# Prespectives in Cosmology, Gravitation and Multidimensions

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# ABSTRACT

Recent developments from the activity of the CGM group are discussed.
Cosmological implications of fundamental approaches to quantization of gravity are presented in order to fix the main issues as well as perspectives for future investigations.
Particular attention will be devoted to the classical and quantum features of the generic inhomogeneous Universe, to the role of reference frame in quantum gravity, and eventually to phenomenological features related with the Kaluza-Klein framework.

SUMMARY OF THE TALK

- Cosmology

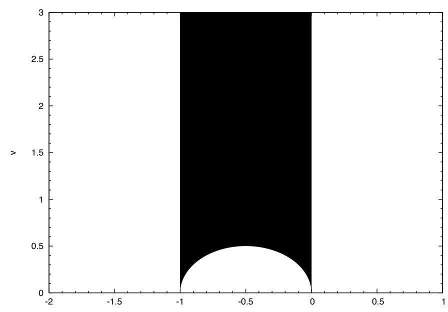
- Quantum Gravity

- Kaluza-Klein

# INHOMOGENEOUS MIXMASTER MODEL

The dynamics of the generic cosmological solution of the Einstein equations can be investigated and reduced, towards the initial singularity, to the sum of $\infty$, decoupled, point Universes, each of them evolving according to the following variational principle in a reduced phase space $\Gamma_Q$

$$S = \int_{\Gamma_Q} d\tau \left( p_u \partial_\tau u + p_v \partial_\tau v + \right. \\ \left. - v\sqrt{p_u^2 + p_v^2} \right)$$




Each of these point Universe (where "point" means of the horizon size) exhibits strong chaotic features that can be characterized by the existence of a stationary measure:

$$d\mu = \frac{1}{\pi}\frac{du dv}{v^2}\frac{d\phi}{2\pi}$$

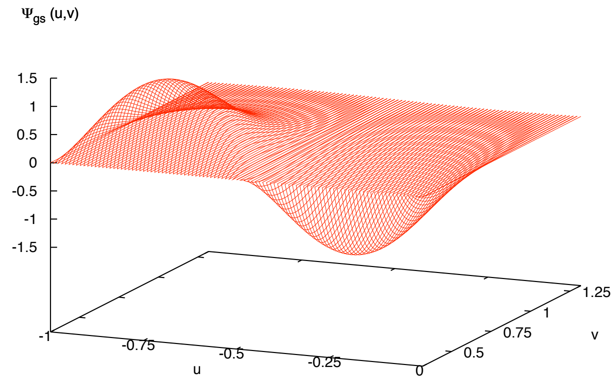
[R.Benini, GM, *Phys. Rev. D*, **70**, (2004) 103527.

# QUANTUM EFFECTS

The quantum regime can be well characterized as

$$\Psi = \sum_n a_n \frac{K_{s-1/2}(2\pi|n|v)}{\sqrt{v}} \times$$
$$\times \sin(2\pi n u)$$

$$(E/\hbar)^2 = t^2 + 1/4 \, .$$




An interesting feature is that $E_0 > 0^a$.

Actually we are working on the implementation of a Weyl description of the quantum dynamics, aiming to get a *quantum phase-space distribution* $\rho^b$

$$H^2 * \rho = E^2 \rho$$

and for evidence of chaos in the wave function of the Universe analyzing the WKB wave function. For the Bianchi II model reads$^b$

$$\Psi_{WKB} = \rho \exp(iS/\hbar); \qquad S \sim k_1 \Omega + k_2 \beta_+ + k_3 \beta_- +$$

$$+ \sqrt{A - 3e^{4(\alpha+\beta_+ + \sqrt{3}\beta_-)}} + \ln[\sqrt{A} - \sqrt{A - 3e^{4(\alpha+\beta_+ + \sqrt{3}\beta_-)}}]$$

[$^a$R.Benini,GM *Class. Quant. Grav.* **4**, (2007) 387.]
[$^b$R.Benini,GM *Weyl Quantum dynamics of the Mixmaster model* in prep.]

# ENERGY MOMENTUM TENSOR OF A VISCOUS SOURCE

Immediate generalization of FRW-scheme $\Rightarrow$ dissipative processes within the fluid dynamics (expected at $T \sim \mathcal{O}(10^{16} GeV)$)

Additional term in the E-M Tensor

$$T_\mu^\nu = \epsilon\,(w+1)\,u_\mu u^\nu - w\,\epsilon\,\delta_\mu^\nu + (\zeta - \tfrac{2}{3}\eta)\,u^\rho_{;\rho}\,(\delta_\mu^\nu - u_\mu u^\nu) + \\ + \eta\,(u_\mu^{;\nu} + u^\nu_{;\mu} - u^\nu u^\rho u_{\mu;\rho} - u_\mu u^\rho u^\nu_{;\rho})$$

$w = p/\epsilon$, where $p$ is the thermostatic pressure and $\epsilon$ the energy density

$\zeta$ *bulk viscosity*: phenomenological issue inherent to the difficulty for a thermodynamical system to follow the equilibrium configuration.

$\eta$ *shear viscosity*: energy dissipation due to displacement of the matter layers with respect to each other.

$$\zeta = \zeta_0\,\rho^{\,s} \qquad\qquad \eta = \eta_0\,\rho^{\,q}$$

- $\epsilon \to \infty$: $\quad 0 \leq s < 1/2 \qquad q \geq 1/2 + s$
- $\epsilon \to 0$: $\quad s \geq 1 \qquad q \geq 1$

[V.A. Belinskii, I.M. Khalatnikov, *Sov. Phys. JETP*, **42** (1976) - **45** (1977)]

# COSMOLOGICAL IMPLEMENTATION OF THE VISCOUS EFFECT

Isotropic (quasi-isotropic) model: only **bulk viscosity**

$\to$ Viscosity induces a negative pressure term: dumping of the cosmological perturbations

• FRLW-Model: asymptotic behavior of the density contrast for $\eta \ll 1$ ($\eta$: conformal time)

$$\frac{\delta\rho}{\rho} \sim \left[C_1 \eta^{3-2/\omega} + C_2 \eta^2 + C_3 \eta^{3-1/\omega} + C_4 \eta^{5-1/\omega}\right]$$

where $\omega = 1 - \chi\,\zeta_0$, $\quad \chi = \sqrt{54\,\pi G}$

Perturbations are damped and for $\zeta_0 > 1/3\chi$ the isotropic and homogeneous Universe *acquires instability* in the direction of the singularity

• Collapsing-Shell: weak field limit (Newtonian analysis), adiabatic behavior of the gas clouds $\frac{4}{3} < \gamma \leq \frac{5}{3}$

$$\delta^{\,ADB} \sim t^{\,\frac{\gamma}{2} - \frac{5}{6} + \frac{\lambda}{6A}} \qquad \lambda = C\,\zeta_0$$

Threshold value: $\lambda > \lambda^*$ no fragmentation process $\delta^{\,ADB} \to 0$

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[N. Carlevaro, GM, *Mod. Phys. Lett. A*, **20** (2007) 1729.]
[N. Carlevaro, GM, *Class. Quant. Grav.*, **22** (2005) 4715.]
[N. Carlevaro, GM, accepted by *Int. J. Mod. Phys. D*]

# DEFORMED MINISUPERSPACE DYNAMICS

Generalized Uncertainty Principle (GUP)

$$\Delta q \Delta p \geq \frac{1}{2}\left(1+\beta(\Delta p)^2+\beta\langle \mathbf{p}\rangle^2\right)$$

String theory leads to this relation

The GUP can be obtained deforming the Heisenberg algebra

$$[\mathbf{q},\mathbf{p}] = i(1+\beta\mathbf{p}^2)$$

A non-vanishing minimal uncertainty in position arises $\Delta q_{min} = \sqrt{\beta} > 0$

Eigenstate of an observable $A$ *implies* $\Delta A = 0$. Therefore, in the GUP framework, **no physical states which are position eigenstates exist at all**

Information on position can be recovered from the quasiposition wave function $\psi(\zeta) \equiv \langle \psi_{\zeta}^{ml}|\psi\rangle$

$$\psi(\zeta) \sim \int \frac{dp}{(1+\beta p^2)^{3/2}} e^{i\frac{\zeta}{\sqrt{\beta}}\tan^{-1}(\sqrt{\beta}p)}\psi(p)$$

This is a generalized Fourier transformation

# FRW MODEL IN THE GUP FRAMEWORK

$$H_g + H_\phi \equiv -9\kappa p_x^2 x + \frac{3}{8\pi}\frac{p_\phi^2}{x} = 0 \quad x \equiv a^3,$$

$a$ is the scale factor and $\phi$ the emergent time: the wave function $\Psi(x,\phi)$ evolves as $\phi$ changes

Quantization: $(\phi, p_\phi)$ are canonically quantized while $(x, p_x)$ are GUP quantized via $[x, p_x] = i(1 + \beta p_x^2)$

Decomposition of the solution into positive and negative frequency: $i\partial_\phi \Psi = -\sqrt{\widehat{\Theta}}\Psi$ (positive frequency)

Computation of the quasiposition wave function of the model and analysis of the wave packets dynamics

The GUP wave packets do not fall in the singularity, but they approach the Planckian region in a stationary way

**The FRW Universe in the GUP scheme appears to be singularity-free in a probabilistic sense.**

[M.V.Battisti, GM, *Phys. Lett. B*, **656**, (2007) 96.]

# TAUB MODEL IN THE GUP FRAMEWORK

The Taub cosmological model is particular case of Bianchi IX ($\gamma_- = 0$)

$$ds^2 = N^2 dt^2 - e^{2\alpha} \left(e^{2\gamma}\right)_{ij} \omega^i \otimes \omega^j$$

$\alpha$ describes the isotropic expansion of the Universe while $\gamma_{ij} = \gamma_{ij}(t)$ determines the anisotropies via $\gamma_\pm$

The classical dynamics of this model resemble the one of a mass-less particle which bounces against a given wall

Quantization: the time variable (namely the volume of the Universe) canonically treated, the anisotropic one quantized in the deformed approach

By the analysis of the GUP wave packets dynamics **the probability to find the Universe at the classical singularity is negligible**

[M.V.Battisti, GM, *Phys. Rev. D*, **77**, (2008) 023518.]

# POLYMER TAUB UNIVERSE

$$[\hat{q}, \hat{p}] = i\hbar\,\hat{\mathbf{1}} \Rightarrow U(\alpha)\cdot V(\beta) = e^{(-i\alpha\,\beta)/\hbar}V(\beta)\cdot U(\alpha)$$

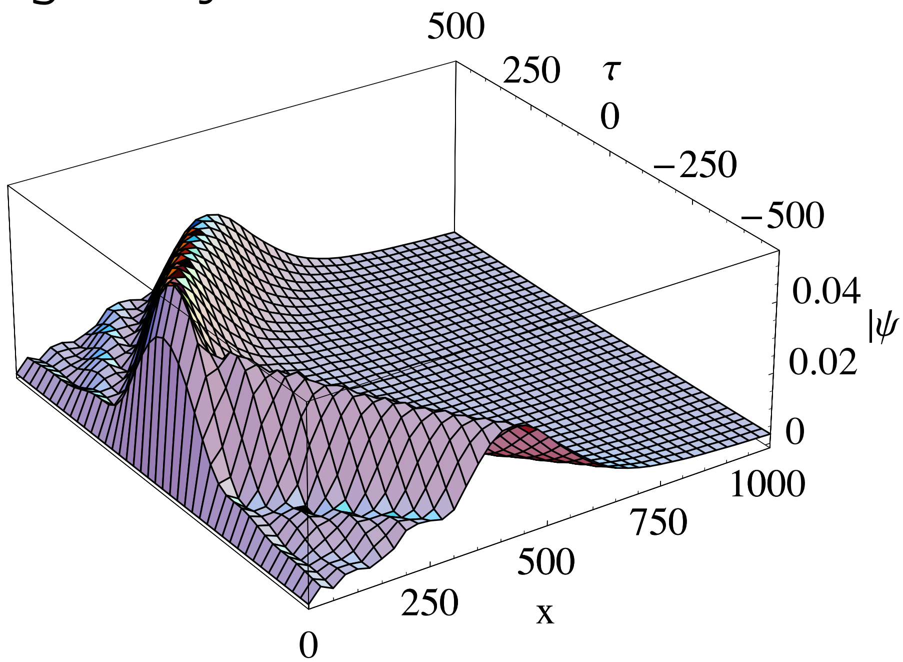
-exponentiated versions of $\hat{q}$ and $\hat{p}$

$$U(\alpha) = e^{i(\alpha\,\hat{q})/\hbar} \quad V(\beta) = e^{i(\beta\,\hat{p})/\hbar}$$

-their expectation values on the vacuum state

$$\hat{U}(\alpha)\cdot\phi(q) := (e^{i\alpha\,q/\hbar}\,\phi)(q), \quad \hat{V}(\beta)\cdot\phi(q) := e^{\frac{\beta}{d^2}(q-\beta/2)}\,\phi(q-\beta)$$

Polymer substitution $p \rightarrow \frac{1}{\mu_0}\sin(\mu_0 p)$.
The application of this paradigm to the Taub Universe $H^T_{ADM} = p_x \equiv p$ does not remove the cosmological singularity




[M.V Battisti, O.M. Lecian, GM, submitted to *Phys.Rev.D.*]

# TIME GAUGE IN QUANTUM GRAVITY

Given a 3+1 slicing of the space-time manifold, 4-bein vectors can be written as

$$e^0_\mu = (N, \chi_a E^a_i) \qquad e^a_\mu = (E^a_i N^i, E^a_i).$$

$\chi_a$ variables give velocity components of $\{e^a_\mu\}$ with respect to spatial hypersurfaces.

Loop Quantum Gravity is based on the time-gauge condition $\chi_a = 0$, by which a SU(2) Gauss constraint is inferred. After the quantization, a discrete spectrum for geometrical operators is predicted.

A quantization procedure without the time gauge would shed light on the behavior of this discrete spatial structure under boosts.

# SECOND-ORDER FORMULATION

In a second-order formulation without the time-gauge the boost constraints are obtained

$$\pi^a - \pi^b \chi_b \chi^a + \delta^{ab} \pi^i_b \chi_c E^c_i = 0$$

As soon as $\chi_a = \bar{\chi}_a$ are fixed, conjugate momenta $\pi^a$ can be evaluated and substituted into other constraints.

In this scheme, $\bar{\chi}_a$ label different sectors where a canonical quantization can be performed.

The invariance under boosts is preserved on a quantum level, since a unitary operator $U_\epsilon$ can be find mapping physical states between $\bar{\chi}_a = 0$ and $\bar{\chi}_a = \epsilon_a$

$$U_\epsilon = I - \frac{i}{4} \int \epsilon^a \epsilon_b (E^b_i \pi^i_a + \pi^i_a E^b_i) d^3x + O(\epsilon^4).$$

[F.Cianfrani, GM, *Class. Quant. Grav.*, **24**, (2007) 4161.]

# FIRST-ORDER FORMULATION

In a first order formulation, some second-class constraints arise, which can be solved by fixing the local Lorentz frame

An extension of Barbero-Immirzi connections has been provided

$$\tilde{A}_i^a = T^{ab}(\omega_{0bi} - {}^{\pi}D\chi_b) - \frac{1}{2\gamma(1+\chi^2)}\epsilon^a{}_{cd}{}^{\pi}\omega^{cf}{}_i T^{-1d}_{\ \ f}$$

such that Gauss constraints can be defined

$$G'_a = \partial_i \tilde{\pi}^i_a - \gamma(1+\chi^2)\epsilon_{abc}T^{cd}\tilde{A}^b_i\tilde{\pi}^i_d$$

$$\{G'_a, G'_b\} = \gamma(1+\chi^2)\epsilon_{abc}T^{cd}G'_d.$$

This formulation enables the use of LQG quantization techniques in a generic Lorentz frame. This way the behavior under boosts of discrete spectra of geometrical operator can be inferred.

# FLUID ENTROPY AS AN EVOLUTION OPERATOR IN CANONICAL QG

Matter/Reference frame duality allows the definition of a time operator via the Schutz' perfect fluid

$$\sqrt{-g}\left[\rho_0\left(\mu - TS\right) + R^{(4)}\right]$$

*($\mu$ is the normalization of the velocity potentials, $T$ the temperature and $S$ the entropy field)* The Kuchař-Brown mechanism allows to solve the super-Hamilotonian constraint

$$\pi - h[H^G, H^G_i, S] = 0$$

*($\pi$ is the momentum conjugate to one of the scalar fields involved, and $G$ denotes matter free GR quantities)*
In the comoving frame one identifies $Sp_S = \theta H^G/T$ so that

$$\{\bar{\mathcal{H}}_{phys}, \mathcal{O}_f(\tau)\} = \frac{\delta}{\delta lnS}\mathcal{O}_f(\tau)$$

So in the Comoving Frame one can identify the Log of $S$ with the time variable for observables.

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[GM, S. Zonetti, *Definition of a time variable with Entropy of a perfect fluid in Canonical Quantum Gravity*, (2008) submitted to Phys.Rev.D]

# GRAVITY AS A GAUGE THEORY

## 4-Dimensional manifold $\rightarrow$ tetrads formalism

Orthonormal basis for the *local Minkowskian tangent* space-time

Recover Lorentz symmetry: tetrad changes defined as local L.tr

$$g_{\mu\nu} = \eta_{ab}\, e_\mu{}^a\, e_\nu{}^b \qquad e_\mu{}^a\, e^\mu{}_b = \delta^a_b \qquad e_\mu{}^a\, e^\nu{}_a = \delta^\nu_\mu$$

## <u>*Lorentz (spin) connections*</u> $\omega_\mu{}^{ab}$: covariant-der. $\mathcal{D}_a$

$$\Gamma^{(L)}_\mu = \frac{1}{2}\,\omega_\mu{}^{cd}\,\Sigma_{cd} \qquad\qquad \omega_\mu{}^{cd} = e^{c\nu}\nabla_\mu e_\nu{}^d$$

($\Sigma_{cd}$: generators of the LG - $\nabla_\mu$: coordinate covariant derivative)

## Description of gravity as a gauge model:

$$\omega_\mu{}^{ab} = e_\mu{}^c\,\gamma^{ab}{}_c \qquad\qquad S = -\frac{1}{4}\int e\, d^4x\;\; e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}$$

$$R_{\mu\nu}{}^{ab} = \partial_\nu\omega_\mu{}^{ab} - \partial_\mu\omega_\nu{}^{ab} + \mathcal{F}_{cd}{}^{ab}{}_{ef}\,\omega_\mu{}^{cd}\omega_\nu{}^{ef}$$

$$\omega_\mu{}^{ab} \rightarrow \omega_\mu{}^{ab} - \partial_\mu\epsilon^{ab} + \frac{1}{4}\mathcal{F}_{cd}{}^{ab}{}_{ef}\,\epsilon^{cd}\omega_\nu{}^{ef}$$

<u>Ambiguity</u>: spin connections can be uniquely determined as functions of tetrad fields in terms of the Ricci rotation coefficients - The model is not based on two independent d.o.f.

# PROPOSAL FOR A GAUGE THEORY OF THE LORENTZ GROUP

Starting point $\rightarrow$ isometric diffeomorphisms induce local Lorentz rotations

$$x'^{\mu} = x^{\mu} + \xi^{\mu}(x) \qquad \nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu} = 0$$

$$\text{Iso. Diff.} \rightarrow e'^{a}_{\mu}(x') = e^{a}_{\mu}(x) + e^{a}_{\rho}(x)\partial\xi^{\rho}/\partial x'^{\mu}$$

$$\text{Inf. Lore.} \rightarrow e'^{a}_{\mu}(x') = e^{a}_{\mu}(x') + e^{b}_{\mu}(x')\epsilon^{a}_{b}$$

$$\epsilon_{ab} = D_{[a}\xi_{b]} - R_{abc}\xi^{c}$$

*If the two transformations overlap: inconsistence*

$\rightarrow$ Spin Connections: vectors or gauge fields?

$\rightarrow$ Fermions: scalars or spinor Lorentz rotated?

New gauge field ${A_{\mu}}^{ab}$ to restore the Lorentz invariance

Flat-space: $\partial_{\mu} \rightarrow \partial_{\mu} - \frac{i}{4}{A_{\mu}}^{ab}\Sigma_{ab}$

$\rightarrow$ fermion dynamics: $\mathcal{L}_{int} = \frac{1}{4}\,\bar{\psi}\,\epsilon^{c}_{abd}\,\gamma_5\,\gamma^{d}\,A^{ab}_{c}\,\psi$

- Curved space-time: connections$\rightarrow \widetilde{\omega}^{a}_{b} = \omega^{a}_{b} + A^{a}_{b}$;

${A_{\mu}}^{ab}$ identified with *torsion* fields:

right hand side of the $2^{nd}$ Cartan equation $de^{a} + \omega^{a}_{b} \wedge e^{b} = \mathcal{T}^{a}$

[N. Carlevaro, O.M. Lecian, G.M., *A. Fond. L. deBroglie*, **32** (2007) 281.]
[N. Carlevaro, O.M. Lecian, G.M., to be submitted to *Eur. Phys. Lett.*]
[O.M. Lecian, G.M., accepted by *Journ. Math. Phys.*]

# $f(R)$ MODIFIED GRAVITY

$$S_G = -\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}f(\mathcal{R})$$
$$f'\mathcal{R}_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu\nabla_\nu f' + \Box f' = 0$$
$$3\Box f' + f'\mathcal{R} - 2f = 0, \quad f'(\mathcal{R}) \equiv df(\mathcal{R})/d\mathcal{R}$$

$$S = -\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\left[f'(A)(\mathcal{R} - A) + f(A)\right], \quad R \equiv A$$

$$g_{\mu\nu} \to e^{\varphi}g_{\mu\nu}, \varphi = -\ln f'(\mathcal{A})$$

$$S = -\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\left[\mathcal{R} - \frac{3}{2}g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi - V(\varphi)\right],$$

$$V(\varphi) = \frac{A}{f'(A)} - \frac{f(A)}{f'(A)^2},$$

# EXPONENTIAL $f(\mathcal{R})$

$f(\mathcal{R}) = \lambda e^{\mu\mathcal{R}}$: only one free parameter available for the model, the 'cosmological constant' $\Lambda = f(0) \neq 0$. $\lambda = 2\Lambda$, $\mu = \frac{1}{2\Lambda}$.

Full non-Einsteinian regime: $\Lambda > 0$

Einsteinian regime at lower orders: the Taylor expansion holds for $\Lambda < 0$ (accelerating deSitter phase)

For a Planckian value of $\Lambda$, a suitable cancellation mechanism has to be hypothesized.

# NON-ANALYTICAL $f(\mathcal{R})$

$f(\mathcal{R}) = \mathcal{R} + \gamma\mathcal{R}^{\beta}$, $2 < \beta < 3$

$ds^2 = (1+\Phi)dt^2 - (1-\Psi)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$

$$R = Ar^{\frac{2}{\beta-2}}, \quad A = \left[-\frac{6\gamma\beta(3\beta-4)(\beta-1)}{(\beta-2)^2}\right]^{\frac{1}{2-\beta}}$$

$$\Phi \equiv \Phi_N + \Phi_C = \sigma + \frac{\delta}{r} + \frac{A(\beta-2)^2}{6(3\beta-4)(\beta-1)} r^{2\frac{\beta-1}{\beta-2}}$$

$$\Psi \equiv \Psi_N + \Psi_C = \frac{\delta}{r} + \frac{A(\beta-2)}{3(3\beta-4)} r^{2\frac{\beta-1}{\beta-2}},$$

very stringent constraints on $\gamma$ imposed by planetary orbital periods but for $\beta \sim 2$

validity range $r_s \ll r \ll r^*$

[O.M.Lecian, GM, *Int. J. Mod. Phys. D*, **17**, (2008) 1111.]
[O.M.Lecian, G.Montani, submitted to *Phys. Rev. D*]

# 5D KALUZA KLEIN MODEL

$$J_{AB}=\begin{pmatrix} g_{\mu\nu}-\phi^2(ek)^2A_\mu A_\nu & -\phi^2(ek)A_\mu \\ -\phi^2(ek)A_\mu & -\phi^2 \end{pmatrix} \quad (ek)^2=4G\,;\; c=1$$

$$S_5=-\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(\phi R-2\Box\phi+\frac{1}{4}(ek)^2\phi^3F_{\mu\nu}F^{\mu\nu})$$

If we put $\phi=1$ *before* the variational procedure we recover Einstein-Maxwell dynamics.

The problem of matter:

$$S_5=-\hat{m}\int ds_5 \qquad \to P_AP^A=\hat{m}^2$$

The 4D reduced particle is characterized by :

$$q=ekP_5 \qquad m^2=\left(\frac{P_5^2}{\phi^2}+\hat{m}^2\right)$$

This result is not consistent with Lorenzian dynamics: it provides a bounded $q/m$ and gives a huge massive modes spectrum, beyond Planck scale, when we consider the compactification of the extra dimension.

[V.Lacquaniti,GM *Int.Journ.Mod.Phys.D* **4**, (2007) 387.]
[V.Lacquaniti,GM *Dynamics of Matter in a 5D KK Model* submitt. *CQG*]

# REVISED APPROACH TO MATTER

$$D_A T^{AB} = 0 \qquad \partial_5 T^{AB} = 0$$

After KK reduction we get a conserved current:

$$5) \quad \rightarrow \nabla_\mu \left(\phi T^\mu_5\right) = 0 \qquad \rightarrow j^\mu = ek\phi T^\mu_5$$

$$\mu) \quad \rightarrow \nabla_\rho (\phi T^{\mu\rho}) = -g^{\mu\rho}\left(\frac{\partial_\rho \phi}{\phi^2}\right) T_{55} + F^\mu_\rho j^\rho$$

For a point-like particle, after a Papapetrou expansion,we have:

$$\boxed{m\frac{Du^\mu}{Ds} = A(u^\rho u^\mu - g^{\mu\rho})\frac{\partial_\rho \phi}{\phi} + qF^{\mu\rho}u_\rho}$$

$$m = \frac{1}{u^0}\int d^3x \sqrt{g}\,\phi T^{00} \quad q = ek\int d^3x \sqrt{g}\,\phi T^0_5 \qquad A = u^0\int d^3x \sqrt{g}\,\frac{T_{55}}{\phi}$$

In this formulation $m$ and $q$ are not correlated via $P_5$; we have no bound on $q/m$; furthermore, analyzing the effective Lagrangian related to such a motion we can recognize that the huge massive modes are suppressed ( no Kaluza-Klein tower ).

[V.Lacquaniti,GM *Dynamics of Matter in a Compactified 5D KK Model* submitted *Class. Quantum Grav.*]

# KALUZA KLEIN THEORY WITH MATTER

Given the consistency of the approach to matter we can consider the full dynamics for matter and fields: From $G^{AB} = 8\pi G T^{AB}$ we get:

$$G^{\mu\nu} = \frac{1}{\phi}\nabla^{\mu}\partial^{\nu}\phi - \frac{1}{\phi}g^{\mu\nu}\Box\phi + 8\pi G\phi^2 T^{\mu\nu}_{em} + 8\pi G\frac{T^{\mu\nu}_{matt}}{\phi}$$

$$\nabla_{\nu}\left(\phi^3 F^{\nu\mu}\right) = 4\pi j^{\mu} \quad \rightarrow j^{\mu} = ek\phi T^{\mu}_{5}$$

$$\frac{1}{2}R + \frac{3}{8}\phi^2 (ek)^2 F^{\mu\nu}F_{\mu\nu} = 8\pi G\frac{T_{55}}{\phi^2}$$

The problem concerning $\phi = 1$ is now removed. Now we are studying omogeneus cosmological solutions and spherical solutions. Interesting perspectives are related to the behaviour of mass

$$\boxed{\frac{dm}{ds} = -\frac{A}{\phi}\frac{d\phi}{ds}}$$

Noticeably, if we assume $A = \alpha m$ we recover FFU (Free Falling Universality) and we have

$$m = m_0 \left(\frac{\phi}{\phi_0}\right)^{-\alpha}$$

[V.Lacquaniti,GM *Geometry and Matter in 5D KK framework* to be submitted to *Class. Quantum Grav.*]

# KALUZA-KLEIN FRAMEWORK

The Kaluza-Klein (KK) approach is based on the identification of gauge symmetries with isometries of an homogeneous extra-dimensional space.

The full metric contains gauge bosons $A_\mu^{\bar{M}}$ as off-diagonal components

$$j_{AB} = \left( \begin{array}{c|c} g_{\mu\nu} - \phi^2 \gamma_{mn} \xi^m_{\bar{M}} \xi^n_{\bar{N}} A^{\bar{M}}_\mu A^{\bar{N}}_\nu & -\phi^2 \gamma_{mn} \xi^m_{\bar{M}} A^{\bar{M}}_\mu \\ \hline -\phi^2 \gamma_{mn} \xi^n_{\bar{N}} A^{\bar{N}}_\nu & -\phi^2 \gamma_{mn} \end{array} \right).$$

Under these hypotheses, the Yang-Mills Lagrangian density comes out by the dimensional reduction of the Einstein-Hilbert action in 4-dimensions

$$S = -\frac{c^3}{16\pi^{(n)}G} \int_{V^4 \otimes B^K} \sqrt{-j}^{(n)} R d^4x d^K y =$$
$$= -\frac{c^3}{16\pi G} \int_{V^4} \sqrt{-g} \phi \left[ R + {}^{(k)}R' + \frac{1}{4} \phi^2 F^{\bar{M}}_{\mu\nu} F^{\bar{M}\mu\nu} \right] d^4x$$

# PHENOMENOLOGICAL POINT OF VIEW

The KK procedure is not able to reproduce non-Abelian gauge bosons transformations

$$\xi'^{m}_{\bar{M}}(y')A'^{\bar{M}}_{\mu} = \xi'^{m}_{\bar{M}}(y')(A^{\bar{M}}_{\mu} - \partial_{\mu}\omega^{\bar{M}}).$$

This issue can be solved by an averaging procedure on the extra-dimensional space[1].

Furthermore, this average allows us to find out a form for spinors, such that KK mass terms can be suppressed by an order parameter $\beta$

$$\chi_{rs} = \frac{1}{\sqrt{V}}e^{-\frac{i}{2}\sigma_{(p)rs}\lambda^{(p)}_{(q)}\Theta^{(q)}(y^m)} ; \quad \Theta^{(p)} = \frac{1}{\beta}c^{(p)}e^{-\beta\eta}$$

Within this scheme the electro-weak model can be geometrized, finding an upper bound for $\beta$ ($\beta > 10^{28}$). Massive neutrinos and a justification for the fine-tuning of the Higgs parameters can be given too[2].

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[[1]F.Cianfrani, GM *Mod. Phys. Lett. A*, **21**, 3, (2006) 265.]
[[2]F.Cianfrani, GM, *IJMPD* **17**, 5, (2008) 785.]