Fluid entropy as time evolution operator in canonical quantum gravity

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ICRA - Internationa Center for Relativistic Astrophysics

III Stueckelberg Workshop
Pescara, 8-18 July 2008

G. Montani, S. Zonetti: *"Definition of a time variable with Entropy of a perfect fluid in Canonical Quantum Gravity"*, submitted to Phys. Rev. D (2008)

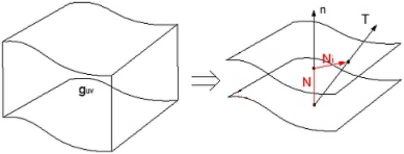
Outline

- The problem of time
- The Brown-Kuchař Mechanism
- The Schutz perfect fluid
- Entropy as a time variable
- Conclusions

Space-Time Foliation

$$S_{EH} = \frac{1}{k}\int d^4x\sqrt{-g}R$$

- Assumption on topology: $\mathcal{M} = \mathbb{R}x\sigma$
- Spacetime foliation with deformation vector: $T_\mu = Nn_\mu + N_\mu$



so the 3+1 Einstein-Hilbert action reads:

$$\mathcal{S}_{EH} = \frac{1}{k}\int d^3x\ dt(P_{ab}\dot{q}^{ab} - H^G N - H^G_a N^a)$$

Constraints and time evolution

Hamiltonian function: $\mathcal{H} = \int d^3x \left(H^G N + H_a^G N^a \right)$

Varying N_a and N:

- 3-diff constraint: $H_a^G = -2P_{ab}{}^{;b} = 0$
- Hamiltonian constraint: $H^G = \frac{1}{\sqrt{q}} \left(P_{ab}P^{ab} - \frac{P^2}{2} \right) - \sqrt{q}R = 0$

Wheeler-DeWitt Equation:

$$-i\hbar \frac{\partial}{\partial t}\psi = \hat{\mathcal{H}}\psi = 0$$

No Time Evolution

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Localization: Reference Fluids

Fluid Particles $\Longrightarrow$ Space-Time points

Coordinate conditions (constraints on phase space) $\Downarrow$ Additional terms in the action $\mathbb{L} = \mathbb{L}_{EH} + \lambda_i \mathcal{V}_i$	$\Longrightarrow$	Matter terms $\Uparrow$ Additional terms in the Einstein Equations $G_{\mu\nu} = \Lambda_{\mu\nu}$

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Explicit proof

Gaussian conditions $\Longleftrightarrow$ Heat conducting dust

Gaussian conditions and broken diff-invariance

Gaussian conditions:

$$g^{00}(X) = 1$$
$$g^{0k}(X) = 0$$

Imposing such conditions on the action:

$$\mathcal{S} = \mathcal{S}_{EH} + \int d^3X dT \sqrt{q}(\bar{\epsilon}(N - N^{-1}) + N\eta_a N^a)$$

where (T, X) are gaussian coordinates, the vacuum constraints are broken:

$$\bar{\epsilon} = \frac{H^G}{2\sqrt{q}} \qquad \eta_a = \frac{H_a^G}{\sqrt{q}}$$

Heat conducting dust

Diff-invariance can be recovered with label coordinates $X^\mu = X^\mu(x_\alpha)$, with the condition:

$$\mathcal{S}(g, \bar{\epsilon}, \eta_a, X^\mu = \delta^\mu_\alpha x^\alpha) = \mathcal{S}(g, \bar{\epsilon}, \eta_a).$$

Varying respect to the metric tensor gives the Einstein equations:

$$G^{\alpha\beta} = \frac{1}{2} T^{\alpha\beta} = \frac{1}{\sqrt{-g}} \frac{\delta \mathcal{S}_F}{\delta g_{\alpha\beta}}.$$

Which give:

$$T^{\alpha\beta} = \epsilon U^\alpha U^\beta + \eta^{(\alpha} U^{\beta)},$$

where $U^\alpha := -g^{\alpha\beta} T_{,\beta}$ is the 4-velocity in the Gaussian frame.

Energy-momentum tensor for a heat conducting dust

The Brown-Kuchař mechanism

Additional matter term in the action:

$$\mathcal{S}_F = \int d^4x\ \sqrt{-g}L_F(-\partial_\mu\phi\partial^\mu\phi) = \int d^4x\sqrt{-g}L_F(\Upsilon)$$

with the ADM formalism

$$\Downarrow$$

Time derivatives $\dot{\phi} = \mathscr{L}_T\phi = \partial_\mu\phi T^\mu$

Separation between spatial and normal d.o.f. $g_{\mu\nu} = -n_\mu n_\nu + q_{\mu\nu}$

$$\Upsilon = (-\partial_\mu\phi n^\mu)^2 - \partial_a\phi\partial^a\phi = (\phi_n)^2 - V$$

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Conjugate momentum: $\pi = \frac{\delta\mathcal{S}_F}{\delta\dot{\phi}} = 2\sqrt{q}\phi_n\frac{\delta L_F}{\delta\Upsilon}$

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Legendre transform

Solving:

$$\pi^2 = \left[\frac{\delta L_F}{\delta \Upsilon}(\Upsilon)\right]^2 4(V - \Upsilon)q$$
$$\Upsilon = (\phi_n)^2 - V$$

One gets:

$$\Upsilon = \tilde{\Upsilon}(\pi, V)$$
$$\phi_n = \tilde{F}(\pi, V) = \frac{\pi}{2\sqrt{q}}\left[\frac{\delta L_F}{\delta \Upsilon}(\tilde{\Upsilon})\right]^{-1}$$

So that the Hamiltonian contains only π and V:

$$\mathbf{H} = \int d^3x \left[\pi \partial_a \phi N^a - \sqrt{q} N \left(L_F - \frac{\pi^2}{2q}\left(\frac{\delta L_F}{\delta \Upsilon}\right)^{-1}\right)\Big|_{\Upsilon = \tilde{\Upsilon}} + N H^G + N^a H^G_a\right]$$

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Contraints

General Relativity + Fluid:

$H_a = H_a^G + \pi\partial_a\phi = 0$

$H = \sqrt{q}\left(L_F - \frac{\pi^2}{2q}\left(\frac{\delta L_F}{\delta\Upsilon}\right)^{-1}\right)|_{\Upsilon=\tilde{\Upsilon}} + H^G = 0$

Squarring the super-momentum:

$$V = \frac{H_a^G H_b^G q^{ab}}{\pi^2} = \frac{d}{\pi^2}$$

One can define a new constraint equivalent to the super-Hamiltonian:

$$\pi - h(H^G, d, q) = 0$$

To be used to define the **Physical Hamiltonian**

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Physical Hamiltonian

$\pi - h(H^G, d, q) = 0$ Schrödinger-like operator

Physical Hamiltonian:

$$\mathbf{H}_{phys} = \int d^3x h(x)$$

Necessary conditions:

- Invariance under the action of the super-Hamiltonian contraint
- Invariance under the action of the 3-diff constraint
- Indipendence from π and ϕ

Allows the definition of a time evolution for observables:

$$-\frac{d\mathcal{O}(t)}{dt} = \{\mathbf{H}_{phys}, \mathcal{O}(t)\}$$

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The perfect fluid model

- velocity potential representation of hydrodynamics
- come directly from termodynamical principles
- 6 scalar fields: ϕ, α, β, θ, S, μ
- S entropy per baryon
- μ specific inertial mass
- No trivial interpretation for the other fields
- 4-velocity:

$$U_\nu = \mu^{-1}(\phi_{,\nu} + \alpha\beta_{,\nu} + \theta S_{,\nu}) = v_\nu \mu^{-1}$$

Equation of state:

$$p = \rho_0(\mu - TS)$$

Reproduces all hydrodynamics

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Canonical formulation 1/2

Schutz fluid action:

$$\mathcal{S}_F = \int d^4x \sqrt{-g}\, p = \int d^4x \sqrt{-g}\, \rho_0 \left(\sqrt{-v^\mu v_\mu} - TS\right),$$

Just one independent conjugate momentum:

$$\begin{aligned} p_\phi &= \sqrt{q}\rho_0\mu^{-1} v_\mu n^\mu = \pi \\ p_\alpha &= 0 \\ p_\beta &= \sqrt{q}\rho_0\mu^{-1}\alpha v_n = \alpha\pi \\ p_\theta &= 0 \\ p_S &= \sqrt{q}\rho_0\mu^{-1}\theta v_n = \theta\pi \end{aligned}$$

$$\Downarrow$$

Constrained theory

No secondary constraints

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Canonical formulation 2/2

Hamiltonian density:

$$\mathbb{H}_F = N\left[\pm\frac{\pi^2\sqrt{V}}{\sqrt{\pi^2 - q\rho_0^2}} \pm \pi\phi_\mu n^\mu \pm v^a\phi_a\sqrt{\frac{\pi^2 - q\rho_0^2}{V}} + \sqrt{q}\rho_0 ST\right] + N_a v^a \pi$$

Cannot complete the Legendre trasfrom without further conditions.

Equations of motion:

$$\dot{\pi} \Longrightarrow (\rho_0 U_\mu)^{;\mu} = 0$$

$$\dot{\alpha} \Longrightarrow U_\mu \alpha^\mu = 0$$

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so that $U_\mu = \mu^{-1}\phi_\mu$ when equations of motion hold

on-shell the Hamiltonian density is well defined.

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Coupling with General Relativity

Constraints:

- Primary constraints ⇒ Union of the primary constraints of uncoupled models
- Secondary constraints:

$$H = \pm\sqrt{\frac{V}{\pi^2 - q\rho_0^2}}\left(\xi\pi^2 + \chi q\rho_0^2\right) + \sqrt{q}\rho_0 ST + H^G = 0$$

$$H_a = \pi\phi_a + H_a^G = 0$$

where $\xi = (3,1)$ e $\chi = \pm 1$

One has to get rid of multiple cases

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Comoving frame and multiple solutions

Solving the super-Hamiltonian respect to π:

$$\pi = h(\xi, \chi \dots)$$

In the comoving frame ($U^\mu = n^\mu$):

$$\bar{\pi} = -\sqrt{q}\rho_0$$
$$\bar{H} = \sqrt{q}\rho_0 ST + H^G = 0$$
$$\bar{H}_a = H_a^G = 0$$

So it must be: $\bar{\pi} = \bar{h}(\xi, \chi \dots)$

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Physical Hamiltonian

The equivalent super-Hamiltonian constraints reads:

$$\pi = -\sqrt{q}\rho_0 \sqrt{\frac{\left(2d + \bar{H}^2 \pm \bar{H}\sqrt{8d + \bar{H}^2}\right)}{2\left(d - \bar{H}^2\right)}} = h$$

where $\bar{H} = \sqrt{q}\rho_0 ST + H^G$

Necessary conditions are satisfied

$\Downarrow$

$$\{\mathbf{H}_{phys}, \mathcal{O}_f(\tau)\} = -\frac{\delta}{\delta\phi}\mathcal{O}_f(\tau),$$

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Evolutionary behaviour of the system

The Entropy field is linked with evolution

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Entropy as a time variable

In the comoving frame:

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From the definition of $p_S = \theta\pi$

$$Sp_S = \frac{\theta H^G}{T} = \tilde{h}$$

So defining $lnS = \tau$

$$-\frac{d\mathcal{O}(\tau)}{dlnS} = \{\mathbf{H}_{phys}, \mathcal{O}(\tau)\} = -\frac{d\mathcal{O}(\tau)}{d\tau}$$

Entropy is the time variable

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Entropy is the time variable

Conclusions

- Schutz model allows the application of the Brown-Kuchař mechanism, so that a physical Hamiltonian appears
- Entropy per baryon is linked with the time evolution of the system
- In the comoving frame Entropy, through its log, IS the time variable
- In the comoving frame the Physical Hamiltonian is rather simple