The Space-time of the Pioneer Anomaly

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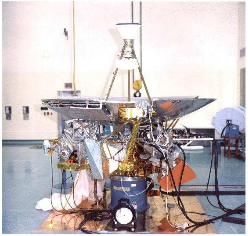
Outline

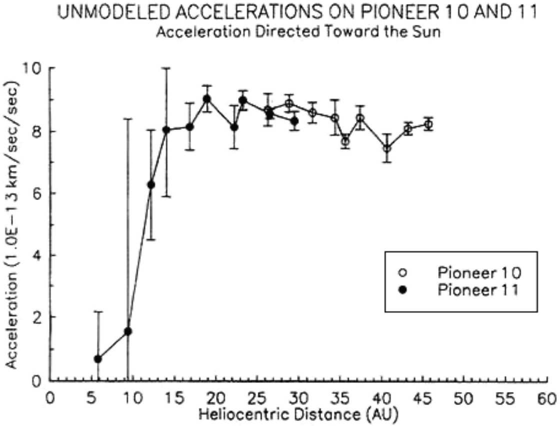
Overview of Pioneer spacecrafts and mission



Main mission: Jupiter (10+11) and Saturn (11) exploration

Pioneer 10 launched on 2 March 1972
Pioneer 11 launched on 4 December 1973
They follow hyperbolic escape orbits near the plane of the ecliptic to opposite sides of the solar system, and can be used for precise celestial mechanics experiments.
Surprisingly there are permanent discrepancy from the theoretical and experimental Doppler tracking results (computational errors are excluded by at least 7 independent analyses).

Pioneer Anomaly



J. Anderson et al, Phys. Rev. D 65 (2002) 082004, gr-qc/0104064

Pioneer Anomaly

- Pioneer 10/11, Galileo, Ulysses in outer Solar system.
- Surprisingly «unmodelled» acceleration was found.
- The acceleration is the same for all spaceships and independent on radius.
- Its value is defined most accurately for Pioneer 10 $a_P = (8{,}74 \pm 1{,}33) \cdot 10^{-10}$ m/s^2.
- The same value of accelerations suggests *metric origin* of them.
- At the same time there are no signatures of such an acceleration in the orbits of outer planets and asteroids (*L. Iorio, gr-qc/0610050*).

Explanations proposed so far: a real deceleration

- Local manifestation of global Universe expansion. But the effect has second order in H so it is negligible.
- Gravitational forces from unidentified sources such as the Kuiper belt or dark matter. But the acceleration is absent in the orbits of the outer planets!
- Drag from the interplanetary medium, including dust, solar wind and cosmic rays. But the densities are too small.
- Gas leaks, radiation pressure of sunlight, radio transmissions, or thermal radiation. At the APS April 2008 meeting Slava Turyshev suggests that differential heating may account for 28 to 36% of the observed acceleration, but not all.
- Electromagnetic forces due to an electric charge on the spacecraft. But charges must decay quckly.

Explanations proposed so far: New physics

- Clock acceleration between coordinate or Ephemeris time and International Atomic Time. Excluded by ephemerides.
- A modification of the gravity theory.
  - The MOND (MOdified Newtonian Dynamics).
  - Aethereal Gravitation Theory (AGT).
  - Scalar-vector-tensor theory of gravitation.
  - Down-scaling of photon frequency as a consequence of integrable Weyl geometry.
  - Extending the Hubble law to the realm of unbounded massive particles.
  - A coupling between the spin of an object and its effective value of G.
  - Many many more theories...

Goal

We determine the space-time, radial motion in which shows Pioneer anomaly without affecting circular orbits.

This possiblity follows from the existance of *two* metric functions in the spherically-symmetric static space-time.

The almost same idea was developed in
M.-T. Jaekel, S. Reynaud, CQG 23 (2006) 7561,
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Spherically-symmetric static space-time and the radial motion in it

Interval in the general coordinates

$$ds^2 = e^{\tau(r)}dt^2 - e^{\rho(r)}dr^2 - e^{\sigma(r)}r^2(d\theta^2 + \cos^2\theta d\varphi^2). \qquad (1)$$

Radial motion can be obtained from the energy $g_{tt}\frac{dt}{ds} = e^{\tau(r)}u^0 = k = const$ and 4-velocity length preservation $e^{\tau(r)}{u^0}^2 - e^{\rho(r)}{u^1}^2 = \varepsilon,\ \varepsilon = 0$ or 1:

$$\frac{dt}{dr} = \frac{e^{\frac{\rho(r)-\tau(r)}{2}}}{\sqrt{1 - \varepsilon e^{\tau(r)}/k^2}}. \qquad (2)$$

Doppler tracking 1

The universal formula for Doppler shift in the geometric optics approximation

$$\frac{\nu_r}{\nu_e} = \frac{s_e}{s_r} = \frac{\vec{u}_r \cdot \vec{k}_r}{\vec{u}_e \cdot \vec{k}_e}, \qquad (3)$$

ν_r and ν_e — received and emitted frequencies, measured by the standard observers with atomic clocks,
s_r and s_e — proper time of one circle of oscillation,
u_r and u_e — 4-velocity of receiver and emitter,
k_r and k_e — tangential null vector (wave vector), paralelly transported along the path of signal.
The signal is emitted from the «fixed» Earth from $r = r_0$, and received on the spaceship, then reemitted to Earth.

Doppler tracking 2

The finally received on Earth frequency ν_r is connected to the initially emitted ν_e as

$$\nu_r = \nu_e \frac{1 - \sqrt{1 - e^{\tau(r)}/k^2}}{1 + \sqrt{1 - e^{\tau(r)}/k^2}} = \nu_e \, e^{-\tau(r)} \left(k - \sqrt{k - e^{\tau(r)}}\right)^2, \quad (4)$$

$$\dot{\nu}_r = \nu_e \frac{e^{\frac{\tau(r)-\rho(r)}{2}} e^{\tau(r)\prime}}{\left(k + \sqrt{k^2 - e^{\tau(r)}}\right)^2}. \quad (5)$$

It depends on $e^{\tau(r)}$ only!
We can determine $e^{\tau(r(t))}$, but we don't know $r(t)$.
There is no information on space part of metric.

Signal time arrival analysis

The time of signal travel is

$$t_p = \int_{r_0}^{r} e^{\frac{\rho(r)-\tau(r)}{2}}\, dr, \tag{6}$$

its time derivative

$$\dot{t}_p = \sqrt{1 - e^{\tau(r)}/k^2}. \tag{7}$$

So dependence of $r = r(t)$ can be choosen *freely* and then from the measured $e^{\tau(t)}$ we can determine $\rho(r)$ for the given Doppler tracking and time arrival results.

But we know that the near-circular motion of outer planets is unperturbed. This gives us a clue for complete metric determination.

Circular motion and radial motion
Choice of coordinates

For the circular motion $\theta = 0,\ \phi = \omega t$ and

$$\omega^2 = \frac{(e^{\tau(r)})'}{(r^2 e^{\sigma(r)})'}. \tag{8}$$

Radial motion

$$\frac{dt}{dr} = \frac{e^{\frac{\rho(r)-\tau(r)}{2}}}{\sqrt{1-\varepsilon e^{\tau(r)}/k^2}}. \tag{9}$$

The maximal simplification suggests null coordinates $\tau(r) \equiv \rho(r)$.

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Null coordinates. Radial motion

$$\frac{dt}{dr} = \frac{1}{\sqrt{1 - \varepsilon e^{\tau(r)}/k^2}} \quad \Rightarrow \quad k^2 = \frac{1}{1 - v^2} \tag{10}$$

$$t_p = r - r_0 \quad \Rightarrow \quad t = \frac{t_r + t_e}{2}, \qquad r = r_0 + \frac{t_r - t_e}{2}, \tag{11}$$

$$z(t) = \frac{\Delta\nu}{\nu} = \frac{2\sqrt{1 - e^{\tau(r(t))}/k^2}}{1 + \sqrt{1 - e^{\tau(r(t))}/k^2}} = \frac{2}{(1 - e^{\tau(r(t))}/k^2)^{-1/2} + 1}, \tag{12}$$

$$e^{\tau(r(t))} = k^2 \left[1 - \left(\frac{z(t)}{2 - z(t)}\right)^2\right], \tag{13}$$

Null coordinates. Circular motion

$$\omega^2 = \frac{(e^{\tau(r)})'}{(r^2 e^{\sigma(r)})'} \quad \Rightarrow \tag{14}$$

$$r^2 e^{\sigma(r)} = r_0^2 e^{\sigma(r_0)} - \int_{r_0}^{r} \frac{4k^2 z(r) z'(r)}{(2 - z(r))^3 \omega^2(r)} dr. \tag{15}$$

Circular motion is the same as in the Schwarzschild field

Radial motion differs slightly from the Schwarzschild field

We can find perturbations in the metric coefficients for the Pioneer anomaly

Null coordinates of Schwarzschild space-time

$$ds^2 = \frac{W\left(e^{\frac{r}{r_g}-1}\right)}{1+W\left(e^{\frac{r}{r_g}-1}\right)}(dt^2 - dr^2) -$$

$$-r_g^2\left(1+W\left(e^{\frac{r}{r_g}-1}\right)\right)^2(d\theta^2 + \cos^2\theta d\varphi^2), \qquad (16)$$

$$e^{\tau(r)} = e^{\rho(r)} = \frac{W\left(e^{\frac{r}{r_g}-1}\right)}{1+W\left(e^{\frac{r}{r_g}-1}\right)} = r_g\, W_r'\left(e^{\frac{r}{r_g}-1}\right), \qquad (17)$$

$$e^{\sigma(r)} = \frac{r_g^2}{r^2}\left(1+W\left(e^{\frac{r}{r_g}-1}\right)\right)^2, \qquad W(x)e^{W(x)} = x. \qquad (18)$$

Space-time determination 1

$$\frac{d}{d\mathrm{ET}}(\nu_r - \nu_m) = -\nu_e \frac{2a_P}{c}, \tag{19}$$

$$\nu_m = \nu_0 \, \frac{1 + 1/W(e^{\frac{r}{r_g}-1})}{1 - v^2} \left(1 - \sqrt{1 - \frac{1 - v^2}{1 + 1/W(e^{\frac{r}{r_g}-1})}}\right)^2, \tag{20}$$

$$\frac{dr}{dt} = v(r) = \sqrt{\frac{v^2 + 1/W(e^{\frac{r}{r_g}-1})}{1 + 1/W(e^{\frac{r}{r_g}-1})}} \tag{21}$$

$$\delta e^{\tau(r)} \simeq -\frac{z(r)\,\delta z(r)}{2} = a_P \, z(r)\Delta t(r), \tag{22}$$

Space-time determination 2

$$t_m(r) = t(r_0) + \int_{r_0}^{r} \frac{dr}{\dot{r}} = t(r_0) + \int_{r_0}^{r} \sqrt{\frac{1 + 1/W(e^{\frac{r}{r_g}-1})}{v^2 + 1/W(e^{\frac{r}{r_g}-1})}}\, dr \simeq$$

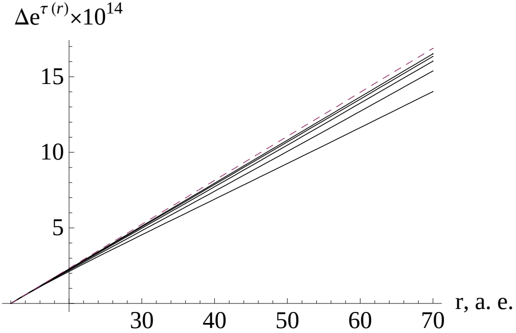
$$\simeq t_0 + \frac{r}{v^2}\sqrt{v^2 + \frac{r_g}{r}} - \frac{r_g}{v^3}\,\mathrm{arcsh}\left(\sqrt{\frac{r}{r_g}}v\right) \quad (23)$$

Metric perturbations 1

$$\delta e^{\tau(r)} = 2a_P\left(r + \frac{r_g}{v^2}\left[1 - \sqrt{1 + \frac{r_g}{r\ v^2}}\,\mathrm{arcsh}\left(\sqrt{\frac{r}{r_g}}v\right)\right] - \right.$$
$$\left. - C\sqrt{v^2 + \frac{r_g}{r}}\right), \quad (24)$$

Non-linear perturbation, but for the precision of measurements the difference can be neglected and linear approximation of $\delta e^{\tau(r)} \simeq 2\eta a_P(r - r_0)$ will be sufficient. Relative deviation from linearity decreases with radial distance.

Metric perturbations 2



$\delta e^{\tau(r)}$, of the Pioneer anomaly for v from 5 km/s to 50 km/s in 5 km/s steps (from down to up) for the metric matching to Schwarzschild's on 12 a. u.

Metric perturbations 3

In the first approximation

$$\delta e^{\sigma(r)} = \frac{4a_P\eta\, r_g^2}{r^2}\int_{r_0}^{r}\left(1+W\left(e^{\frac{r}{r_g}-1}\right)\right)^3 dr = \\ = \frac{4a_P\eta\, r_g^2}{r^2}\int_{r_0}^{r}\left(\frac{r}{r_g}\right)^3 dr = \frac{4a_P\eta(r^4-r_0^4)}{r^2 r_g}. \quad (25)$$

Note the gravitational radius of the source r_g in the answer!
So there is no equivalence principle violation (opposite to statements of *L. Iorio, Found. Phys., V. 37, N. 6, pp. 897-918*)!

Einstein Equations

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = \varkappa T_{ij}, \qquad \varkappa = \frac{8\pi G}{c^4} \tag{26}$$

$$ds^2 = e^{\tau(r)} dt^2 - e^{\rho(r)} dr^2 - e^{\sigma(r)} (d\theta^2 + \sin^2\theta d\phi^2) \tag{27}$$

$$G_{ij} = \frac{e^{-\rho}}{4} \Big(\lambda_t T_i \otimes T_j - \lambda_s S_i \otimes S_j - \lambda g_{ij} \Big), \tag{28}$$

$$S_i = \left\{ 0, e^{\frac{\rho}{2}}, 0, 0 \right\}, \qquad T_i = \left\{ e^{\frac{\tau}{2}}, 0, 0, 0 \right\}, \qquad -S_i S^i = T_i T^i = 1, \tag{29}$$

$$\lambda_t = 4 e^{\rho - \sigma} + \left(\rho' - 2\sigma' - \tau' \right) \left(\sigma' - \tau' \right) + 2 \left(\tau'' - \sigma'' \right), \tag{30}$$

$$\lambda_s = 4 e^{\rho - \sigma} - \tau' \left(\sigma' - \tau' \right) - \rho' \left(\sigma' + \tau' \right) + 2 \left(\tau'' + \sigma'' \right), \tag{31}$$

$$\lambda = \sigma'^2 + \sigma' \tau' + \tau'^2 - \rho' \left(\sigma' + \tau' \right) + 2 \left(\tau'' + \sigma'' \right). \tag{32}$$

To the first power of a_P

$$\lambda_t = -96\,\frac{a_P\eta}{r_g}, \quad \lambda_s = -32\,\frac{a_P\eta}{r_g}\,\frac{r_0^4}{r^4}, \quad \lambda = 16\,\frac{a_P\eta}{r_g}\left(3 - \frac{r_0^4}{r^4}\right). \tag{33}$$

so the algebraic type of EMT at spatial infinity is that of ideal fluid (by $\lambda_s \to 0$) with

$$\text{pressure} \qquad p = \frac{e^{-\tau}}{4\varkappa}\lambda \to 12\,\frac{a_P\eta}{\varkappa r_g} > 0, \tag{34}$$

$$\text{energy density} \qquad \rho = \frac{e^{-\tau}}{4\varkappa}(\lambda_t - \lambda) \to -36\,\frac{a_P\eta}{\varkappa r_g} < 0. \tag{35}$$

It is worth noting that relation between p and ρ is as for ultrarelativistic fluid besides the sign: instead $p = \rho/3$ we have asymptotically $p = -\rho/3$.

Conclusion

- The perturbation of time metric coefficient is nearly linear in r and that of transversal space coefficient is proportional to r^2. Non-linearity in $e^{\tau(r)}$ cannot be found from the current measurements.
- The model proposed must be carefully studied by the «Grand-fit» investigations, but direct measurements from the planned missions for testing General Relativity in space are preferable.
- The energy-momentum tensor of matter, that can form such a metric, violates energy dominance condition, which strongly suggests non-metric origin of the Pioneer Anomaly.

Thank You
for Your kindly attention.