Threshold Energies of Pions from pp Interactions

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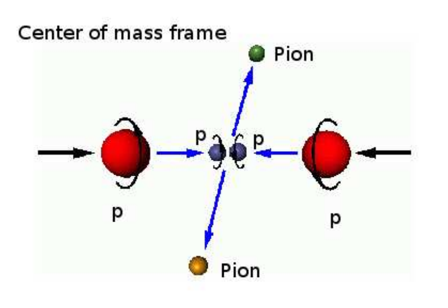
Introduction

In this work we studied the threshold energies of pions produced by pp interactions. We obtained the maximum and minimum energy possible of created pion in function of energy of incident proton. We analysed the threshold energies of the work of Blattnig 2000 [1]. From this study we can obtain the quantity of gamma ray, because $\pi^0 \to 2\gamma$.



Lorentz Invariance

The Lorentz invariance can be defined as: "The square of the 4-momentum of a particles system (isolated) is invariant under Lorentz Transformations, rotation and time, $P^\mu P_\mu = \text{constant}$". The modulus of 4-momentum is equal to the energy of the system in the center of mass (c.m.) frame, $\sqrt{|P_\mu P^\mu|} = \sqrt{s}$. Then

$$(P^\mu_{0initial})^2 = (P^\mu_{0final})^2 = (P^\mu_{initial})^2 = (P^\mu_{final})^2 = s, \qquad (1)$$

where $\sqrt{s}$ is the total energy of the system in the c.m. frame, the notation "$\sqrt{s}$" is standard, the index "0" indicate c.m. frame and without the index "0" indicate that it is in laboratory frame (LF), we use $c = 1$.

The square of the total 4-momentum of the a system is defined as

$$(P^{\mu})^2 = |(E_{tot}, \vec{P}_{tot})|^2 = (E_{tot})^2 - |\vec{P}_{tot}|^2. \quad (2)$$

Thus we can get the c.m. energy of a system of two particles colliding (a and b), where the particle "a" has energy and momentum, respectively, E_a and $\vec{p}_a$, and the particle in rest "b" (target), m_b and $\vec{p}_b = \vec{0}$. In the c.m. frame we have, the particle "a" has E_{0a} and $\vec{p}_{0a}$, and the particle "b", E_{0b} and $\vec{p}_{0b}$ (where $\vec{p}_{0a} + \vec{p}_{0b} = \vec{0}$). Then in the LF

$$\begin{aligned} (P^{\mu}_{tot})^2 = |P^{\mu}_a + P^{\mu}_b|^2 = |(E_a + m_b, \vec{p}_a + \vec{0})|^2, \\ s = 2m_b E_a + m_a^2 + m_b^2, \end{aligned} \quad (3)$$

and in the c.m. frame,

$$(P^{\mu}_{tot})^2 = |P^{\mu}_{0a} + P^{\mu}_{0b}|^2 = |(E_{0a} + E_{0b}, \vec{p}_{0a} + \vec{p}_{0b})|^2, \tag{4}$$

where the total 3-momentum ($\vec{p}_{0tot} = \vec{p}_{0a} + \vec{p}_{0b}$) of the system in the c.m. frame always vanishes, $\vec{p}_{0a} + \vec{p}_{0b} = \vec{0}$. Then

$$s = (E_{0a} + E_{0b})^2. \tag{5}$$

If the two particles are protons, the Eqs. (3) and (5) are rewritten as

$$s = 2m_p(E_p + m_p) = 4E_{0p}^2. \tag{6}$$

Maximum energy of the pion

From the Lorentz Tranformation, we can get the energy of a pion in the laboratory frame in function of its energy in the c.m. frame [2], as

$$E_\pi = \gamma(E_{0\pi} + vp_{0\pi}cos\theta), \tag{7}$$

where γ is the Lorentz factor, "v" the velocity of the pion, $E_{0\pi}$ the pion energy in the c.m. frame and $p_{0\pi}$ the pion momentum in the c.m. frame. If $cos\theta = 1$ the pion energy is maximum (E_π^{max}), and if $cos\theta = -1$ the pion energy is minimum (E_π^{min}). Then the maximum and minimum energy of the pion are,

$$E_\pi{}^{\substack{max \\ min}} = \gamma(E_{0\pi} \pm vp_{0\pi}). \tag{8}$$

In following we will obtain $E_\pi{}^{\substack{max \\ min}}$ in function of m_π, m_p, E_p.

In the c.m. frame it is known $E_{0tot} = 2E_{0p} = \sqrt{s}$ and $\vec{P}_{0tot} = \vec{0}$. It is important use the c.m. energy because we can use the Lorentz invariance. In the LF we have,

$$E_{tot} = \gamma\sqrt{s}, \qquad \vec{P}_{tot} = \gamma\sqrt{s}\vec{v}. \tag{9}$$

We have the condition $(M + m_\pi)^2 \leq s \leq E_{tot}^2$ (where $M = 2m_p$), this is the condition needed to obtain only a pion after the collision. Substituting Eq. (9) in (8),

$$E_\pi^{\substack{max \\ min}} = \frac{1}{\sqrt{s}}(E_{0\pi}E_{tot} \pm p_{0\pi}|\vec{p}_{tot}|), \tag{10}$$

where

$$p_{0\pi} = \sqrt{E_{0\pi}^2 - m_\pi^2}. \tag{11}$$

It is our interest to obtain E_π^{min} and E_π^{max} in function of m_π, m_p and s; which we need to obtain the terms between parentheses in Eq. (10) in function of m_π, m_p and s.
The square of the c.m. energy is given by,

$$s = |(E'^{p}_{0tot}, \vec{p'}^{p}_{0tot}) + (E^{\pi}_{0tot}, \vec{p}^{\pi}_{0tot})|^2 = |(2E'_{0p} + E_{0\pi}, \vec{p'}_{0tot})|^2,$$
$$s = (2E'_{0p} + E_{0\pi})^2 - |\vec{p'}_{0tot}|^2, \quad (12)$$

where we assumed that after of the collision, the two remaining protons have the same energy, thus as we told before, in the c.m. frame, the total momentum vanishes ($\vec{p'}_{0tot} = \vec{0}$). The index ' says that the process (instant) is after the interaction, but we will not put this index in the case of the pions, because had not pions before the collision. Then,

$$\sqrt{s} = 2E'_{0p} + E_{0\pi}. \quad (13)$$

Considering $E'_{0p} = \sqrt{m_p^2 + p'^2_{0p}}$, the Eq. (12) can be wroten as

$$s = 4(M^2/4 + p'^2_{0p}) + E^2_{0\pi} + 4E'_{0p}E_{0\pi}. \tag{14}$$

We have,

$$\begin{aligned} \vec{p'}_{0tot} &= \vec{p}_{0\pi} + 2\vec{p'}_{0p} = \vec{0}, \\ 4|\vec{p'}_{0p}|^2 &= |\vec{p}_{0\pi}|^2 = E^2_{0\pi} - m_\pi^2. \end{aligned} \tag{15}$$

Substituting Eq. (15) in (14), using the Eq. (13), as

$$s = M^2 + 2E^2_{0\pi} - m_\pi^2 + 4E'_{0p}E_{0\pi}, \tag{16}$$

$$E_{0\pi} = \frac{1}{2\sqrt{s}}(s + m_\pi^2 - M^2), \tag{17}$$

Substituting Eq. (17) in (10) we get

$$E_\pi{}^{max}_{min} = \frac{1}{2s}\left[E_{tot}(s+m_\pi^2-M^2) \pm |\vec{p}_{tot}|R\right], \tag{18}$$

where

$$R = 2\sqrt{s}p_{0\pi} = 2\sqrt{s}\sqrt{E_{0\pi}^2-m_\pi^2}, \tag{19}$$

$$R^2 = [s-(M+m_\pi)^2][s-(M-m_\pi)^2]. \tag{20}$$

It is known

$$E_{tot} = E_p + m_p = \frac{s}{2m_p}, \tag{21}$$

$$|\vec{p}_{tot}| = p_{tot} = \sqrt{E_{tot}^2 - s}. \tag{22}$$

Substituting Eqs. (21) and (22) in the Eq. (18),

$$E_\pi{}^{max}_{min} = \frac{1}{2s}\left[\frac{s}{2m_p}(s + m_\pi^2 - M^2) \pm R\sqrt{(s/2m_p)^2 - s}\right]. \quad (23)$$

In following we will get $E_\pi^{max}(E_p)$, which is the maximum energy of the ejected pion in function of the incident proton energy E_p. Substituting Eq. (6) in the Eq. (23), doing $M = 2m_p$ and considering only the maximum energy $(+)$,

$$E_\pi^{max} = \frac{1}{4m_p}\left(2m_pE_p - 2m_p^2 + m_\pi^2 + R\sqrt{1 - \frac{2m_p}{E_p + m_p}}\right). \quad (24)$$

Substituting Eq. (6) in (20),

$$R^2 = (2m_pE_p - 2m_p^2 - m_\pi^2)^2 - 16m_p^2m_\pi^2, \tag{25}$$

where we considered $(4m_p^2 - m_\pi^2)^2 = (4m_p^2 + m_\pi^2)^2 - 16m_p^2m_\pi^2$. Rewriting the Eq. (24)

$$E_\pi^{max} = \frac{1}{4m_p}(2m_pE_p - 2m_p^2 + m_\pi^2 + R_p), \tag{26}$$

where

$$R_p^2 = \frac{(E_p - m_p)[(2m_pE_p - 2m_p^2 - m_\pi^2)^2 - 16m_p^2m_\pi^2]}{E_p + m_p}. \tag{27}$$

Then the Eqs. (26) describe the maximum energy via pions physically possible (from the Eq. 8), in function of the incident proton energy.

Case $E_p \gg m_p$

Multiplying and dividing Eq. (27) per $E_p - m_p$ and considering $m_p E_p + m_\pi^2 \simeq m_p E_p$ and $E_p^2 + m_p^2 \simeq E_p^2$, as

$$R_p^2 = \frac{(E_p - m_p)^2 (2m_p E_p - 2m_p^2)^2}{E_p^2 - m_p^2} = \frac{4m_p^2 (E_p^2 - 2m_p E_p)^2}{E_p^2},$$

$$R_p = 2m_p (E_p - 2m_p). \tag{28}$$

Substituting Eq. (28) in (26) and neglecting m_π^2, we get

$$E_\pi^{max} = E_p - \frac{3}{2} m_p. \tag{29}$$

It is interesting to stress how in the last equation the two protons remain with a total kinetic energy equivalent to $m_p/2$.

Minimum energy of the pion

From the Eqs. (23) and (26) we can get the case of minimum ejected energy via pions,

$$E_\pi^{min} = \begin{cases} \frac{1}{4m_p}(2m_pE_p - 2m_p^2 + m_\pi^2 - R_p) & \text{if} \quad \mathrm{E_p > E_p^*} \\ m_\pi & \text{if} \quad \mathrm{E_p \leq E_p^*} \end{cases}, \quad (30)$$

where E_p^* is the threshold energy in the case that $E_\pi = m_\pi$. In following we will get E_p^*. We use the Lorentz invariance, as

$$|P^\mu_{initial}|^2 = |P^\mu_{final}|^2,$$
$$|(E_p^* + m_p, \vec{p}_p)|^2 = |(2E'_p + m_\pi, 2\vec{p'}_p)|^2, \quad (31)$$

where the calculation is for $E_\pi = m_\pi$, which $E_p^* = E_p(m_\pi)$.

From the Eq. (31) we get,

$$2m_pE_p^* = 2m_p^2 + m_\pi^2 + 4m_\pi E_p'. \tag{32}$$

From energy conservation,

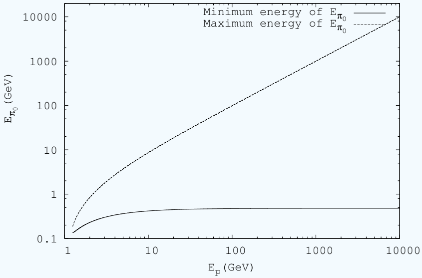
$$\begin{aligned} E_p^* + m_p &= 2E_p' + m_p - m_\pi, \\ E_p' &= \frac{1}{2}(E_p^* + m_p - m_\pi). \end{aligned} \tag{33}$$

Substituting Eq. (33) in (32) and developing, we get

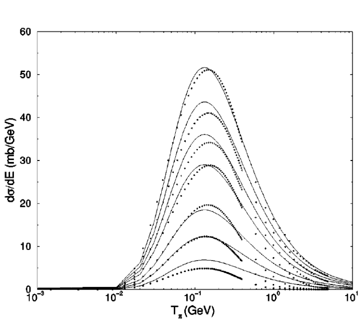
$$E_p^* = \frac{2m_p^2 + 2m_pm_\pi - m_\pi^2}{2(m_p - m_\pi)} \simeq 1.242 \qquad \mathrm{GeV}. \tag{34}$$

Results

In the figure below, we can see the behavior of the Eqs. (26) and (30), where they are the maximum and minimum limits of the neutral pion energy created. We can note that the minimum limit is approximated of $E_{\pi^0}^{min} = 0.478$ GeV for $E_p = 10^4$ GeV, where $E_{\pi^0}^{min}$ increases very softly. The maximum limit tends to Eq. (29).



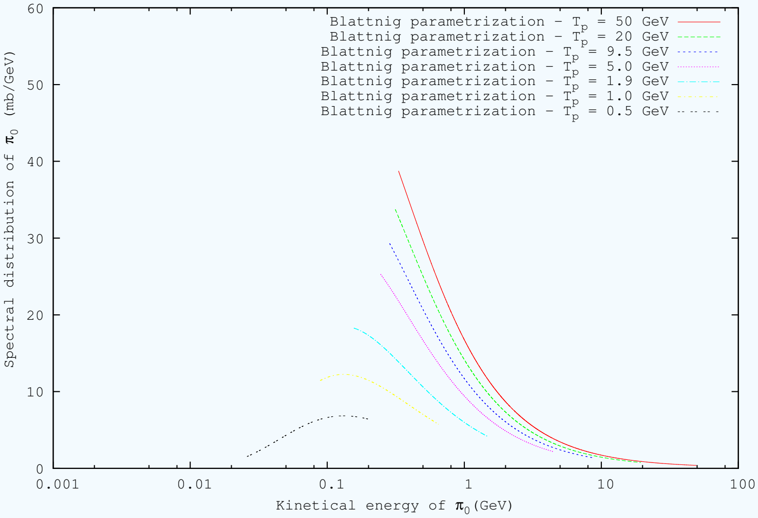
In the figure below, we can see the analytical fit (solid lines) of the numerical integration (points), where Blattnig obtains the spectral distribution of π^0 $[d\sigma/dE_p(\mathrm{mb/GeV})]$ in function of the kinetic energy of π^0 created (T_{π^0}), for energy of incident proton $E_p \leq 50$ GeV. Blattnig obtains the spectral distribution and total cross section of π^0 for seven different kinetic energies of incident protons, $T_p = 0.5, 1.0, 1.9, 5.0, 9.5, 20, 50$ GeV.



The analytical function that Blattnig got is

$$\frac{d\sigma}{dE_p} = \exp\left(K_1 + \frac{K_2}{T_p^{0.4}} + \frac{K_3}{T_\pi^{0.2}} + \frac{K_4}{T_\pi^{0.4}}\right), \tag{35}$$

where $K_1 = -5.8$, $K_2 = -1.82$, $K_3 = 13.5$, and $K_4 = -4.5$. The last expression is the analytical fit of a numerical integration [1]. In the next figure we apply the maximum (26) and minimum (30) limit energies of the π^0 in the spectral distribution of Blattnig et al. 2000 (last figure), and our results contradicts their fit.


Blattnig parametrization – Tp = 50 GeV
Blattnig parametrization – Tp = 20 GeV
Blattnig parametrization – Tp = 9.5 GeV
Blattnig parametrization – Tp = 5.0 GeV
Blattnig parametrization – Tp = 1.9 GeV
Blattnig parametrization – Tp = 1.0 GeV
Blattnig parametrization – Tp = 0.5 GeV
Spectral distribution of π0 (mb/GeV)
Kinetical energy of π0 (GeV)
0
10
20
30
40
50
60
0.001
0.01
0.1
1
10
100

Conclusion

We studied the thresholds energy of created pions via pp interactions and we applied it in the work of Blattnig et al. 2000, we obtained results that contradict their limits of energy, where it shows problems in minimum and maximum energies.

- S.R. Blattnig et al. 2000, PRD, **62**, 094030.
- S.R. Kelner and F.A. Aharonian 2008, arXiv:0803.0688v1.

GRAZIE

THANK YOU

DANKE SCHÖN

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etc