of time. The maximum is $31;36$ min in Scorpio $8° - 9°$ and the minimum is 0;0 min in Aquarius $18° - 19°$. The argument is the solar longitude in intervals of one degree and the table starts with Aries. The maximum corresponds to the value $7;54° \times 4$ where $7;54°$ is the maximum for the equation of time in the Toledan Tables and the factor 4 comes from the conversion from units of arc to units of time (we have to multiply the entries in time-degrees by 4). In the Toledan Tables, in the zīj of al-Battānī and in the *editio princeps* of the Alfonsine Tables the equation of time is together with the right ascension.



☉	♈		♉		♊		♋		♌		♍	
g̃	m̄	2	m̄	2	m̄	2	m̄	2	m̄	2	m̄	2
1	9	0	18	44	22	12	17	4	12	16	15	12
2	9	20	18	56	22	12	16	56	12	16	15	24
3	9	40	19	12	22	12	16	40	12	16	15	36
4	10	0	19	24	22	12	16	28	12	16	15	48
5	10	20	19	40	22	12	16	12	12	16	16	4
6	10	44	19	56	22	8	16	0	12	16	16	24
7	11	4	20	8	21	56	15	48	12	16	16	48
8	11	28	20	20	21	44	15	36	12	16	17	8
9	11	48	20	32	21	32	15	24	12	16	17	24
10	12	12	20	40	21	20	15	12	12	20	17	44
11	12	32	20	52	21	8	15	0	12	24	18	4
12	12	56	21	4	21	0	14	48	12	28	18	24
13	13	16	21	12	20	48	14	36	12	32	18	44
14	13	36	21	20	20	40	14	24	12	36	19	4
15	13	56	21	28	20	32	14	12	12	40	19	24
16	14	16	21	32	20	20	14	0	12	44	19	44
17	14	36	21	40	20	12	13	48	12	52	20	4
18	14	56	21	48	20	0	13	40	12	56	20	24
19	15	16	21	52	19	44	13	32	13	4	20	44
20	15	36	21	56	19	32	13	24	13	8	21	4
21	16	0	22	0	19	16	13	16	13	16	21	24
22	16	20	22	4	19	4	13	8	13	14	21	44
23	16	44	22	4	18	52	13	0	13	56	22	4
24	17	4	22	8	18	40	12	52	13	48	22	24
25	17	20	22	8	18	29	12	44	14	0	22	44
26	17	32	22	8	18	12	12	36	14	12	23	0
27	17	48	22	8	18	0	12	32	14	24	23	20
28	18	4	22	12	17	48	12	28	14	36	23	40
29	18	16	22	12	17	36	12	24	14	48	23	56
30	18	32	22	12	17	20	12	20	15	0	24	56
	eq̃tio		eq̃tio		eq̃tio		eq̃tio		eq̃tio		eq̃tio	

Figure 51: Giovanni Bianchini's equation of time table part I in Edition 1526.

Tabula equationis dierum

☉	♎		♏		♐		♑		♒		♓	
g	m	2	m	2	m	2	m	2	m	2	m	2
1	24	36	31	16	27	56	14	44	2	28	0	48
2	24	56	31	20	27	40	14	12	2	12	1	0
3	25	12	31	24	27	20	13	40	2	0	1	8
4	25	28	31	28	26	56	13	12	1	44	1	20
5	25	48	31	28	26	32	12	48	1	52	1	28
6	26	8	31	32	26	8	12	16	1	20	1	40
7	26	24	31	32	25	40	11	48	1	12	1	52
8	26	48	31	36	25	16	11	20	1	0	2	4
9	27	0	31	36	24	48	10	52	0	52	2	20
10	27	16	31	32	24	24	10	28	0	40	2	32
11	27	32	31	28	23	56	10	4	0	32	2	44
12	27	48	31	24	25	32	9	36	0	24	3	0
13	28	4	31	20	23	12	9	8	0	20	3	16
14	28	24	31	16	22	52	8	40	0	16	3	36
15	28	40	31	8	22	36	8	12	0	52	3	52
16	29	0	31	0	22	8	7	44	0	8	4	12
17	29	16	30	52	21	40	7	16	0	4	4	28
18	29	32	30	44	21	12	6	52	0	0	4	48
19	29	48	30	36	20	44	6	28	0	0	5	4
20	30	4	30	28	20	16	6	4	0	4	5	24
21	30	16	30	16	19	48	5	40	0	4	5	40
22	30	24	30	4	19	16	5	16	0	4	6	0
23	30	32	29	48	18	48	4	56	0	8	6	16
24	30	36	29	32	18	16	4	36	0	8	6	36
25	30	44	29	20	17	48	4	16	0	12	6	56
26	30	48	29	4	17	16	3	56	0	16	7	16
27	30	56	28	48	16	48	3	36	0	24	7	36
28	31	0	28	42	16	16	3	16	0	28	8	0
29	31	8	28	20	15	48	2	56	0	32	8	20
30	31	12	28	8	15	16	2	44	0	40	8	40

Figure 52: Giovanni Bianchini's equation of time table part II in Edition 1526.

In May 2018 I visited the Vatican Library to consult the manuscript Pal. Lat. 1375 in which we find two equation of time tables. The first one is similar to the one in Edition 1526:

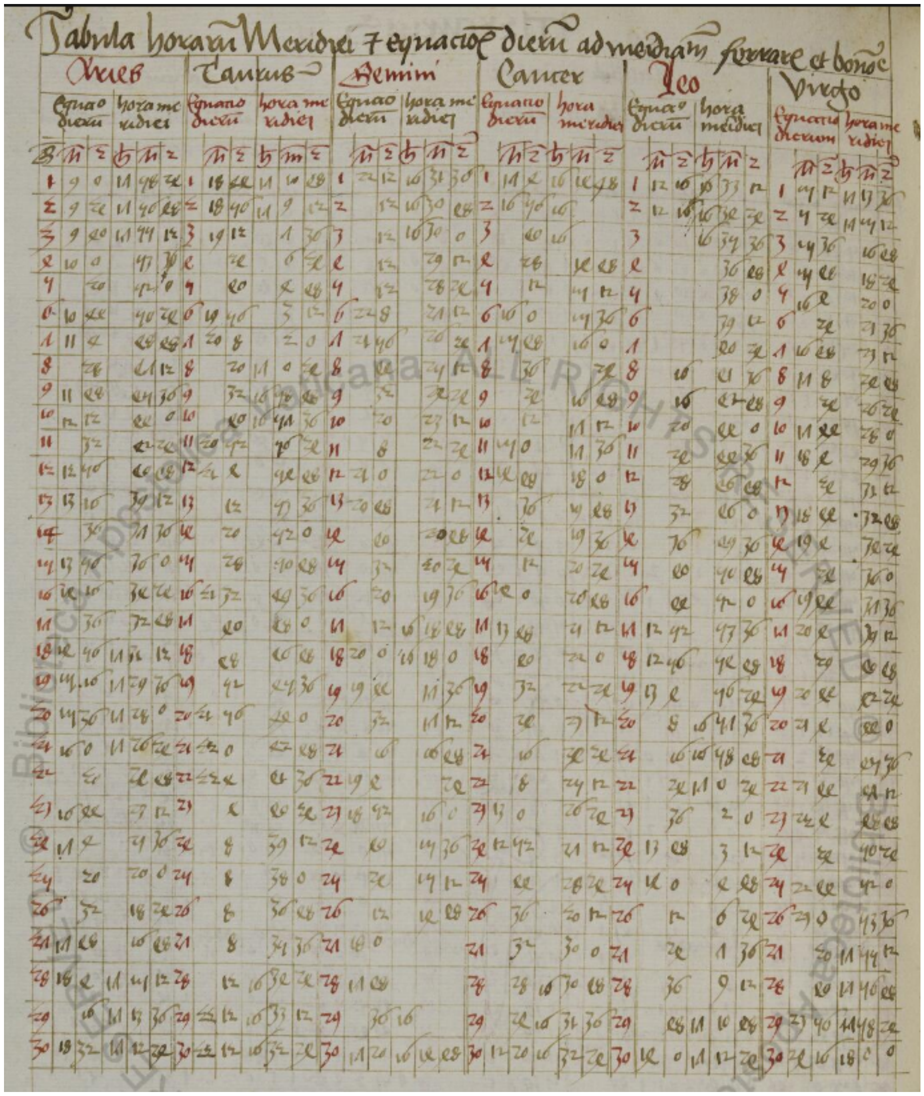


Figure 53: Giovanni Bianchini's equation of time table in Pal. Lat. 1375, 258v.

The second one, in f. 90r, it is different:

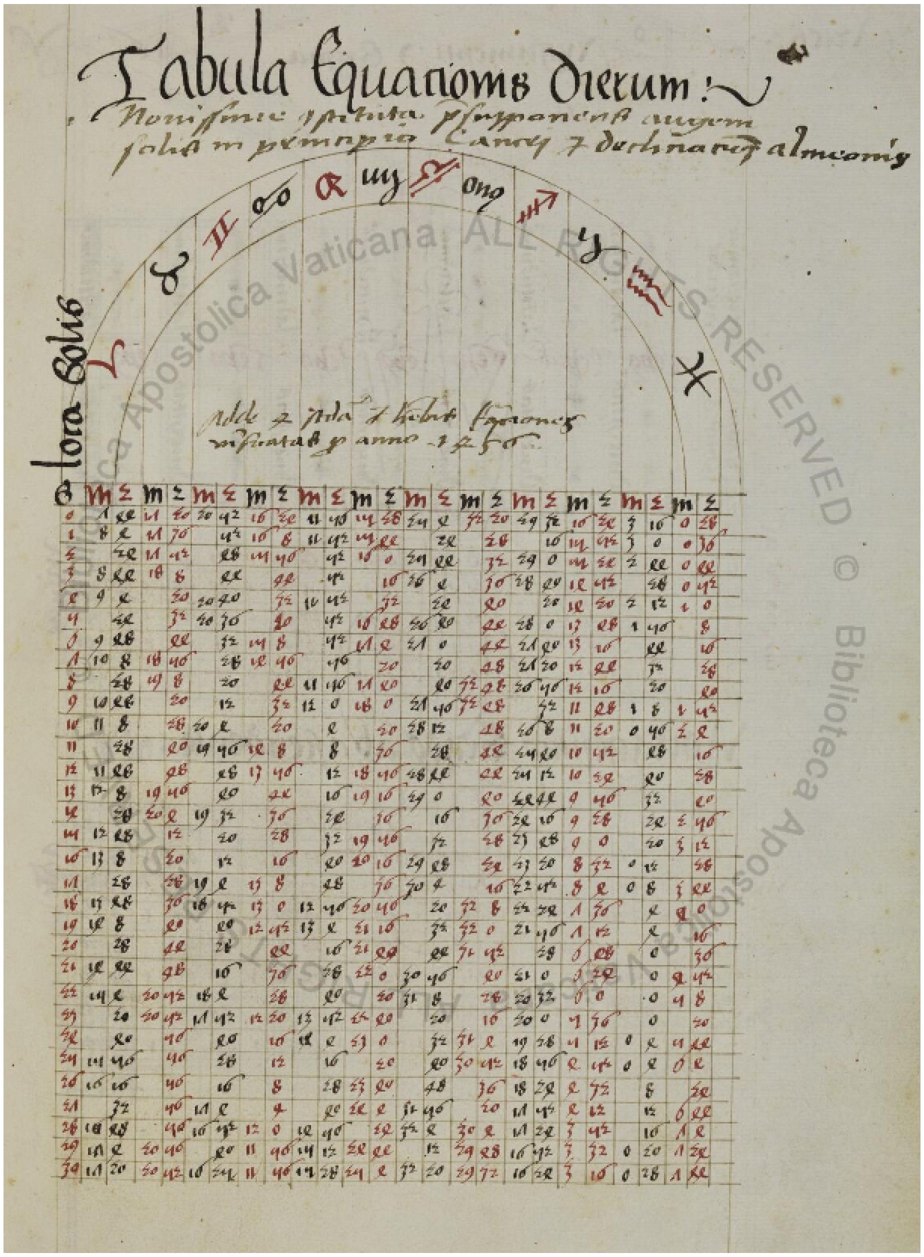


Figure 54: Giovanni Bianchini's equation of time table in Pal. Lat. 1375, 90r.

In this manuscript we find also three solar equation tables, the first in f. 25r (*Tabula equationis solis prima*) the second in ff. 83r-v, 84r-v (*Tabula equationis solis prima*) and the third in f. 187r ((*Tabula equationis solis*).

The solar apogee table (*Tabula augis solis*) is in f. 56v, the solar declination table (*Tabula declinationum*) is in f. 116r and the right ascension (*Ascentionis recte*) table is in f. 128r.

10.3 Analyses of the table

For the modern recomputation the set of parameters, from LS, is:

- $\varepsilon = 23;34,42°$ (LS);
- $e = 2;3,41$ parts (LS);
- $\lambda_A = 82;32,54°$ (LS);
- $E_0 = 4;5,42°$ (LS).

In the case of the recomputation by the use of tables we used the right ascension table and the solar equation table available in the *Toledan Tables*. We also used the solar equation in PAT to show that it was not the one used by Bianchini.

10.3.1 Recomputation by modern method

From our analyses by the use of modern formula with parameters from LS we get $\sigma = 7.8363$, $E_M = 0,5222$ seconds of time. The shape and the values of the differences

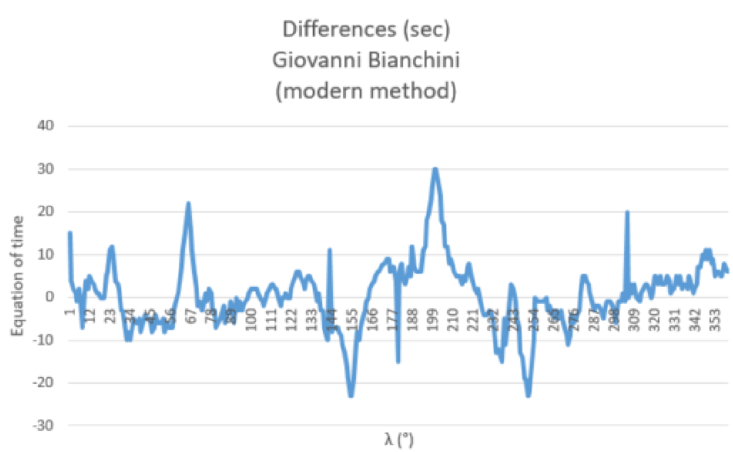


Figure 55: Giovanni Bianchini's equation of time differences, given in seconds.

are close to the one that we get from the recomputation by the use of tables, see next subsection.

10.3.2 Recomputation by the use of tables

In the following table there is a synthesis of our analyses giving the values for the solar apogee and the epoch constant with the standard deviation σ and the average E_M of the differences for our recomputation of the equation of time.

Giovanni Bianchini
$\lambda_A = 82;15°$
$E_0 = 4;5,46°$
$\sigma = 8.6112$
$E_M = 0,1631$

Table 62: Standard deviations σ and averages E_M given in seconds of time for our recomputation with the estimation of λ_A and E_0.

We can state that the best fit is for the method with the double interpolation for the solar equation.

We want to underline, by the way, that the difference between the two methods is really small.

We want also to underline the different values for the epoch that we find in our recomputation according to the minimum for the equation of time:

- $\lambda = 318°$: $E_0 = 4;6,48°$ using the interpolated solar equation and $E_0 = 4;8,41°$ using the provided solar equation table;
- $\lambda = 319°$:$E_0 = 4;6,57°$ using the interpolated solar equation and $E_0 = 4;8,46°$ using the provided solar equation table.

Playing by numbers we find the best fit for $E_0 = 4;5,46°$.

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
1	9	10	8	52	18
2	9	20	9	13	7
3	9	40	9	34	6
4	10	0	9	56	4
5	10	20	10	18	2
6	10	44	10	40	4
7	11	4	11	2	2
8	11	18	11	24	-6
9	11	48	11	42	6
10	12	12	12	5	7
11	12	32	12	27	5
12	12	56	12	46	10
13	13	16	13	9	7
14	13	36	13	32	4
15	13	56	13	51	5
16	14	16	14	11	5
17	14	36	14	34	2
18	14	56	14	54	2
19	15	16	15	13	3
20	15	36	15	33	3
21	16	0	15	53	7
22	16	20	16	14	6
23	16	44	16	30	14
24	17	4	16	50	14
25	17	20	17	7	13
26	17	32	17	27	5
27	17	48	17	43	5
28	18	4	18	0	4
29	18	16	18	17	-1
30	18	32	18	34	-2
31	18	44	18	47	-3
32	18	56	19	4	-8
33	19	12	19	18	-6
34	19	24	19	31	-7
35	19	40	19	45	-5
36	19	56	19	59	-3
37	20	8	20	12	-4
38	20	20	20	22	-2
39	20	32	20	36	-4
40	20	40	20	47	-7

Table 63: Giovanni Bianchini: data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
41	20	52	20	57	-5
42	21	4	21	7	-3
43	21	12	21	17	-5
44	21	20	21	24	-4
45	21	28	21	30	-2
46	21	32	21	37	-5
47	21	40	21	44	-4
48	21	48	21	51	-3
49	21	52	21	58	-6
50	21	56	22	1	-5
51	22	0	22	4	-4
52	22	4	22	7	-3
53	22	4	22	10	-6
54	22	8	22	14	-6
55	22	8	22	17	-9
56	22	8	22	16	-8
57	22	8	22	16	-8
58	22	12	22	15	-3
59	22	12	22	15	-3
60	22	12	22	11	1
61	22	12	22	10	2
62	22	12	22	6	6
63	22	12	22	2	10
64	22	12	21	58	14
65	22	12	21	54	18
66	22	8	21	46	22
67	21	56	21	42	14
68	21	44	21	34	10
69	21	32	21	26	6
70	21	20	21	18	2
71	21	8	21	10	-2
72	21	0	21	2	-2
73	20	48	20	54	-6
74	20	40	20	43	-3
75	20	32	20	35	-3
76	20	20	20	23	-3
77	20	12	20	11	1
78	20	0	20	0	0
79	19	44	19	48	-4
80	19	32	19	36	-4

Table 64: Giovanni Bianchini: data from manuscript, recomputation and differences, part II.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
81	19	16	19	24	-8
82	19	4	19	13	-9
83	18	52	19	3	-11
84	18	40	18	46	-6
85	18	29	18	34	-5
86	18	12	18	22	-10
87	18	0	18	6	-6
88	17	48	17	55	-7
89	17	36	17	39	-3
90	17	20	17	27	-7
91	17	4	17	15	-11
92	16	56	17	0	-4
93	16	40	16	48	-8
94	16	28	16	32	-4
95	16	12	16	20	-8
96	16	0	16	8	-8
97	15	48	15	52	-4
98	15	36	15	40	-4
99	15	24	15	28	-4
100	15	12	15	16	-4
101	15	0	15	4	-4
102	14	48	14	52	-4
103	14	36	14	40	-4
104	14	24	14	27	-3
105	14	12	14	15	-3
106	14	0	14	7	-7
107	13	48	13	54	-6
108	13	40	13	46	-6
109	13	32	13	37	-5
110	13	24	13	29	-5
111	13	16	13	20	-4
112	13	8	13	11	-3
113	13	0	13	3	-3
114	12	52	12	58	-6
115	12	44	12	49	-5
116	12	36	12	44	-8
117	12	32	12	39	-7
118	12	28	12	33	-5
119	12	24	12	28	-4
120	12	20	12	27	-7

Table 65: Giovanni Bianchini: data from manuscript, recomputation and differences, part III.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
121	12	16	12	21	-5
122	12	16	12	20	-4
123	12	16	12	18	-2
124	12	16	12	16	0
125	12	16	12	14	2
126	12	16	12	12	4
127	12	16	12	14	2
128	12	16	12	16	0
129	12	16	12	18	-2
130	12	20	12	20	0
131	12	24	12	25	-1
132	12	28	12	31	-3
133	12	32	12	36	-4
134	12	36	12	41	-5
135	12	40	12	46	-6
136	12	44	12	51	-7
137	12	52	12	56	-4
138	12	56	13	4	-8
139	13	4	13	13	-9
140	13	8	13	21	-13
141	13	16	13	30	-14
142	13	24	13	42	-18
143	13	56	13	50	6
144	13	48	14	2	-14
145	14	0	14	14	-14
146	14	12	14	26	-14
147	14	24	14	38	-14
148	14	36	14	49	-13
149	14	48	15	5	-17
150	15	0	15	16	-16
151	15	12	15	31	-19
152	15	24	15	46	-22
153	15	36	16	1	-25
154	15	48	16	15	-27
155	16	4	16	34	-30
156	16	24	16	48	-24
157	16	48	17	6	-18
158	17	8	17	21	-13
159	17	24	17	39	-15
160	17	44	17	56	-12

Table 66: Giovanni Bianchini: data from manuscript, recomputation and differences, part IV.

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
161	18	4	18	14	-10
162	18	24	18	32	-8
163	18	44	18	49	-5
164	19	4	19	10	-6
165	19	24	19	27	-3
166	19	44	19	44	0
167	20	4	20	5	-1
168	20	24	20	26	-2
169	20	44	20	42	2
170	21	4	21	3	1
171	21	24	21	23	1
172	21	44	21	39	5
173	22	4	21	59	5
174	22	24	22	19	5
175	22	44	22	38	6
176	23	0	22	58	2
177	23	20	23	17	3
178	23	40	23	37	3
179	23	56	23	56	0
180	23	56	24	15	-19
181	24	36	24	33	3
182	24	56	24	52	4
183	25	12	25	11	1
184	25	28	25	29	-1
185	25	48	25	47	1
186	26	8	26	5	3
187	26	24	26	23	1
188	26	48	26	41	7
189	27	0	26	54	6
190	27	16	27	12	4
191	27	32	27	29	3
192	27	48	27	43	5
193	28	4	28	0	4
194	28	24	28	17	7
195	28	40	28	30	10
196	29	0	28	42	18
197	29	16	28	59	17
198	29	32	29	11	21
199	29	48	29	24	24
200	30	4	29	36	28

Table 67: Giovanni Bianchini: data from manuscript, recomputation and differences, part V.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
201	30	16	29	48	28
202	30	24	30	0	24
203	30	32	30	8	24
204	30	36	30	20	16
205	30	44	30	27	17
206	30	48	30	39	9
207	30	56	30	46	10
208	31	0	30	53	7
209	31	8	31	0	8
210	31	12	31	7	5
211	31	16	31	10	6
212	31	20	31	17	3
213	31	24	31	19	5
214	31	28	31	21	7
215	31	28	31	24	4
216	31	32	31	26	6
217	31	32	31	28	4
218	31	36	31	26	10
219	31	36	31	28	8
220	31	32	31	26	6
221	31	28	31	24	4
222	31	24	31	22	2
223	31	20	31	20	0
224	31	16	31	13	3
225	31	8	31	7	1
226	31	0	31	0	0
227	30	52	30	53	-1
228	30	44	30	46	-2
229	30	36	30	39	-3
230	30	28	30	28	0
231	30	16	30	17	-1
232	30	4	30	6	-2
233	29	48	29	55	-7
234	29	32	29	43	-11
235	29	20	29	32	-12
236	29	4	29	16	-12
237	28	48	29	0	-12
238	28	42	28	45	-3
239	28	20	28	29	-9
240	28	8	28	10	-2

Table 68: Giovanni Bianchini: data from manuscript, recomputation and differences, part VI.

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
241	27	56	27	54	2
242	27	40	27	34	6
243	27	20	27	14	6
244	26	56	26	55	1
245	26	32	26	35	-3
246	26	8	26	11	-3
247	25	40	25	51	-11
248	25	16	25	27	-11
249	24	48	25	3	-15
250	24	24	24	39	-15
251	23	56	24	15	-19
252	23	32	23	51	-19
253	23	12	23	27	-15
254	22	52	22	58	-6
255	22	36	22	34	2
256	22	8	22	6	2
257	21	40	21	38	2
258	21	12	21	9	3
259	20	44	20	41	3
260	20	16	20	13	3
261	19	48	19	45	3
262	19	16	19	16	0
263	18	48	18	48	0
264	18	16	18	16	0
265	17	48	17	47	1
266	17	16	17	19	-3
267	16	48	16	47	1
268	16	16	16	18	-2
269	15	48	15	46	2
270	15	16	15	18	-2
271	14	44	14	50	-6
272	14	12	14	18	-6
273	13	40	13	49	-9
274	13	12	13	17	-5
275	12	48	12	49	-1
276	12	16	12	21	-5
277	11	48	11	49	-1
278	11	20	11	21	-1
279	10	52	10	53	-1
280	10	28	10	26	2

Table 69: Giovanni Bianchini: data from manuscript, recomputation and differences, part VII.

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
281	10	4	9	58	6
282	9	36	9	30	6
283	9	8	9	2	6
284	8	40	8	35	5
285	8	12	8	7	5
286	7	44	7	43	1
287	7	16	7	16	0
288	6	52	6	52	0
289	6	28	6	29	-1
290	6	4	6	5	-1
291	5	40	5	41	-1
292	5	16	5	18	-2
293	4	56	4	55	1
294	4	36	4	36	0
295	4	16	4	13	3
296	3	56	3	54	2
297	3	36	3	35	1
298	3	16	3	16	0
299	2	56	2	57	-1
300	2	44	2	43	1
301	2	28	2	24	4
302	2	12	2	10	2
303	2	0	1	55	5
304	1	44	1	41	3
305	1	52	1	27	25
306	1	20	1	17	3
307	1	12	1	7	5
308	1	0	0	53	7
309	0	52	0	44	8
310	0	40	0	34	6
311	0	32	0	28	4
312	0	24	0	23	1
313	0	20	0	17	3
314	0	16	0	12	4
315	0	12	0	7	5
316	0	8	0	2	6
317	0	4	0	2	2
318	0	0	0	3	-3
319	0	0	0	3	-3
320	0	4	0	4	0

Table 70: Giovanni Bianchini: data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
321	0	4	0	4	0
322	0	4	0	0	4
323	0	8	0	0	8
324	0	8	0	3	5
325	0	12	0	7	5
326	0	16	0	11	5
327	0	24	0	16	8
328	0	28	0	20	8
329	0	32	0	29	3
330	0	40	0	34	6
331	0	48	0	43	5
332	1	0	0	52	8
333	1	8	1	1	7
334	1	20	1	10	10
335	1	28	1	24	4
336	1	40	1	33	7
337	1	52	1	47	5
338	2	4	1	57	7
339	2	20	2	11	9
340	2	32	2	25	7
341	2	44	2	39	5
342	3	0	2	53	7
343	3	16	3	8	8
344	3	36	3	27	9
345	3	52	3	42	10
346	4	12	3	57	15
347	4	28	4	16	12
348	4	48	4	35	13
349	5	4	4	51	13
350	5	24	5	10	14
351	5	40	5	30	10
352	6	0	5	46	14
353	6	16	6	6	10
354	6	36	6	26	10
355	6	56	6	47	9
356	7	16	7	7	9
357	7	36	7	28	8
358	8	0	7	48	12
359	8	20	8	9	11
360	8	40	8	30	10

Table 71: Giovanni Bianchini: data from manuscript, recomputation and differences, part IX.

The pattern of the differences is given by the graphic:

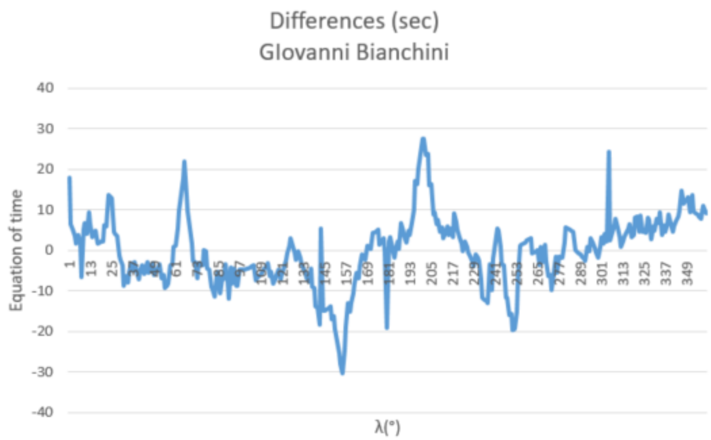


Figure 56: Giovanni Bianchini's equation of time differences, given in seconds.

and this pattern is close to the one we get from the modern recomputation even if the parameters are different.

10.4 Conclusions

From our recomputation we find out, for the solar apogee, a value that is the same in al-Battānī. Also the value for the initial constant is close to the one in al-Battānī, especially the one calculated from the minimum of the equation of time by the use of the interpolated solar equation.

We also tried different methods to reconstruct the extracted solar equation underlying the equation of time table according to Benno van Dalen's method with Eq. **??** and we can state that Giovanni Bianchini did not used the solar equation table in the Toledan Tables. Further investigation is needed by the use of the chords table.

From the reconstruction of the extracted right ascension underlying the equation of time table according to Benno van Dalen's method with Eq. **??** we can state that Giovanni Bianchini did not used the right ascension table in the Toledan Tables. Further investigation is needed by the use of linear interpolation between some corrected values (given for multiples of 10 degrees) calculated in the *Almagest*.

The procedure which considers the equation of time as a function of the mean solar longitude as argument have not given yet good results due to an inadequate reconstruction of the right ascension through interpolation used to pass from $\alpha(\lambda)$ to $\alpha(\bar{\lambda})$.

10.5 List of Manuscripts

- Naples, Biblioteca Nazionale, MSS VIII.C.34;
- Nuremberg, Stadtbibliothek, Cent V 57;
- Rome, Biblioteca Casanatense, MS 1673;
- Vatican, Biblioteca Apostolica, MS Pal. lat. 1375.

11 Georg Puerbach

Georg Peurbach (Austria, 1423-1461) was an astronomer and a mathematician [14]. He graduated in Arts and it is not well known where he studied astronomy. He was a teacher in Ferrara where he met Giovanni Bianchini. In 1457 he became the court astrologer to the Emperor Frederick III in Wiener Neustadt. Regiomontanus was his student and associate and together they calculated ephemerides from Bianchini's tables, they observed the Halley's Comet in June 1456, lunar eclipses and they found the value of $23;28°$ for the obliquity of the ecliptic. He found the value of $48;22°$ for Vienna's latitude (the true value is $48;13°$).

11.1 Overview

Among his works we have *Theoricae novae planetarum*, a textbook on planetary theory on Ptolemaic model; the original version was completed in 1454 and contained also the motion of the 8th sphere according to the Alphonsine Tables. This model remained valid until Tycho Brahe disproved the model of the spheres. This work was translated into French, Italian and Hebrew.

His most important work is *Tabulae eclipsium* [18] completed in 1459, based on Alphonsine Tables in which underlying parameters are Alphonsine. We have two versions of these tables: one calculated for the meridian of Vienna and the other version (known as *Tabulae Waradienses*) with the epoch position shifted 0;30 hours to adapt the tables to the meridian of Grosswardein (the actual Oradea in Hungary).

He also wrote the first book of *Epitoma Almagesti Ptolemaei*, a textbook on astronomy. The other books where completed by Regiomontanus after his death.

See Appendix I for primary and secondary sources.

11.2 Presentation of the Table

In *Tabulae eclipsium* [18] we have an equation of time table (*Tabula equationis dierum*). The table starts with Aries, the argument is the solar longitude in intervals of one degree, the entries are given in minutes and seconds of time. The maximum is $32;48$ min in Scorpio $7° - 10°$ and the minimum is 0;0 min in Aquarius $21° - 23°$.

Tabula Equationis dierum.

Tabula Equationis dierum nouissime constituta Presupponens Augem solis in principio Cancri Et Declinationem Almeonis.

Loci solis	Aries ♈	Taurus ♉	Gemini ♊	Cancer ♋	Leo ♌	Virgo ♍	Libra ♎	Scorpio ♏	Sagitarius ♐	Capricornus ♑	Aquarius ♒	Pisces ♓
g	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa	m̄ sa
0	7 44	17 20	20 52	16 24	11 56	15 28	25 4	32 20	29 32	16 24	3 16	0 28
1	8 4	17 36	20 52	16 8	11 52	15 44	25 24	32 28	29 16	15 52	3 0	0 36
2	8 24	17 52	20 48	15 56	11 52	16 0	25 44	32 32	29 0	15 24	2 44	0 44
3	8 44	18 8	20 44	15 44	11 52	16 16	26 4	32 36	28 40	14 52	2 28	0 52
4	9 4	18 20	20 40	15 32	11 52	16 32	26 24	32 40	28 20	14 20	2 12	1 0
5	9 24	18 32	20 36	15 20	11 52	16 48	26 40	32 44	28 0	13 48	1 56	1 8
6	9 48	18 44	20 32	15 8	11 52	17 4	27 0	32 44	27 40	13 16	1 44	1 16
7	10 8	18 56	20 28	14 56	11 56	17 20	27 20	32 48	27 20	12 44	1 32	1 28
8	10 28	19 8	20 20	14 44	11 56	17 40	27 40	32 48	26 56	12 16	1 20	1 40
9	10 48	19 20	20 12	14 32	12 0	18 0	27 56	32 48	26 32	11 48	1 8	1 52
10	11 8	19 28	20 4	14 20	12 4	18 20	28 12	32 48	26 8	11 20	0 56	2 4
11	11 28	19 40	19 56	14 8	12 8	18 36	28 28	32 44	25 40	10 52	0 48	2 16
12	11 48	19 48	19 48	13 56	12 12	18 56	28 44	32 44	25 12	10 24	0 40	2 28
13	12 8	19 56	19 40	13 44	12 16	19 16	29 0	32 40	24 44	9 56	0 32	2 40
14	12 28	20 4	19 32	13 36	12 24	19 36	29 16	32 36	24 16	9 28	0 24	2 56
15	12 48	20 12	19 20	13 28	12 32	19 56	29 32	32 28	23 48	9 0	0 20	3 12
16	13 8	20 20	19 12	13 16	12 40	20 16	29 48	32 24	23 20	8 32	0 12	3 28
17	13 28	20 28	19 4	13 8	12 48	20 36	30 4	32 16	22 52	8 4	0 8	3 44
18	13 48	20 36	18 52	13 0	12 56	20 56	30 20	32 8	22 24	7 36	0 4	4 0
19	14 8	20 40	18 40	12 52	13 4	21 16	30 32	32 0	21 56	7 12	0 4	4 16
20	14 28	20 44	18 28	12 44	13 16	21 40	30 44	31 52	21 28	6 48	0 0	4 36
21	14 44	20 48	18 16	12 36	13 28	22 0	30 56	31 40	21 0	6 24	0 0	5 52
22	15 4	20 52	18 4	12 28	13 40	22 20	31 8	31 28	20 32	6 0	0 0	5 8
23	15 20	20 52	17 52	12 20	13 52	22 40	31 20	31 16	20 0	5 36	0 0	5 20
24	15 40	20 56	17 40	12 16	14 4	23 0	31 32	31 4	19 28	5 12	0 4	5 44
25	15 56	20 56	17 28	12 12	14 16	23 20	31 40	30 52	18 56	4 52	0 4	6 4
26	16 16	20 56	17 16	12 8	14 28	23 44	31 48	30 36	18 24	4 32	0 8	6 24
27	16 32	20 56	17 4	12 4	14 40	24 4	31 56	30 20	17 52	4 12	0 12	6 44
28	16 48	20 56	16 52	12 0	14 56	24 24	32 4	30 4	17 24	3 52	0 16	7 4
29	17 4	20 56	16 40	11 56	15 12	24 44	32 12	29 48	16 52	3 32	0 20	7 24
30	17 20	20 56	16 24	11 56	15 28	25 4	32 20	29 32	16 24	3 16	0 28	7 44

Figure 57: Georg Puerbach's equation of time in Tabulae eclipsium.

11.3 Analyses of the table

We focus only on a preliminary analyses by recomputating the equation of time using the modern formula with the underlying parameters from LS:

- $\varepsilon = 23;33,44°$ (LS);
- $e = 2;16,42$ parts (LS);
- $\lambda_A = 91;2,4°$ (LS);
- $E_0 = 4;5,36°$ (LS).

11.3.1 Recomputation by modern method

In the following table we can see the original data from manuscript, the recomputation and the differences:

$\lambda(^{\circ})$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
1	8	4	8	0	4
2	8	24	8	20	4
3	8	44	8	40	4
4	9	4	9	0	4
5	9	24	9	20	4
6	9	48	9	41	7
7	10	8	10	1	7
8	10	28	10	22	6
9	10	48	10	42	6
10	11	8	11	3	5
11	11	28	11	23	5
12	11	48	11	43	5
13	12	8	12	3	5
14	12	28	12	23	5
15	12	48	12	43	5
16	13	8	13	3	5
17	13	28	13	23	5
18	13	48	13	42	6
19	14	8	14	1	7
20	14	28	14	20	8
21	14	44	14	39	5
22	15	4	14	57	7
23	15	20	15	15	5
24	15	40	15	33	7
25	15	56	15	50	6
26	16	16	16	7	9
27	16	32	16	24	8
28	16	48	16	40	8
29	17	4	16	56	8
30	17	20	17	12	8
31	17	36	17	27	9
32	17	52	17	41	11
33	18	8	17	56	12
34	18	20	18	9	11
35	18	32	18	22	10
36	18	44	18	35	9
37	18	56	18	47	9
38	19	8	18	59	9
39	19	20	19	10	10
40	19	28	19	20	8

Table 72: Georg Puerbach: data from manuscript, recomputation and differences, part I.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
41	19	40	19	30	10
42	19	48	19	39	9
43	19	56	19	48	8
44	20	4	19	56	8
45	20	12	20	4	8
46	20	20	20	11	9
47	20	28	20	17	11
48	20	36	20	23	13
49	20	40	20	28	12
50	20	44	20	32	12
51	20	48	20	36	12
52	20	52	20	39	13
53	20	52	20	42	10
54	20	56	20	44	12
55	20	56	20	45	11
56	20	56	20	46	10
57	20	56	20	46	10
58	20	56	20	45	11
59	20	56	20	44	12
60	20	56	20	42	14
61	20	52	20	40	12
62	20	48	20	37	11
63	20	44	20	33	11
64	20	40	20	29	11
65	20	36	20	24	12
66	20	32	20	19	13
67	20	28	20	13	15
68	20	20	20	6	14
69	20	12	19	59	13
70	20	4	19	52	12
71	19	56	19	44	12
72	19	48	19	36	12
73	19	40	19	27	13
74	19	32	19	18	14
75	19	20	19	8	12
76	19	12	18	58	14
77	19	4	18	48	16
78	18	52	18	37	15
79	18	40	18	27	13
80	18	28	18	15	13

Table 73: Georg Puerbach: data from manuscript, recomputation and differences, part II.

$\lambda(°)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
81	18	16	18	4	12
82	18	4	17	52	12
83	17	52	17	40	12
84	17	40	17	28	12
85	17	28	17	16	12
86	17	16	17	3	13
87	17	4	16	51	13
88	16	52	16	38	14
89	16	40	16	25	15
90	16	24	16	12	12
91	16	8	16	0	8
92	15	56	15	47	9
93	15	44	15	34	10
94	15	32	15	22	10
95	15	20	15	9	11
96	15	8	14	57	11
97	14	56	14	45	11
98	14	44	14	33	11
99	14	32	14	21	11
100	14	20	14	10	10
101	14	8	13	59	9
102	13	56	13	48	8
103	13	44	13	37	7
104	13	36	13	27	9
105	13	28	13	17	11
106	13	16	13	8	8
107	13	8	12	59	9
108	13	0	12	50	10
109	12	52	12	42	10
110	12	44	12	34	10
111	12	36	12	27	9
112	12	28	12	20	8
113	12	20	12	14	6
114	12	16	12	8	8
115	12	12	12	3	9
116	12	8	11	58	10
117	12	4	11	54	10
118	12	0	11	51	9
119	11	56	11	48	8
120	11	56	11	45	11

Table 74: Georg Puerbach: data from manuscript, recomputation and differences, part III.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
121	11	52	11	44	8
122	11	52	11	43	9
123	11	52	11	42	10
124	11	52	11	43	9
125	11	52	11	43	9
126	11	52	11	45	7
127	11	56	11	47	9
128	11	56	11	50	6
129	12	0	11	53	7
130	12	4	11	57	7
131	12	8	12	2	6
132	12	12	12	7	5
133	12	16	12	13	3
134	12	24	12	19	5
135	12	32	12	27	5
136	12	40	12	34	6
137	12	48	12	43	5
138	12	56	12	52	4
139	13	4	13	1	3
140	13	16	13	12	4
141	13	28	13	22	6
142	13	40	13	34	6
143	13	52	13	45	7
144	14	4	13	58	6
145	14	16	14	11	5
146	14	28	14	24	4
147	14	40	14	38	2
148	14	56	14	52	4
149	15	12	15	7	5
150	15	28	15	23	5
151	15	44	15	38	6
152	16	0	15	55	5
153	16	16	16	11	5
154	16	32	16	28	4
155	16	48	16	46	2
156	17	4	17	3	1
157	17	20	17	21	-1
158	17	40	17	40	0
159	18	0	17	58	2
160	18	20	18	17	3

Table 75: Georg Puerbach: data from manuscript, recomputation and differences, part IV.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
161	18	36	18	37	-1
162	18	56	18	56	0
163	19	16	19	16	0
164	19	36	19	36	0
165	19	56	19	56	0
166	20	16	20	16	0
167	20	36	20	36	0
168	20	56	20	57	-1
169	21	16	21	17	-1
170	21	40	21	38	2
171	22	0	21	59	1
172	22	20	22	19	1
173	22	40	22	40	0
174	23	0	23	1	-1
175	23	20	23	22	-2
176	23	44	23	42	2
177	24	4	24	3	1
178	24	24	24	23	1
179	24	44	24	44	0
180	25	4	25	4	0
181	25	24	25	24	0
182	25	44	25	44	0
183	26	4	26	4	0
184	26	24	26	23	1
185	26	40	26	43	-3
186	27	0	27	2	-2
187	27	20	27	20	0
188	27	40	27	39	1
189	27	56	27	57	-1
190	28	12	28	14	-2
191	28	28	28	32	-4
192	28	44	28	49	-5
193	29	0	29	5	-5
194	29	16	29	21	-5
195	29	32	29	37	-5
196	29	48	29	52	-4
197	30	4	30	7	-3
198	30	20	30	21	-1
199	30	32	30	35	-3
200	30	44	30	48	-4

Table 76: Georg Puerbach: data from manuscript, recomputation and differences, part V.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
201	30	56	31	0	-4
202	31	8	31	12	-4
203	31	20	31	24	-4
204	31	32	31	35	-3
205	31	40	31	45	-5
206	31	48	31	54	-6
207	31	56	32	3	-7
208	32	4	32	11	-7
209	32	12	32	19	-7
210	32	20	32	26	-6
211	32	28	32	32	-4
212	32	32	32	37	-5
213	32	36	32	42	-6
214	32	40	32	45	-5
215	32	44	32	49	-5
216	32	44	32	51	-7
217	32	48	32	52	-4
218	32	48	32	53	-5
219	32	48	32	53	-5
220	32	48	32	52	-4
221	32	44	32	51	-7
222	32	44	32	48	-4
223	32	40	32	45	-5
224	32	36	32	41	-5
225	32	28	32	36	-8
226	32	24	32	30	-6
227	32	16	32	23	-7
228	32	8	32	16	-8
229	32	0	32	7	-7
230	31	52	31	58	-6
231	31	40	31	48	-8
232	31	28	31	37	-9
233	31	16	31	25	-9
234	31	4	31	13	-9
235	30	52	30	59	-7
236	30	36	30	45	-9
237	30	20	30	30	-10
238	30	4	30	14	-10
239	29	48	29	58	-10
240	29	32	29	40	-8

Table 77: Georg Puerbach: data from manuscript, recomputation and differences, part VI.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
241	29	16	29	22	-6
242	29	0	29	3	-3
243	28	40	28	44	-4
244	28	20	28	23	-3
245	28	0	28	2	-2
246	27	40	27	41	-1
247	27	20	27	18	2
248	26	56	26	55	1
249	26	32	26	31	1
250	26	8	26	7	1
251	25	40	25	42	-2
252	25	12	25	17	-5
253	24	44	24	50	-6
254	24	16	24	24	-8
255	23	48	23	57	-9
256	23	20	23	29	-9
257	22	52	23	1	-9
258	22	24	22	33	-9
259	21	56	22	4	-8
260	21	28	21	35	-7
261	21	0	21	6	-6
262	20	32	20	36	-4
263	20	0	20	6	-6
264	19	28	19	36	-8
265	18	56	19	5	-9
266	18	24	18	35	-11
267	17	52	18	4	-12
268	17	24	17	33	-9
269	16	52	17	2	-10
270	16	24	16	31	-7
271	15	52	16	0	-8
272	15	24	15	29	-5
273	14	52	14	59	-7
274	14	20	14	28	-8
275	13	48	13	57	-9
276	13	16	13	27	-11
277	12	44	12	57	-13
278	12	16	12	27	-11
279	11	48	11	57	-9
280	11	20	11	27	-7

Table 78: Georg Puerbach: data from manuscript, recomputation and differences, part VII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
281	10	52	10	58	-6
282	10	24	10	29	-5
283	9	56	10	1	-5
284	9	28	9	33	-5
285	9	0	9	5	-5
286	8	32	8	38	-6
287	8	4	8	11	-7
288	7	36	7	45	-9
289	7	12	7	20	-8
290	6	48	6	55	-7
291	6	24	6	30	-6
292	6	0	6	6	-6
293	5	36	5	43	-7
294	5	12	5	20	-8
295	4	52	4	59	-7
296	4	32	4	37	-5
297	4	12	4	17	-5
298	3	52	3	57	-5
299	3	32	3	38	-6
300	3	16	3	20	-4
301	3	0	3	2	-2
302	2	44	2	45	-1
303	2	28	2	29	-1
304	2	12	2	14	-2
305	1	56	2	0	-4
306	1	44	1	46	-2
307	1	32	1	34	-2
308	1	20	1	22	-2
309	1	8	1	11	-3
310	0	56	1	0	-4
311	0	48	0	51	-3
312	0	40	0	42	-2
313	0	32	0	35	-3
314	0	24	0	28	-4
315	0	20	0	22	-2
316	0	12	0	16	-4
317	0	8	0	12	-4
318	0	4	0	8	-4
319	0	4	0	6	-2
320	0	0	0	4	-4

Table 79: Georg Puerbach: data from manuscript, recomputation and differences, part VIII.

$\lambda(^\circ)$	**Manuscript** minutes	**(sexagesimal)** seconds	**Recomputation** minutes	**(sexagesimal)** seconds	**Differences (sec)**
321	0	0	0	3	-3
322	0	0	0	2	-2
323	0	0	0	3	-3
324	0	4	0	4	0
325	0	4	0	6	-2
326	0	8	0	9	-1
327	0	12	0	12	0
328	0	16	0	17	-1
329	0	20	0	22	-2
330	0	28	0	28	0
331	0	36	0	34	2
332	0	44	0	41	3
333	0	52	0	49	3
334	1	0	0	58	2
335	1	8	1	7	1
336	1	16	1	17	-1
337	1	28	1	27	1
338	1	40	1	38	2
339	1	52	1	50	2
340	2	4	2	2	2
341	2	16	2	15	1
342	2	28	2	29	-1
343	2	40	2	42	-2
344	2	56	2	57	-1
345	3	12	3	12	0
346	3	28	3	27	1
347	3	44	3	43	1
348	4	0	3	59	1
349	4	16	4	16	0
350	4	36	4	33	3
351	4	52	4	50	2
352	5	8	5	8	0
353	5	20	5	26	-6
354	5	44	5	44	0
355	6	4	6	3	1
356	6	24	6	22	2
357	6	44	6	41	3
358	7	4	7	0	4
359	7	24	7	20	4
360	7	44	7	40	4

Table 80: Georg Puerbach: data from manuscript, recomputation and differences, part IX.

Our results give the best fit with: $\sigma = 6,9954$, $E_M = 1,8055$ seconds of time and this is the pattern of the differences:

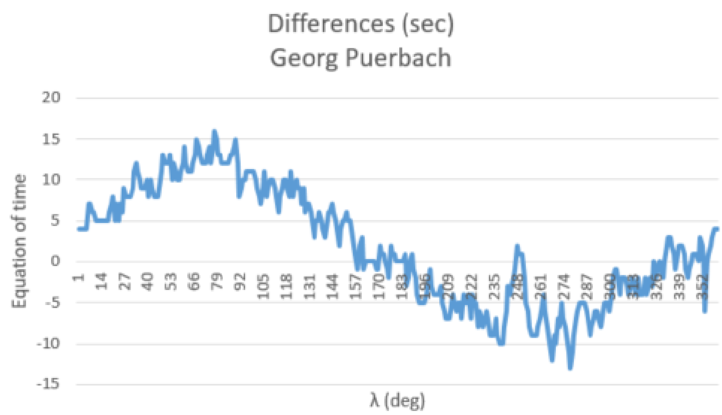


Figure 58: Georg Puerbach's equation of time differences, given in seconds.

We want to point out that we also compared Puerbach's equation of time table with the one of Levi ben Gerson multiplied by 4. In the following graphic we can see the differences between the two tables:

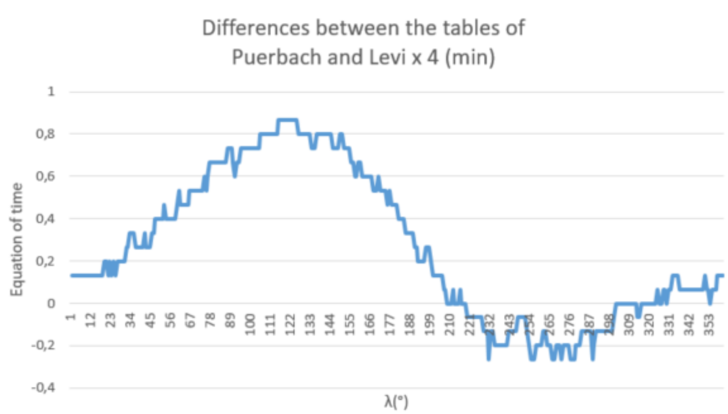


Figure 59: Differences between Georg Puerbach and Levi ben Gerson (multiplied by 4)'s equation of time tables, given in minutes.

then we recompute also the equation of time for Puerbach using Levi's parameters but without reaching a good result.

11.4 Conclusions

From our preliminary analyses we can state that parameters from LS are close to the historical values for the obliquity of the ecliptic and for the solar eccentricity, by the way the result of our recomputation is not excellent even if most part of the values have and error below 10 seconds.

By the comparison of Puerbach and Levi ben Gerson's table multiplied by 4 we can state that there is a large error in the first half of the table, that is one reason way the recomputation by the use of Levi's parameters does not give better results.

11.5 List of manuscripts

- Vienna, ÖNB, 5299, fol. 112r-117r; 124r-125v;
- Vienna, ÖNB, 5291, fol 100v-163r;
- Vienna, ÖNB, 5334, fol 9r-7v, 95r-137v;
- Munich, Clm, 214, fol 115r-160r;
- Vienna, ÖNB, 2380, fol 30v-33v;
- Munich, Clm, 19550, fol. 151r-213v;
- Munich, Clm, 51, fol. 37r-45v;
- Vienna, ÖNB, 5412, fol 175r-236v.

12 Abraham Zacut

Abraham Zacut was born in Salamanca, Spain, in 1452 and died in Portugal in 1515. He was a Jew and studied medicine and astrology. He left Spain for Portugal in 1492 (after the law of the expulsion) [19].

His disciples were Joseph Vizinus and Augustin Ricius.

12.1 Overview

In 1478 he wrote his first book on astronomy in Hebrew, the *ha-Hibbur ha-gadol (The Great Composition)* with canons and tables. He also wrote *Tratado breve en las ynfluencias del cielo* in 1486 (published by Carvalho in 1927) and *De los eclipses del sol y la luna*. In 1496 he composed the *Almanach perpetuum* that is a set of canons (different frome the *Hibbur*) and astronomical tables (mostly taken from the *Hibbur*) in the form of an almanac, with the entries computed from the Alfonsine Tables. In Portugal he prepared the tables for the voyage of Vasco da Gama, fragments of them can be found in *Livro de marinharia* by Andre Pires and they were computed basing on *Almanach perpetuum* [14].

The use of the Alfonsine Tables in Spain comes from 1460 in Salamanca, shortly before Zacut's astronomical activity. Their Castilian canons survived but not the tables. The diffusion of the tables came from Paris with a revision by John of Lignères, John of Murs and John of Saxony.

See Appendix J for primary and secondary sources.

12.2 Presentation of the table

In the equation of time table in the *Almanach perpetuum*, identical to the one in the *Hibbur* in Lyon and in Warsaw, the argument is the day of the year and the entries are given in minutes of time for each day of the year that begins in March. The maximum is 0.32h in 20-24 October and the minimum is 0;0h in 22 January-6 February. These values are compatible with the ones in the Alfonsine Tables, in al-Battānī and in the Toledan Tables.

Tabula equacionis diez

dies mēsium	mārtius	aprilis	mayus	yunius	yulius	augustus	septembre	octobre	nouembre	decembre	ianuarius	februarius
1	6	16	22	20	14	13	20	29	31	21	6	0
2	6	16	22	20	14	13	20	29	31	20	5	0
3	6	17	22	19	13	13	21	30	30	20	5	0
4	6	17	22	19	13	13	21	30	30	19	5	0
5	7	17	22	19	13	13	21	30	30	19	4	0
6	7	17	22	19	13	13	22	30	30	18	4	0
7	7	18	22	18	13	13	22	31	30	18	4	1
8	8	18	22	18	13	14	22	31	29	17	3	1
9	8	18	22	18	13	14	23	31	29	17	3	1
10	8	18	22	18	13	14	23	31	29	16	3	1
11	9	19	22	18	13	14	23	31	29	16	3	1
12	9	19	22	17	12	15	24	31	28	15	2	1
13	9	19	22	17	12	15	24	31	28	15	2	1
14	10	19	22	17	12	15	24	31	28	14	2	1
15	10	20	22	17	12	15	25	31	28	14	2	2
16	10	20	22	17	12	16	25	31	27	13	1	2
17	11	20	22	16	12	16	25	31	27	13	1	2
18	11	20	22	16	12	16	25	31	27	12	1	2
19	12	20	22	16	12	16	26	31	26	12	1	3
20	12	21	22	16	12	17	26	32	26	11	1	3
21	12	21	22	16	12	17	26	32	25	11	1	3
22	13	21	21	15	12	17	27	32	25	10	0	3
23	13	21	21	15	12	17	27	32	24	10	0	4
24	13	21	21	15	12	18	27	32	24	10	0	4
25	14	21	21	15	12	18	28	31	23	9	0	4
26	14	21	21	15	12	18	28	31	23	9	0	4
27	14	22	21	14	13	19	28	31	23	8	0	5
28	15	22	20	14	13	19	28	31	22	8	0	5
29	15	22	20	14	13	19	29	31	22	7	0	5
30	15	22	20	14	13	20	29	31	21	7	0	0
31	16	0	20	0	13	20	0	31	0	6	0	0

Figure 60: Abraham Zacut's equation of time in Almanach perpetuum.

12.3 Analyses of the table

In this section the difference between such approximation and the correct result should be further evaluated, being the average solar motion very close to $1°/day$. We focus only on a preliminary analyses by recomputating the equation of time using the modern formula with the underlying parameters from LS (obliquity of the ecliptic is given in the Almanach perpetuum):

- $\varepsilon = 23;33°$ (historical);
- $e = 1;57,16$ parts (LS);
- $\lambda_A = 88;27,6°$ (LS);
- $E_0 = 4;7,11°$ (LS).

12.4 Recomputation by modern method

Due to the shift of the original table that starts at March the 1st, we shifted the table of 21 days so that the table starts with Aries 1°. We get $\sigma = 1,4251$, $E_M = 0,5218$ minutes of time and here are the differences:

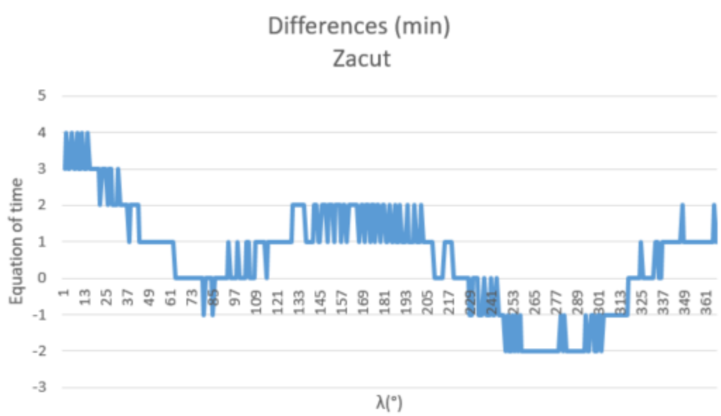


Figure 61: Abraham Zacut's equation of time differences, given in minutes.

12.5 Conclusions

According to the not excellent results from our preliminary recomputation we decided to put aside Zacut from our analyses.

12.6 List of Manuscripts

- Lisbon, Biblioteca Nacional, MS Cast. 124;
- Madrid, Biblioteca Nacional, MS Latin 3385, ff. 1-113;

- Lyon, Bibliothéque Municipale, MS Heb.14;
- Warsaw, ZIH, MS Heb. 245.

13 Topics For Further Research

An initial version of this work was prepared in the context of the ERC project ALFA (CoG 723085) on Alfonsine astronomy during my first year of PhD in 2017/2018 at the Observatoire de Paris. I worked on the evolution of the equation of time from the Greek tradition of Ptolemy in the second century to Abraham Zacut in the first half of the sixteenth century.

In this survey I made a first round of recomputation by the use of modern formulas using the underlying parameters from LS.

After that I tried to recompute everything by the use of tables in a first superficial analyses, using only the solar equation table given from the primary and secondary sources without making an interpolation to express it as a function of the true solar anomaly. For the parameters we made some historical considerations in order to improve the results given by LS.

In a third round of recomputation I focused on Ptolemy and Levi ben Gerson as referents for the equation of time given in units of time (precision to seconds) and in units of degrees (precision to minutes of degree) respectively: Levi's case was successful, Ptolemy's case not completely. On Ptolemy I tried also the extraction of the solar equation and right ascension underlying the equation of time table: this procedure is still opened to improvements. I also tried to reconstruct solar equation from the chord table and right ascension from interpolation between some values given in the *Almagest* and using them for a new recomputation: also this procedure is still opened.

Then I focused on the Alfonsine astronomy trying to recompute 3 astronomers never investigated before: Peter of Saint Omer, John of Murs and Giovanni Bianchini: the goal is to reproduce the same precision of errors already achieved in the secondary sources.

The case of Peter of Saint Omer is the best one: we get a result with an error that is mainly below 2 minutes of degree with some peaks of 3 minutes of degree.

In the case of Bianchini we get an error that is mainly below 10 seconds but with some peaks of 20-30 seconds.

The case of John of Murs is particular because his tables are really hard to recompute. We get an error about one minute of degree for the *Kalendarium*, around 10 seconds (with some peaks of 20-30 seconds) in *Tables of 1321* in days and approximately the same for *Tables of 1321* in λ.

As I pointed out several times in this survey, further investigation is needed by the use of the chords table for the reconstruction of the solar equation and by the use of linear interpolation between some corrected values (given for multiples of 10 degrees) calculated in the *Almagest* for the right ascension.

I did not make a specific statistical analyses apart the standard deviation and the average value: further investigation is needed.

I did not work on different ways of rounding: starting from the original table I converted the values of the equation of time in decimal degrees because my recomputation is in decimal degrees. I worked with all the decimal digits in my Excel files to get the result that, at the end, is reconverted in the original units of the table without rounding, then I convert this last one in sexagesimals so there is a sort of truncation. The differences are given from the difference between the original table in sexagesimals and my final result in sexagesimals.

Appendices

A Ptolemy

A.1 Primary and secondary sources

- The *Almagest*[7], Book III [on the Sun];
- PhD thesis by W.D.Stahlman [The Astronomical Tables of Codex Vaticanus Graecus 1291, 1959] [8];
- The critical edition by Tihon and Mercier [9].

A.2 How to get the Equation of Time

The tropical year, as defined by Ptolemy, is the period of time in which the true Sun S (with variable apparent speed on the ecliptic) returns to an equinox or solstice and for him was a constant [3]. The ecliptical mean Sun $\bar{S}$ (on the ecliptic) and the equatorial mean Sun M (on the equator) have uniform motion. λ is the ecliptical longitude of the true Sun, $\bar{\lambda}$ is the ecliptical longitude of the ecliptical mean Sun, μ is the right ascension of the equatorial mean Sun and they are measured from the vernal point.

$\bar{\lambda}$ and μ have uniform motion so they increase linearly at the same rate: $\bar{\lambda} + E_0 = \mu$ and $\bar{\lambda} = \lambda + c$ where c is the solar equation.

Time is counted from midday and the true solar time is defined as the hour angle $h(S)$ of S and the mean solar time is defined as the hour angle $h(M)$ of M.

The equation of time is (in degrees)

$$E_d = h(S) - h(M) \tag{6}$$

and it is not a constant because S has a variable speed and moves on the ecliptic while M has an uniform speed and moves on the equator. At any moment the equation of time equals the difference between the right ascension of M and S

$$E_d = \mu - \alpha(\lambda) = \bar{\lambda} - \alpha(\lambda) + E_0 \tag{7}$$

where $\alpha(\lambda)$ is the right ascension of S which depends on the obliquity of the ecliptic ε.

For $\lambda \in [0^\circ, 90^\circ\rangle$ we use the modern formula

$$\alpha(\lambda) = \arctan(\cos\varepsilon \tan\lambda) \tag{8}$$

and for $\lambda \in [90^\circ, 360^\circ]$ we use, for reasons of symmetry, $\alpha(180^\circ - \lambda) = 180^\circ - \alpha(\lambda)$ and $\alpha(180^\circ + \lambda) = 180^\circ + \alpha(\lambda)$.

The solar equation c depends on the longitude of the solar apogee λ_A and on the solar eccentricity e so it can be expressed by the formula

$$c(\lambda) = \arcsin\left(\frac{e}{60}\sin(\lambda - \lambda_A)\right) \tag{9}$$

and in the case of the mean solar longitude we have

$$c(\bar{\lambda}) = \arctan\left(\frac{e\sin(\bar{\lambda} - \lambda_A)}{60 + e\cos(\bar{\lambda} - \lambda_A)}\right). \tag{10}$$

The constant E_0 for the solar equation determines the synchronization of $\bar{S}$ and M and in modern astronomy is taken to be zero ($\bar{S}$ and M passes the vernal equinox simultaneously) but in ancient astronomers used to fix an "epoch" (the starting point of the mean motion tables) in order to determine it.

At the time of the *Handy Tables* the equation of time was close to its minimum so the constant E_0 was chosen to have the minimum at $0;0,0^h$ and it is called "epoch constant".

In the *Handy Tables* the tabulated quantity is E_h (equation of time in hours) and to convert E_d to E_h we have to divide by 15 ($24^h = 360°$) but the most accurate conversion factor is $(360;0+0;59,8)/24 \approx 15;2,28°$ because the daily motion of the Sun on the ecliptic is about $0;59,8°$/day. Generally we can write, for the conversion, $E_h = \frac{1}{D}E_d$ where D can take the value of $15°$ or $15;2,28°$ so

$$E_h(\lambda) = \frac{1}{D}(\lambda + c(\lambda) - \alpha(\lambda) + E_0) \tag{11}$$

and in the case of the mean solar longitude we use

$$\bar{E_h}(\bar{\lambda}) = \frac{1}{D}(\bar{\lambda} - \alpha(\bar{\lambda} - \bar{c}(\bar{\lambda})) + E_0). \tag{12}$$

A.3 Analysis of the equation of time table by Benno van Dalen

To understand an equation of time table we need to know the independent variable (λ or $\bar{\lambda}$), the values of the underlying parameters (obliquity of ecliptic ε, longitude of the solar apogee λ_A , solar eccentricity e, the epoch constant E_0, the conversion factor D), the accuracy for solar equation and right ascension tables and if interpolation was used.

If we do not have these information we need to use some statistical methods for recovering the underlying parameters.

To see if interpolation was used we have to check the first or second order differences. Tabular values calculated without interpolation are called *nodes*, the other values are called *internodal values* and they are disregarded. If we have interpolation we will see, in the tabular differences, two consecutive nodal values differ at most by a single unit.

If we find, in a table, that the final sexagesimal digit of all values or nodes is a multiple of 4 we can conclude that $D = 15(°/h)$.

Using the symmetry relations between the right ascension and the solar equation we get

$$E_h(\lambda_A) = E_h(\lambda + 180°) \tag{13}$$

$$\bar{E}_h(\lambda_A) = \bar{E}_h(\lambda + 180°) \tag{14}$$

from which we can get λ_A, then

$$\sum_{i=1}^{2n} E_h\left(\frac{180° i}{n}\right) = \frac{2nE_0}{D} \tag{15}$$

for every n (half the number of available equation of time values) and

$$\int_0^{2\pi} \bar{E}_h(\bar{\lambda}) d\bar{\lambda} = \frac{2\pi E_0}{D} \tag{16}$$

and

$$E_h(\lambda) + E_h(180^\circ - \lambda) + E_h(\lambda + 180^\circ) + E_h(360^\circ - \lambda) = \frac{4E_0}{D} \tag{17}$$

for every λ from which we can get E_0.

From

$$\alpha(\lambda) = \lambda + E_0 - \frac{1}{2}D \tag{18}$$

for every λ we get the right ascension and, at the end from

$$c(\lambda) = \frac{1}{2}D(E_h(\lambda) - E_h(\lambda + 180^\circ)) \tag{19}$$

for every λ from which we get the solar equation.

Using a least square estimation we can get ε, λ_A, e, E_0. In order to estimate the unknown parameters we have to know the formula used for computing the table so we have to perform an estimation from the minimum standard deviation of tabular errors and if it is much larger than the one of a corrected table it means that we are using the wrong formula.

We can use least square estimation only if tabular errors are independent, have mean zero and identical standard deviation.

A.3.1 Mathematical Analysis

Benno van Dalen investigated the tabular differences and he concluded that Ptolemy used linear interpolation within intervals of 6 degrees and that they are distributed between the nodal values in an irregular way.

He found, through a least square estimation, that the argument used was the true solar longitude (using the standard deviation of the supernodes, tabular values for multiples of 30 degrees) and that the 95% confidence intervals of the underlying parameters are (using $D = 15(^\circ/h)$ because it fits better with the historical values)

- $\varepsilon = \langle 23;51,50 - 23;52,26 \rangle$
- $e = \langle 2;29,51 - 2;30,0 \rangle$
- $\lambda_A = \langle 65;57,25 - 66;0,38 \rangle$
- $E_0 = \langle 3;34,3 - 3;34,9 \rangle$.

Through an analysis of the errors he concludes that the errors derive from systematic errors in the underlying right ascension values.

Using Eq.19 we can get $c(\lambda)$ with the condition that the tabulations of right ascension and solar equation satisfy the symmetry relations (and this is true for the *Almagest*) but we have to distinguish basing on the value of D.

For $D = 15(^\circ/h)$ corrected values of the equation of time contain a rounding error of at most $30'''$ so solar equation values extracted from correct values of the equation of time contain errors with a maximum absolute value of $\frac{1}{2}D * 30''' \approx 7\frac{1}{2}'''$, the standard deviation of these error is approximately $3''$.

He concluded that the value for the longitude or the solar apogee used by Ptolemy was $\lambda_A = 66^\circ, 0'$ with both values of D and for the solar eccentricity the value was $e = 2;30$ with $D = 15(^\circ/h)$.

In the *Almagest* and in the *Handy Tables* we find tables or the solar equation as a function of the mean solar anomaly and to pass to a function of the true solar anomaly Ptolemy used inverse linear interpolation.

For the right ascension values used by Ptolemy to calculate the equation of time there are systematic errors and Benno van Dalen extracted the right ascension using Eq.18 but there is the problem of the epoch constant that was chosen in order to get the minimum value of the equation of time so he used a range of $E_0 = \langle 3;34,0 - 3;34,15 \rangle$. For the Ptolemaic value for the obliquity $\varepsilon = 23;51,20^\circ$ the right ascension values contain the same error pattern as the equation of time (small errors for supernodes) so these values were by exact linear interpolation between the *Almagest* values without rounding and we can see it from the number of errors and from the standard deviations. This result is independent of the value of E_0 (between the range considered) but the number of errors is minimized for $E_0 = 3;34,7,30^\circ$. By minimizing the sum of the squares of the differences between the extracted right ascension and the values computed as we said above we get $E_0 = 3;34,6,26^\circ$. At the end there are some considerations about the outliers (values that are out of the expected range of error), some of them came from an error in the transmission of table, other are due to scribal mistakes, we remind that all these errors were made before interpolation was done.

A.3.2 Final Conclusions

- The independent variable used for the equation of time is λ;
- the epoch constant E_0 was chosen in order to minimize the equation of time (zero);
- Ptolemy used Eq.1 for his computation;
- The conversion factor is $D = 15(^\circ/h)$; The underlying solar equation is based on the value of $e = 2;30$ and was not determined by inverse linear interpolation;
- the value for the longitude of the solar apogee used is $\lambda_A = 66;0^\circ$;

- the underlying right ascension is based on the values for multiples of 10 degrees given in the *Almagest* and the intermediate values are determined by exact linear interpolation without rounding;

- the value of the obliquity is $\varepsilon = 23;51,20°$;

- it is not possible to make numerical and historical consideration about the epoch constant so the value that minimize the number of errors is $E_0 = 3;34,7,30°$;

- linear interpolation was used to determine the tabular values for non-multiple of 6 degrees.

So we can conclude that Ptolemy rounded the longitude of the solar apogee in order to avoid interpolation between his solar equation values, he used linear interpolation between right ascension values for every 10 degrees and he used linear interpolation between equation of time values for multiples of 6 degrees.

A.3.3 Some considerations on the table for the equation of time in the papyrus P. London 1278

The papyrus P. London 1278 is Greek and is kept in the British Museum, it contains six fragments of numerical tables; Neugebauer, after some analysis on this papyrus, stated that this tables may be recensions of the tables in the *Handy Tables*. The right ascension values are identical to the ones in the *Handy Tables*, the equation of time was calculated for the Era Nabonassar (used in the *Almagest*, so the papyrus probably can be put between this and the *Handy Tables*); the minumum for the equation of time occurs in Aquarius and the values are given in minutes of a hour. By applying the least square estimation, assuming $D = 15(°/h)$ and that the independent variable is λ we find out that the equation of time is calculated according to Eq.1 and that the values for the solar eccentricity and the longitude of the solar apogee are respectively $e = 2;30$ and $\lambda_A = 66,0°$ as in the *Handy Tables*.

Another method is shown by Benno van Dalen: maybe the papyrus were computed by substracting the values in the *Handy Tables* from a constant and rounding the result to minutes according to the formula

$$T_{PL}(\lambda) = r_1(C - T_{HT}(\lambda)) \tag{20}$$

for every λ, C is a constant, T_{PL} are the values in the papyrus, T_{HT} are the values in the *Handy Tables* and r_1 indicates the modern rounding to minutes. From this analysis is possible to conclude that the papyrus was calculated according to the *Almagest* theory and from the equation of time in the *Handy Tables* and it is based on the values $\varepsilon = 23;51,20°$, $\lambda_A = 66;0°$, $e = 2;30$ and the constant was taken to be $C = 0;32$.

A.3.4 Statistics

- *Tabular errors*: defined as $e_\theta(x) = T(x) - f_\theta(x)$ where T(x) is the tabular value of T (the table to investigate) and $f_\theta(x)$ is a functional value of the tabulated function. $T(x)$ is said to be correct if $T(x) = r_k(f(x))$ where r_k denotes a rounding procedure to k sexagesimal fractional digits. If the number of sexagesimal digits of the tabular values of a corrected table is sufficiently large then the tabular errors can be assumed independent with uniform distribution.

- *Outliers* are tabular errors significantly larger that the others and they can result from computational errors or scribal mistakes.

- *Least squares estimation*: let the objective function $\Phi(\theta)$, where θ is a parameter vector) be defined as the sum of the squares of the tabular errors: $\Phi(\theta) = \Sigma_x \in_x (T(x) - f_\theta(x))^2$. A least squares estimation for a parameter vector θ is a vector $\hat{\theta}$ that minimizes $\Phi(\theta)$ and we need an iterative method such that of Gauss-Newton.

  The variance is given by $\sigma^2 = \Phi(\hat{\theta})/n$.

B Al-Khwārizmī

B.1 Primary and secondary sources

- *The Astronomical Tables of al-Khwārizmī*, O. Neugebauer and H. Suter [20];
- *Islamic astronomical tables: mathematical analysis andhistorical investigation*, Benno van Dalen [5];
- *Die astronomischen tafeln des Muammed ibn Mūs āa al-Khwārizmī in derbearbeitung des Maslama ibn Amed al-Madjriīi und der latein. uebersetzungdes Athelhard von Bath auf grund der vorarbeiten von A. Bjørnbo undR. Besthorn in Kopenhagen...*, M.M. Khuwārizmī, M.A. Majrīi, R.O. Besthorn, A.A. Bjørnboo, and H. Suter [21].

B.2 Brief Description of some Set of Tables

- SOLAR EQUATION TABLES: computed according to the method of declinations described by al-Bīrūī, the Indians used the method of sines. Al-Majrīṭī found the value $2°14'$ for the maximum solar equation, $77°55'$ for the longitude of the solar apogee.
- SOLAR DECLINATION: we have two tables, in one al-Khwārizmī used the Ptolemy's value for the obliquity ($23°51'20''$) in the other he used $24°$ as in the Indian tradition. In al-Majrīṭī's recension we only have the Ptolemaic table.
- RIGHT ASCENSION: one table in the original zīj starting with Capricorn with $23°51'20''$ (as in the Handy Tables).
- EQUATION OF TIME: in the al-Muthannā's commentary there is no mention of this table but al-Majrīṭī's recension contains one table given in seconds of an hour, the argument of the tables is the true solar longitude and the equation of time must be added to the mean solar time to obtain the true solar time.

We can divide the tables into five groups:

1) tables deriving from al-Khwārizmī's original zīj based on Indian methods and/or parameter value (for example parallax tables);
2) tables deriving from al-Khwārizmī's original zīj based on Persian methods and/or parameter values;
3) tables deriving from al-Khwārizmī's original zīj based on Ptolemaic methods and/or parameter values (for example solar declination and right ascension tables);
4) tables modified by al-Majrīṭī;
5) tables replaced or added by al-Majrīṭī.

The equation of time table, as we can see at the end of this survey, belongs to the third group.

B.3 Suter's Survey

In Suter 1914 [21] we can find a complete transcription of the equation of time table in the Latin recension of al-Khwārizmī's Sindhind zīj by the Spanish astronomer al-Majrīṭī (A.D. 1000) translated by Adelard of Bath (12th century). He gave a critical edition of the Latin text and a commentary but not a translation.

B.4 Neugebauer's Survey

Neugebauer's purpose [20] is to extend the Suter's survey giving first of all a translation. He used the incomplete manuscript in Corpus Christi College, Oxoford and the cod. Vindob. 2385.

Al-Khwārizmī used noon of July 14 A.D. 622 as epoch, he reckoned Muharram 1 (the month) of the first year of the era Hijra (its beginning is noon day 4) from Wednesday July 14 (julian day 1948438) to noon of Thursday July 15 (julian day 1948439) but in the civil calendar Thursday begins at sunset of July 14 to sunset of July 15 so Muslim astronomers reckoned the era Hijra from AD 622 July 15 which corresponds to Muharram 1.

The Arabic year is regulated on the motion of the Moon and has 354 days plus 11/30 days but the additional fraction are not counted if there are less than 1/2 so the year has 354 days or 355 days if more than 1/2 day has accumulated, in that case it is called elkebice. The Arabic year consist of 12 months the first (elmuharram) has 30 days, the second (zafar) has 29 days and so on alternatively but the last (dulheia) has 30 days.

In the Al-Khwārizmī 's tables we find the rule that the *examinatio dierum* which transform true to mean solar days is always negative.

In the table we have the first column called *gradus regulares* in O (centre of deferent) or *gradus equales* in C (center of the epicycle). The second column for each zodiacal sign (the first is Aries) gives the equation of time in minutes and seconds: the minimum 0 occurs at Aquarius 22° and the maximum $0;34,28^h$ at Scorpio $(8-9)°$.

B.5 Benno van Dalen's Survey

The conversion factor [5] is $D = 15°/h$ because all the tabular values are multiples of four seconds, the independent variable is the true solar position and the equation must be subtracted from true solar time to get the mean solar time (we have, in the resulting function, a local maximum, a local minimum, a global maximum and a global minimum). But to verify if it is used the true or mean solar position is not possible, we can only derive some properties to investigate which variable was used.

From the symmetry relations in the right ascension and solar equation we can reconstruct these functions from a table of the equation of time as a function of the true solar longitude.

B.5.1 Reconstruction of the Underlying Right Ascension and Solar Equation

We know that there holds some symmetry relations for the right ascension α

$$\alpha(180^\circ - \lambda) = 180^\circ - \alpha(\lambda); \alpha(180^\circ + \lambda) = 180^\circ + \alpha(\lambda) \tag{21}$$

(so that the rising time for a sphaera recta of Aries is the same in Virgo and in Libras) and for the solar equation c

$$c(\lambda_A + \kappa) = -c(\lambda_A - \kappa); c(\lambda_A + 180^\circ + \kappa) = -c(\lambda_A + \kappa) \tag{22}$$

(the absolute value of the solar equation only depends on the distance of the Sun from apogee or perigee with the different sign depending on which side of the apse's line) where κ is the true solar anomaly, λ is the true solar position and λ_A is the longitude of the solar apogee ($\kappa = \lambda - \lambda_A$).

Then we know that the equation of time (for $D = 15(^\circ/h)$) is

$$E(\lambda) = 1/15 * (\lambda + c(\lambda - \lambda_A) - \alpha(\lambda) + E_0) \tag{23}$$

$$E(180 + \lambda) = 1/15 * (\lambda - c(\lambda - \lambda_A) - \alpha(\lambda) + E_0) \tag{24}$$

and by adding the two equations above we get

$$\alpha(\lambda) = \lambda + E_0 - 7.5 * (E(\lambda) + E(180 + \lambda)) \tag{25}$$

from which we can reconstruct the right ascension underlying an equation of time table as a function of λ if we know the epoch constant E_0.

By subtracting Eq. 32 and Eq. 24 we find

$$c(\lambda - \lambda_A) = 7.5 * (E(\lambda) - E(180^\circ + \lambda)) \tag{26}$$

from which we can reconstruct the solar equation underlying an equation of time table as a function of λ even if we do not know E_0. The last formulas do not hold for the equation of time as a function of the mean solar longitude, for this reason we assume that in al-Khwārizmī's table the argument is the true solar position. Then, from Eq. 26 we can reconstruct the solar equation and van Dalen found that the values are close to the ones calculated by Ptolemy for $e = 2;30$ and $\lambda_A = 84^\circ 40'$. If we used the mean solar longitude we would get systematic divergences between the reconstructed and computed values for any parameter.

To reconstruct the underlying right ascension we need to know E_0 using Eq. 32, Eq. 24 with $E(180^\circ - \lambda)$ and $E(360^\circ - \lambda)$ from which we get

$$E(\lambda) = n/4 * (4/15 * E_0) \tag{27}$$

where n are the values of the equation of time, so

$$E_0 = (15/n) * \sum_{i=1}^{n} E(i * 360^\circ/n) \tag{28}$$

but we have to use (also for the reconstruction of solar equation and right ascension) the tabular values $T(\lambda)$ instead of $E(\lambda)$.

For the al-Khwārizmī's table we find $E_0 = 4;30,3°$ from Eq. 28 and since he used and accuracy to minute probably he rounded to $E_0 = 4;30°$. Using this value for the epoch constant and assuming that the argument is λ, we can get $\alpha(\lambda)$ from Eq. 25. Using al-Khwārizmī's obliquity $\varepsilon = 23°51'$ we get bad results so that probably the argument used is not λ.

To try to find it out we use the least square method: we apply it to the tabular values from al-Majrīṭī's recension using the historical parameters $\varepsilon = 23°51'$, $e = 2;20$, $\lambda_A = 77°55'$, $E_0 = 4;30°$.

At first we can see from the differences (that are up to more than a couple of units of the final sexagesimal position and their pattern is non-random) that both the parameters or the assumption of true solar longitude are wrong.

If we calculate the standard deviation of the differences we can see that our computation is good if the standard deviation is approximately $0;0,0,17$ and this is not our case.

By summarizing:

1) the standard deviation of the differences between the given historical values and the tabulated values should be reasonably small;

2) the differences between the historical table and the computed table based on least mean square method should be random and without any regular pattern;

3) the least square estimates should be close to the historical values.

The historically plausible values can be made on the basis of the 95% confidence intervals for the underlying parameters.

From the least square estimation (using $D = 15(°/h)$ and λ) we find some historically plausible values for the obliquity and for the solar eccentricity but the standard deviation is 20 times larger than the value of $0;0,0,17$ so we conclude that we used the uncorrected underlying function that means that the variable is not the true solar longitude.

If we perform the least squares estimation assuming that the argument is the mean solar longitude we obtain historically plausible value for the obliquity, an impossible value for the solar eccentricity, a large standard deviation so we conclude that also the mean solar longitude is not the correct argument.

At the end van Dalen resumed Kūshyār's approach to made his solar equation always additive by subtracting it from 2° and obtaining a displaced solar equation defined by

$$c_{md}(\bar{\kappa}) = 2° - c_m(\bar{\kappa}) \tag{29}$$

where $\bar{\kappa} = \bar{\lambda} - \lambda_A$ is the mean solar anomaly.

By adding this last equation to the mean solar longitude we obtain $\lambda + 2°$ instead of λ so Kūshyār replaced $\bar{\lambda}$ with a shifted mean solar longitude $\lambda_{ms} = \bar{\lambda} - 2°$. The addition of the displaced solar equation to the corresponding λ_{ms}

has given λ. To tabulate the displaced solar equation as a function of λ_{ms} he shifted all the values two degrees backwards.

By applying this approach to al-Khwārizmī's table we can obtain, for any given shift Δ and shifted true solar longitude $\lambda_s = \lambda - \Delta$, a corresponding equation of time $E_s(\lambda_s) = E(\lambda_s + \Delta)$.

To determine the shift we can consider it as a fifth parameter of the equation of time and we can obtain it from a least squares estimation. After performing the estimation we get a small standard deviation, we find close historically plausible values for the underlying parameters ($\varepsilon = 23°51'$, $e = 2;20$, $\lambda_A = 82°39'$, $E_0 = 4;30°$ and a shift close to $\Delta = -2°$).

B.6 Conclusions

- the independent variable is λ;
- the conversion factor is $D = 15°/h$;
- the obliquity of the ecliptic is $\varepsilon = 23°51'$;
- the solar eccentricity is $e = 2;20$;
- the longitude of the solar apogee is $\lambda_A = 82°39'$;
- the epoch constant is $E_0 = 4;30°$;
- the shift is $\Delta = -2°$.

C Al-Battānī

C.1 Primary and secondary sources

- *Al-Battānī sive Albatenii Opus astronomicum*, C.A. Nallino [11];
- *The Toledan tables: a review of the manuscripts and the textual versions with an edition*, Fritz Saaby Pedersen [12];
- *Tabule astronomice illustrissimi Alfontij regis castelle*, Erhard Ratdolt [10].

C.2 Ratdolt: Tabule Astronomice illustrissimi Alfontij regis castelle (1483)

Erhard Ratdolt (1442–1528) was an early German printer from Augsburg. He was active as a printer in Venice from 1476 to 1486, and afterwards in Augsburg. In his *Tabule Astronomice illustrissimi Alfontij regis castelle* [10] we can find an equation of time table. The heading is *Tabula elevationu Signoru in circulo directo*, the first column is *Gradus equales* for the solar longitudes in intervals of one degree, then we find other two columns for each sign (four zodiacal signs per table) starting with Capricornus: the *Ascensiones* and the *Aequatio dierum* both given in grades and minutes.

The ascension table is entitled *Tabula elevationu Signoru in primo climate* and *Tabula elevationu Signoru in secundo climate*: in the first column *Gradus equales* we find the solar longitudes in intervals of one degree, then we find other two columns for each sign (four zodiacal signs per table) starting with Aries: the *Ascensiones* and the *Partes horarum* both given in grades and minutes.

C.3 Nallino: Al-Battani Opus Astronomicum (1903)

Carlo Alfonso Nallino (1872–1938) was an Italian Islamist, Arabist and academic; his most famous work was the one on al-Battānī. He was the only one to make a full study of the only surviving complete Arabic manuscript of al-Battānī's zīj kept in the Escorial Library near Madrid and it consists in a Latin translation, an edition of the Arabic text and a transcription of the tables. He corrected the tables by recomputation and by adopting values from the Almagest and the Handy Tables of Ptolemy. In his equation of time (called *nychthemeron*) table [11] we find on the heading *Initium tabularum ascensionum signorum in sphaera recta et aequationis nychthemeron.* The argument, the solar longitude, is in the first column and it is given in intervals of one degree; the table starts with Capricorn in which we find, in the second column, the ascension given in degrees and minutes; in the third column we find the *aequatio nychthemeron* in grades and minutes. We find three zodiacal signs per each table.

The heading for the solar declination table is *Initium tabularum declinationis Solis a circulo aequinoctiali, in meridiano*: in the first and third columns we have the *declinatio* in grades, minutes and seconds; in the second and fourth columns, *Numeri quadruplices zodiaci*, divided in four zodiacal signs (Aries, Virgo, Libra, Pisces and Taurus, Leo, Scorpio, Aquarius), we find the argument in interval of one degree from 1 to 360. The last part of the table in entitled *Finis tabularum declinationis Solis et [arcuum] zodiaci* and we have the first column with the *declinatio* from which we take the value for the obliquity of the ecliptic $23°35'$ calculated in 880 in al-Raqqa, and the last column with Gemini, Cancer, Sagittarius and Capricornus.

For the longitude of the solar apogee, we find the value of $\lambda_A = 82°15'$ at the beginning of Aries. The maximum of the equation of time is $7;54°$ in Scorpio $(8-10)°$ and the minimum is $0;0°$ in Aquarius $(18-19)°$.

Al-Battānī, as a formula for the declination δ, used

$$\sin\delta = \frac{\sin\lambda\sin\varepsilon}{R} \tag{30}$$

where λ is the solar longitude, R is the radius of the circle and ε is the obliquity of the ecliptic.

For the right ascension α

$$\sin\alpha = R\tan\delta\cot\varepsilon \tag{31}$$

For the *Aequatio Solis* he used

$$\tan aequat. = \frac{e_1\sin m}{1+e_1\cos m} \tag{32}$$

where m is the solar mean anomaly, e_1 is the radius of the epicycle or the orbit eccentricity and is equal to $\frac{e}{60}$; the value for the solar eccentricity is $e = 2;4;45$.

C.4 Pedersen: The Toledan Tables (2002)

The most successful Arabic astronomical tables translated into Latin during the 12th century were the *Toledan Tables* that were used during the 13th and the early 14th century for astronomical calculations. The Arabic original are probably lost so the tradition is Latin. The Toledan Tables comprise the tables as in many standard Arabic zījes, the conversion between dates are done according to the Islamic era (Hijra) and the computation is based on Ptolemaic astronomy. The tables are accompanied by canons and to be said Toledan they must include mean motion tables with mean longitudes of the sun, moon and planets, syzygy tables, they are calculated for the meridian of Toledo, they are based on the Islamic calendar and the longitudes are sidereal. They were used for astrological purposes. The most prolific manuscripts in Paris originated in the 1270s-1280s and after 1320 they went in competition with the Alfonsine tables.

Most part of the table collections were by al-Khwārizmī (first half of 9th century, Hindu and Persian sources) and by al-Battānī (900, based on the Handy Tables).

From Arabic texts we have that the Toledan tables were divided into two versions: one based on al-Battānī and one based on al-Khwārizmī's Zij al-Sindhind in the structure but with some from al-Battānī with Ptolemaic methods. The transmission if both Latin versions were independent of each other; the latter translation had Christian adaptations and this is the "vulgate" version from which the Parisian manuscript of the late 13th century is a late branch.

Al-Battānī's tables survived in one Western Arabic manuscript. The Latin canons of the tables based on al-Battānī are probably from Azarchel (Ibn al-Zarquali, member of the astronomers of Toledo) and other canons brought the name of Gerard of Cremona (d. 1187) as the translator. The Toledan Tables were in competition with the al-Khwārizmī's work translated by Adelard of Bath in the 1120s - 1130s.

C.4.1 About Right Ascension and Equation of Time Table

The table [12] is cited in all the canons and it is called *tabula circula directi*, for the ascension values we find *ascensiones* or *elevationes*, for the equation of time we find *aequatio dierum cum noctibus suis*. The tables normally starts with Capricorn. The table is the same as al-Battānī in Nallino II p. 61-64, and it shows the values for single degrees. The value used for the obliquity is $\varepsilon = 23°35'$, there are systematic errors present in the al-Battānī's tradition. To recompute the table Pedersen used this value for the ecliptic and the Nallino's value for the solar eccentricity $e = 2;4,45$, for the solar apogee $\lambda_A = 82;15°$, and $E_0 = 4;6,30°$ for the epoch constant chosen to make the tabular values positive (van Dalen).

D John of Murs

D.1 Primary and secondary sources

- *John of Murs Revisited: The Kalendarium Solis Et Lune for 1321*, José Chabás and Bernard R. Goldstein [22];
- *John of Murs's Tables of 1321*, José Chabás and Bernard R. Goldstein [23];
- *Dictionary of Scientific Biography*, C.C. Gillispie and American Council of Learned Societies [14].

D.2 Chabás and Goldstein: John of Murs revisited: the *Kalendarium Solis et Lune* for 1321

Another work by John of Murs was found in Brussels, Bibliothèque Royale and it consists in a calendar with the daily positions of the Sun for 1321, with canons and tables. The purpose of the authors in the paper [22] is to reproduce some excerpts of the *Kalendarium Solis et Lune*: the calendar, the tables for the Sun and the tables for the Moon.

- CALENDAR TABLE: in the first column we have the meridian altitude of the Sun for each month, the second column is for the day of the month, the third for the weekday, the fourth for the names of the saints and festivities, the fifth for the true longitude of the Sun (in zodiacal signs, degrees and minutes). The longitudes were calculated in Toledo using the Alfonsine Tables for 1321. Under the table we find small tables for each month for the length of daylight and nighttime that are calculated for the latitude of Paris.
- TABLES FOR THE SUN: we have four tables. In the first table (*tabula solis*) we find a cycle of four years, the argument is the month and the day within the month, the year begins in January, the entries give the increment to be added or subtracted from the true position of the Sun, in the last column we find the equation of time in minutes of time and this table is calculated for Paris. In the second table (*tabula mensium latinorum*) the year begins in March, in the first column we have the months with its number of days, than we find the excess of daylight in relation to the nighttime, the excess of daylight in relation to that on the following day, the excess of nighttime in relation to that on the following day, in the last column we find the day of the month when the Sun enters a zodiacal sign (for Paris). In the third table (*medius motus Lunae in hora*), despite the title, we find the first columns for the year and the second column for the minutes to be added to the solar longitude to obtain that for a given year. In the fourth table (*tabula veri motus solis*) is a multiplication table, in the headings we find the true solar velocity, the entries are the distance travelled by the Sun.

- TABLES FOR THE MOON: in general we have a main table with some sub-tables. In the main table we have the time and mean longitude for the Sun and the Moon for mean conjunctions, the year begins in January and we find signs of 30 degrees, it was computed for Toledo; in its first sub-table we find the daily and the hourly mean motion of the Moon; in its second sub-table we find the number of lunations as argument and the mean anomaly as entry. Then we have another table for the lunar mean motion in longitude, another for the equation of anomaly.

  Then we find some multiplication tables to determine the time of true conjunction from mean conjunction and a shadow table. A true lunar position for each day between successive syzygies are given in a double argument (lunar anomaly, number of days after mean conjunction or opposition); then we find a table for the daily lunar velocity with two sub-tables that are multiplication tables. Then we find a lunar latitude table as a function of the argument of latitude, tables for lunar and solar eclipses with a table for interpolation with coefficients to correct eclipses tables.

D.3 Chabás and Goldstein: John of Murs's tables of 1321

In this paper [23] the authors studied in detail, for the first time, John of Murs's tables of 1321 and Patefit to demonstrate that they are based on the model of Parisian Alfonsine Tables. This tables survived in two manuscripts, one in Lisbon and the other in Oxford, and both present physical signs of 60 degrees and they are accompanied by canons.

D.3.1 John of Murs's tables of 1321

John of Murs's tables of 1321 are tables on the planets, the Sun and the Moon. The tables for mean conjunctions of planets with the Sun present the mean motions of the planets and their equations are given in double argument in order to simplify computation and this is new in Europe.

Brief description of the tables:

- TABLES OF MEAN CONJUNCTIONS OF THE SUN AND THE PLANETS: in these tables (each one defined as a *tabula principalis*) we have the dates of the conjunctions of the Sun and planets, the mean motion, the mean argument of centre and the true longitude of the planet. In the title we find that the initial time was verified in Toledo by King Alfonso X and that Toledo is 48 minutes of a hour in the West of Paris (so John of Murs took the values in Toledo and then he converted them to the meridian of Paris). From this table we understand that the epoch is 1320 and that the entries were calculated using the solar model with a maximum value of the solar equation of $2;10°$. We need another table (called *contratabula*) to find the true longitude of the planet when is not in conjunction with the Sun: it is a double argument table that gives the "age of the planet" and the correction to be added or subtracted to the mean motion of the planet to obtain its true longitude. In a text referred to these tables we can find the periods of the anomaly of the planets that are consistent with the ones in Alfonsine astronomy (as in the *editio princeps* of 1483).

- TABLES OF MEAN CONJUNCTIONS OF THE SUN AND THE MOON: they have the same format of the tables described above showing the mean position and the mean lunar anomaly and they begin in 1322. As in the previous case we have a *tabula principalis* and a *contratabula*. Also here we can find parameters later used in Parisian Alfonsine Tables. If we compare these tables for the mean conjunctions of the Moon and the Sun with the table of John of Vimond that are calculated in the epoch of 1320 for Paris we find some identical values: John of Murs probably used the principles of John of Vimond or the other option is that both have taken another unknown table.

- TABLES FOR THE PLANETS: three tables for each planet, one for the argument of anomaly and the argument of centre, one for the equation of center and the stations, one for the latitude.

  For the first type of table in the first column we find the argument in days within a period of anomaly; in the second column we find mean motion of the argument of anomaly (in degrees); in the third column we find the mean motion of the argument of centre (in degrees and minutes).

  For the second type of table the first two columns are for the mean argument of centre (in degrees) and the third is for the equation of the centre of planets (in degrees and minutes). In general all the tables of the of planets except for Jupiter and Venus follow the tradition of the Toledan Tables.

  The third type of table (latitudes) is presented with a double argument, superior and inferior planets are divided. For the superior planets the entries are in degrees and minutes, the vertical argument is the argument of centre of the planet then we have the half of the difference (in minutes of arc) between the entries of two successive columns for a fixed argument of center.

  For the inferior planets we have a double argument table for latitude: one for the inclination (*declinatio*) and one for the slant (*reflexio*). The entries are in degrees and minutes. The first column is for the argument.

- TABLE OF TRUE POSITION OF THE SUN: the radices where computed by King Alfonso X for Toledo, the entries are given in physical signs, degrees, minutes and seconds, the table is valid for 1321. We have a *tabula principalis* and a *contratabula* (in which there is no double argument). The argument is the day of the year in intervals of 6 days (beginning in January), in the second column we find the correction to be subtracted from the true position of the Sun for 1321 to find its true position for years after 1321; in the third column we find the hourly velocity of the Sun in minutes and seconds per hours, in the fourth columns we find the equation of time. The external values of the equation of time are unprecedented in astronomy and are: minimum at $0;0°$ between the 24 jan-12 Feb, maximum at $5;14,30°$ the 6th of May, the other minimum is $3;0°$ between 18-24 July and the other maximum is $8;9,30°$ the 24th October (the entries in the table are in hours). All the parameters for the Sun are found in Parisian Alfonsine Tables.

- 5 TABLES OF CONJUNCTIONS OF THE SUN AND THE PLANETS: they are different from the previous ones, the entries correspond to 1452 and were computed for Paris. We have only the *tabula principalis* and surely these tables do not belong to John of Murs's Tables of 1321.

D.3.2 Patefit

John of Murs composed another work in 1321 called *Patefit* that survived in few manuscripts: London (British Library), Erfurt (Biblioteca Amploniana), Vatican (Biblioteca Apostolica). The tables are on syzygies and two of them have a double argument. The authors followed the manuscript of London which is presented with zodiacal signs. The Patefit years refer to the current year. The first table is for mean conjunctions and opposition from 1321 to 1396: we have time in days hours and minutes; mean motion of the Sun in zodiacal signs, degrees and minutes; mean lunar anomaly in zodiacal signs, degrees and minutes; mean argument of of lunar latitude in zodiacal signs, degrees and minutes and was computed for the meridian of Toledo.

The second table is for true conjunctions and opposition from 1321 to 1396 with time given in days, hours and minutes and was computed for the meridian of Toledo.

The third table is for true conjunctions in which we find true longitude of the Moon at mean conjunction, lunar time correction, hourly lunar velocity, true longitude of the Sun at mean conjunction and solar time correction. The time correction of the Moon and for the Sun are the two terms in which the time from mean to true syzygy is divided.

The fourth table is for the true lunar position for each day between successive syzygies and is a double argument table.

The fifth table is a double argument table: *Tabula invencionis veri loci lune incipiendo a coniunctione eius a sole.*

E Peter of Saint Omer & John of Lignères

E.1 Primary and secondary sources

- *A survey of European astronomical tables in the late Middle Ages*, José Chabás and Bernard R. Goldstein [6];
- *Dictionary of Scientific Biography*, C.C. Gillispie and American Council of Learned Societies [14];
- *Les canons de Jean de Lignres sur les tables astronomiques de 1321* [15], Marie-Madeleine Saby-Rousset.

E.2 Peter of Saint Omer

Peter of Saint Omer, known also as Petrus de Sancto Ademaro [13], revised the *Tractatus novi quadrantis* in 1293 in Paris, a treatise on the "new quadrant" invented by Profatius Judaeus: it is a translation from Hebrew that explain this new device that is an astrolabe transformed into a quadrant. He wrote also *Tractatus eclipsis solis et lune* on how computing eclipses [14] and *Tractatus de semissis* in 1293 on the use of an equatorium for calculating planetary longitudes. In the *Toledan Tables* (Pedersen 2002, [12]) we find an equation of time table (*Tabula aequationis dierum cum noctibus suis*) that should belong to Peter of Saint Omer and could have been a source for John of Lignères.

E.3 John of Lignères

John of Lignères was born in 1290 [15] and we do not know his date of death; he lived in Paris from 1320 to 1335 [14] and he published astronomical tables and canons (often confused among the other astronomers of the period like John of Murs, John of Saxony), theory of planets, treatises on instruments and mathematical works which contributed to the diffusion of Alfonsine astronomy in Latin West.

We have three canons [14]:

- *Canones super tabulas magnas* of 1320 with the daily and annual variation of the mean motions and mean arguments of the planets. The tables are with double argument (mean argument and mean center) and they give an equation (to add to the mean motion to obtain the true position) that is the sum of the equation of center and the corrected equation of the argument. They also permit to calculate mean and true conjunctions and oppositions of the Sun and the Moon.
- the second set of tables and canons is dated 1322 and is divided into three parts:
  - *primum mobile* is the trigonometric part of the tables and it deals with the daily movement of the Sun and with astronomical instruments. The corresponding tables are sines and declination tables (maximum declination $23;33,30°$), right and oblique ascension tables for the latitude of Cremona and Paris;

- *canons of the movement of planets* of 1322 on conversion of eras, determination of true positions and latitudes of planets, mean and true conjunctions and oppositions of the Sun and the Moon, eclipses. The corresponding tables are chronological, mean motions and mean arguments of the planets and of equations.
- *Diversitatem aspectus lune in longitudine et latitudine* of 1322 on eclipses;

• the third set of canons (*Quia ad inveniendum loca planetarum*) to treat the Alfonsine tables on mean motions and mean arguments of planets of the sexagesimal multiples of the motion during the day. This sexagesimal form is not the original state of Alfonsine tables so probably the transformation were made by John of Murs and John of Lignères in Paris in the 1320's.

He wrote a treatise on the theory of the planets, *Spera concentrica vel circulus concentricus dicitur* especially on the motion of the eighth sphere and other treatises on instruments (the saphea, the equatorium, the directorium). On the equatorium in particular he wrote *Quia nobilissima scientia astronomie non potest* and *Primo linea recta que est in medio regule*.

We find equation of time tables in *Tractatus diversi de scientiis* of 1401-1500 (Paris, Bibliothèque nationale de France, MS 7282, 110v) where the solar longitude is the argument in intervals of one degree, the table starts with Capricorn, the minimum value of the equation of time in degrees and minutes is $0;0°$ in Aquarius $18° - 25°$ and the maximum value is $7;57°$ in Scorpio $8° - 9°$.

In *Canones Joannis de Ligneriis, aliàs, de Lineriis* of 1401-1500 (Paris, Bibliothèque nationale de France, MS 7295A, 163v) the solar longitude is the argument in intervals of one degree, the table starts with Capricorn, the minimum value of the equation of time in degrees and minutes is $0;0°$ in Aquarius $18° - 25°$ and the maximum value is $7;57°$ in Scorpio $8° - 9°$.

In *Johannes de Lineriis: Tabulae aequationum planetarum* (Vatican, Biblioteca Apostolica, MS Pal. lat. 1412, 101v) the solar longitude is the argument in intervals of one degree, the table starts with Capricorn, the minimum value of the equation of time in degrees and minutes is $0;0°$ in Aquarius $18° - 25°$ and the maximum value is $7;57°$ in Scorpio $8° - 9°$.

F Levi ben Gerson

F.1 Primary and secondary sources

- *The astronomical tables of Levi ben Gerson*, B.R. Goldstein and L. Gershom [4] ;
- *Dictionary of Scientific Biography*, C.C. Gillispie and American Council of Learned Societies [14].

F.1.1 The equation of time's Table

In the *Almagest* the equation of time is

$$E = \alpha - \alpha_0 - (\bar{\lambda} - \bar{\lambda_0}) \tag{33}$$

where α is the right ascension, α_0 is the right ascension calculated in the true solar longitude of the Sun at the epoch t_0, $\bar{\lambda}$ is the mean longitude of the Sun, $\bar{\lambda_0}$ is the mean longitude of the Sun at the epoch t_0. Levi has no epoch, he fixed the true longitude of the Sun in Aquarius 20° so when the Sun reaches this longitude the equation of time is equal to zero and

$$E = E_0 - (\alpha - \bar{\lambda}) \tag{34}$$

where $E_0 = \alpha_0 - \bar{\lambda_0}$, in this way the apparent time is equal to the mean time when the equation of time is equal to zero and in general the apparent noon precedes the mean noon and this is close to the modern definition $E = \bar{\lambda} - \alpha$. The entries are in time degrees and the parameters are

- $\lambda_A =$ Cancer 3°;
- $e = 2;14$;
- $\varepsilon = 23;33°$;
- E_0 at Aquarius 20° is $4;2°$.

The table is presented with the solar longitude (there are no indications if it is the mean or the true but probably is the true solar longitude) as argument in intervals of one degree, it starts with Aries, the maximum is $8;13°$ in Scorpio $8° - 9°$ and the minimum is $0;0°$ in Aquarius $20° - 25°$ [4].

G John Holbroke

G.1 Primary and secondary sources

- *Egerton 889* MS London, British Library; John of Ligneres John Walter William Rede Simon Bredon, John Maudithand John Holbroke [16];
- *John Holbroke, the Tables of Cambridge, and the "true length of the year": a forgotten episode in fifteenth-century astronomy*, C. Philipp E. Nothaft [24].

H Giovanni Bianchini

H.1 Primary and secondary sources

- *The astronomical tables of Giovanni Bianchini*, José Chabás and Bernard R. Goldstein [25].

H.2 Tabulae astronomiae

I referred to *The Astronomical Table of Giovanni Bianchini* by Chabás and Goldstein that analyzed the *Tabulae astronomiae* [17].

We find equation of time tables in Naples, Biblioteca Nazionale, MSS VIII.C.34 (*Tabula horarum meridiei et equationis dierum ad meridianum ferarrie et bononie*), Rome, Biblioteca Casanatense, MS 1673 (*Tabula horarum meridiei ad meridianum ferarrie et bononie et equationis dierum*), Vatican, Biblioteca Apostolica, MS Pal. lat. 1375 (*Tabula horarum meridiei et equationis dierum ad meridianum ferarrie et bononie*) and in Edition 1526 (*Tabula equationis dierum*). In the manuscripts the equation of time is presented with the length of daylight.

In the Toledan Tables, in the zīj of al-Battānī and in the *editio princeps* of the Alfonsine Tables the equation of time is together with the right ascension.

I Georg Puerbach

I.1 Primary and secondary sources

- *Tabulae eclypsiu[m] magistri Georgij Peurbachij: tabulaprimi mobilis Joannis de Monteregio*, G. von Peurbach, A. Stiborius, G. Collimitius, L. Alantsee, L. Alantsee, and J. Winterburger [18];
- *Dictionary of Scientific Biography*, C.C. Gillispie and American Council of Learned Societies [14].

J Abraham Zacut

J.1 Primary and secondary sources

- *Astronomy in the Iberian Peninsula: Abraham Zacut and the transition from manuscript to print* by Chabás and Goldstein [19];

- *Dictionary of Scientific Biography*, C.C. Gillispie and American Council of Learned Societies [14];

J.2 The equation of time table in the *Almanach perpetuum*

I referred to *Astronomy in the Iberian Peninsula: Abraham Zacut and the transition from manuscript to print* by Chabás and Goldstein [19] that analyzed, in a monograph, Zacut's life and his main two works: the *ha-Hibbur ha-gadol* and the *Almanach perpetuum*.

Of the *Almanach perpetuum* were printed canons in Castilian and in Latin. The authors of the monograph [19] consulted the manuscript in Lisbon, Biblioteca National for the Castilian version and the manuscript in Madrid, Biblioteca Nacional for the Latin version.

In the table for the solar declination in the *Almanach perpetuum* we find that the maximum entry, corresponding to the value of the obliquity of the ecliptic, is $23;33°$.

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