New Issues in Lorentz Gauge Theories

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Abstract: This talk is devoted to introduce a gauge theory of the Lorentz Group based on the ambiguity emerging in dealing with isometric diffeomorphism-induced Lorentz transformations. The behaviors under local transformations of fermion fields and spin connections (assumed to be coordinate vectors) are analyzed in flat space-time and the role of the torsion field within the generalization to curved space-time is briefly discussed. The fermion dynamics including the new gauge field is then analyzed assuming time-gauge and stationary solutions in the non-relativistic limit are founded.

Outline:

1. Internal space-time symmetries: the standard approach to a Lorentz Gauge Theory
2. Diffeomorphism induced Lorentz transformations and new connections
3. Formulation of the theory on flat space-time
4. Fermion dynamics in the non-relativistic limit: a generalized Pauli Equation
5. Generalization to curved space-time and the role of the torsion field

Internal space-time symmetries: the standard approach to a Lorentz Gauge Theory

Internal symmetries of the space-time: we focus on the description of GR as a gauge model (underling the ambiguity that arises from this approach).

Tetrad formalism ($e_\mu{}^a$) for the local Minkowskian tangent space-time can recover the Lorentz symmetry
$\longrightarrow$ tetrad changes are defined as local Lorentz *trs* between inertial references

$$g_{\mu\nu} = \eta_{ab}\, e_\mu{}^a\, e_\nu{}^b \qquad e_\mu{}^a\, e^\mu{}_b = \delta^a_b \qquad e_\mu{}^a\, e^\nu{}_a = \delta^\nu_\mu$$

$\mu = 0, 1, 2, 3$ coordinate indices $\qquad a = 0, 1, 2, 3$ Lorentz indices

$$e_\mu{}^a \longrightarrow \Lambda^a_b e_\mu{}^b$$

Local Lorentz invariance of the scheme $\rightarrow$ Covariant derivative

$\partial_a \psi = e_a^{\ \mu} \partial_\mu \psi$ (coordinate scalar) $\quad \rightarrow \quad \mathcal{D}_a \psi = (\partial_a + \Gamma^{(L)}_a)\psi$ (Lorentz vector)

<u>*Lorentz connections*</u> $\omega_\mu^{\ ab}$:

$$\Gamma^{(L)}_a = e_a^{\ \mu} \Gamma^{(L)}_\mu \qquad \Gamma^{(L)}_\mu = \frac{1}{2}\, \omega_\mu^{\ cd}\, \Sigma_{cd} \qquad \boxed{\boldsymbol{\omega_\mu^{\ cd} = e^{c\nu} \nabla_\mu e_\nu^{\ d} = e_\mu^{\ a} \gamma_a^{\ cd}}}$$

Σ_{cd}: generators of the Lorentz Group (LG) $\qquad \nabla_\mu$: coordinate covariant derivative

- Connections are called *spin connections*:

  $\rightarrow$ they restore the correct Dirac algebra in curved space-time.

  $\rightarrow$ the correct treatment of spinors leads to the introduction of that connections which guarantee a suitable gauge model for the Lorentz group even on flat space-time.

  $\rightarrow$ spinors are a particular representation of the LG.

This picture suggests *(in appearance)* the description of GR as a **gauge model**

Lorentz connections are the projected *Ricci rotation coefficients* $\omega_\mu{}^{ab} = e_\mu{}^c \gamma^{ab}{}_c$:

$$R_{\mu\nu}{}^{ab} = \partial_\nu \omega_\mu{}^{ab} - \partial_\mu \omega_\nu{}^{ab} + \mathcal{F}_{cd}{}^{ab}{}_{ef} \omega_\mu{}^{cd} \omega_\nu{}^{ef}$$

$\mathcal{F}_{cd}{}^{ab}{}_{ef}$: LG structure constants

The Hilbert-Einstein action for GR can be written in the form

$$S(e,\omega) = -\tfrac{1}{4}\int e\, d^4x \;\; e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}$$

$\rightarrow$ Variation wrt connections leads to the *II Cartan structure equation*

$$\partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a - \omega_\mu{}^{ab} e_{\nu b} + \omega_\nu{}^{ab} e_{\mu b} = 0$$

$\rightarrow$ Variation wrt tetrads gives the Einstein equations

In the usual approach, ${\omega_\mu}^{ab}$ transform like Lorentz gauge vectors under infinitesimal local Lorentz *trs* - ϵ^{ab}: infinitesimal parameter $\rightarrow$ $\Lambda_a^b = \delta_a^b + \epsilon_a^b$

$${\omega_\mu}^{ab} \rightarrow {\omega_\mu}^{ab} - \partial_\mu \epsilon^{ab} + \frac{1}{4} {{\mathcal{F}_{cd}}^{ab}}_{ef} \epsilon^{cd} {\omega_\nu}^{ef}$$

Riemann tensor is preserved by such a change
(in flat space-time, we deal with non-zero gauge connections, but a vanishing curvature)

- Ambiguity: ${\omega_\mu}^{ab}$ exhibit the right behavior to play the role of Lorentz gauge fields, and GR assume the features of a gauge theory.
  But:
  $\rightarrow$ spin connections can be uniquely determined as functions of tetrad fields in terms of the Ricci rotation coefficients: non fundamental gauge fields

  $\rightarrow$ Tetrad fields (Principle of General Covariance): two dependent dof

Diffeomorphism induced Lorentz transformations and new connections

The introduction of fermions into the dynamics requires to treat local Lorentz *trs* as the real independent gauge of GR. Because of the spinor behavior, it is crucial to investigate if diffeomorphisms can be reinterpreted as local Lorentz transformations.

Transformation laws:

$\omega_\mu{}^{ab}$
- $\longrightarrow$ Gauge potential under Lorentz *trs* - (Lorentz indices)
- $\longrightarrow$ Coordinate vector under diffeomorphism - (Coordinate indices)

ψ
- $\longrightarrow$ Representation of the LG - $\psi(x) \to \psi'(x') = S\,\psi(x)$ (tangent bundle)
- $\longrightarrow$ Coordinate scalar - (no world indices)

If the two *trs* overlap: inconsistence - what is the nature of $\omega_\mu{}^{ab}$ and ψ ?

An isometric diffeomorphism induces orthonormal transformed basis ${e_\mu}^a$:
in this sense an isometry generates a local Lorentz transformation of the basis.

- Infinitesimal isometric diffeomorphism:

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x) \qquad \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0 \qquad \text{(isometry condition)}$$

$$ {e_\mu}^a(x) \xrightarrow{D} {e_\mu}^a(x) + {e_\rho}^a(x)\, \partial\xi^\rho \,/\, \partial x'^\mu $$

- Infinitesimal Lorentz transformation:

$$\Lambda_a^b(x) = \delta_a^b + \epsilon_a^b \qquad {e_\mu}^a(x) \xrightarrow{L} {e_\mu}^a(x) + {e_\mu}^b(x)\epsilon_b^a$$

The two *trs* overlap if: $\epsilon_{ab} = D_{[a}\xi_{b]} - R_{abc}\xi^c$

Isometry condition $\nabla_{(\mu}\xi_{\nu)} = 0$ must hold in order to have $\epsilon_{ab} = -\epsilon_{ba}$.

If isometric diffeomorphism are allowed:

Diffeomorphism induced Lorentz transformation: a new gauge field $A_\mu{}^{ab} \neq \omega_\mu{}^{ab}$ *must be introduced to restore the Lorentz invariance*

→ *Flat space-time:* in the case $e_\mu{}^a = \delta_\mu{}^a$ spin connections vanish and they remain identically zero under diffeomorphisms.

Coordinate transformations = Lorentz rotations (gauge transformations): $\omega_\mu{}^{ab}$ are unappropriate to restore local Lorentz invariance.

→ *Curved space-time*: $\omega_\mu{}^{ab}$ are assumed to behave like tensors under diffeomorphism induced rotations.

obs. If $\omega_\mu{}^{ab}$ behave like gauge vectors, the standard approach can be recovered (ambiguity of tetrads dependence)

→ *Arbitrary choice*

obs. Spinor ψ can noway be a Lorentz scalar

Formulation of the theory on flat space-time

Construction of a diffeo-induced Lorentz gauge model on a Minkowski space.

Riemann curvature tensor vanishes: spin connections ${\omega_\mu}^{ab}$ can be set to zero
(in general, they are allowed to be non-vanishing quantity in view of local Lorentz invariance)

$\rightarrow$ The introduction of Lorentz connections ${A_\mu}^{ab}$ as the gauge field of local LG on flat space-time (as far as the correspondence between an infinitesimal diffeomorphism and a local Local rotation is recovered)

The *metric tensor* can be expressed as: $g_{\mu\nu} = \eta_{ab} {e_\mu}^a {e_\nu}^b$

Infinitesimal diffeomorphism

$$x^a \xrightarrow{D} x'^a = x^a + \xi^a(x^c)$$

Infinitesimal local Lorentz *tr*

$$x^a \xrightarrow{L} x'^a = x^a + \epsilon^a_b(x^c)\, x^b$$

obs. If *vector fields* are treated no inconsistence arises if the two *trs* overlap.

If $\epsilon_a^b \equiv \partial^b \xi_a(x^c)$ the two transformation laws are the same

$$^D V'_a(x'^c) = V_a(x^c) + \partial_a \xi^b(x^c)\, V_b(x^c) \qquad {}^L V'_a(x'^c) = V_a(x^c) + \epsilon_a^b\, V_b(x^c)$$

and the LG loses its status of independent gauge group. To restore the proper number of degrees of freedom of a Lorentz *tr*, 10, out of that of generic diffeo., the ***isometry condition*** $\partial_b \xi_a + \partial_a \xi_b = 0$ has to be imposed.

Spin-$1/2$ fields are is described by the Lagrangian density on a 4D flat manifold

$$\mathcal{L} = \frac{i}{2}\, \bar{\psi} \gamma^a e^\mu{}_a \partial_\mu \psi - \frac{i}{2}\, e^\mu{}_a \partial_\mu \bar{\psi} \gamma^a \psi - m\, \bar{\psi} \psi$$

which is invariant under global Lorentz *trs*

Let us introduce a local Lorentz *tr*: $S = S(\Lambda(x))$

$$\psi(x) \rightarrow S\,\psi(x) \qquad \bar{\psi}(x) \rightarrow \bar{\psi}(x) S^{-1}$$

where S is in every point a *non-singular matrix*.

Infinitesimal transformations $\epsilon^a_b(x) \ll 1 \;\; (\epsilon^{ab} = -\epsilon^{ba}) \quad \rightarrow \quad \Lambda^a_b = \delta^a_b + \epsilon^a_b$

$$S = I - \tfrac{i}{4}\,\epsilon^{ab}\,\Sigma_{ab} \qquad \Sigma_{ab} = -\tfrac{1}{2}[\gamma_a, \gamma_b] \qquad [\Sigma_{cd}, \Sigma_{ef}] = i\mathcal{F}_{cdef}{}^{ab}\,\Sigma_{ab}$$

- A spinor can not be a Lorentz scalar $\rightarrow$ for assumption the connections ${\omega_\mu}^{ab}$ do not follows Lorentz gauge transformations.

  New connections have to be introduced to restore Lorentz invariance

  (Differently from *vector fields*, spinors have to recognize the isometric components of the diffeomorphism as a local Lorentz *tr*, if accelerated coordinates are taken into account)

Let us assume that γ matrices transform like Lorentz vectors $S\,\gamma^a\,S^{-1} = (\Lambda^{-1})^a_b\,\gamma^b$

- In particular, if ${\omega_\mu}^{ab} = 0$ they still vanish under a Lorentz gauge *tr* which now can be seen as a diffeomorphism.

This way $\mathcal{L}$ invariance is restored by the ***new*** covariant derivative

$$D_\mu\psi = (\partial_\mu - ig\,A_\mu)\,\psi = (\partial_\mu - ig\,{A_\mu}^{ab}\,\Sigma_{ab})\,\psi$$

The gauge invariance $\gamma^a {e^\mu}_a D_\mu\psi \to S\gamma^a {e^\mu}_a D_\mu\psi$ is provided by the *gauge field*

$A_\mu = {A_\mu}^{ab}\Sigma_{ab}\ \ (\neq {\omega_\mu}^{ab})$ which transforms like $A_\mu \to S\,A_\mu\,S^{-1} - 4i\,S\,\partial_\mu\,S^{-1}$

$$ {A_\mu}^{ab} \to {A_\mu}^{ab} - \partial_\mu\epsilon^{ab} + 4{\mathcal{F}^{ab}}_{cdef}\ \epsilon^{ef}\,{A_\mu}^{cd} \qquad (\epsilon^{ef}(x) \ll 1)$$

i.e. as a **natural Yang-Mill field** associated to the ***Lorentz gauge Group***, living in the tangent bundle.

Fermion dynamics in the non-relativistic limit: a generalized Pauli Equation

For the 4-spinor ψ , the implementation of the *local Lorentz symmetry* ($\partial_\mu \to D_\mu$), in a flat space, leads to the Lagrangian density:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \qquad \mathcal{L}_0 = \frac{i}{2}\,\bar{\psi}\gamma^a e^\mu{}_a \partial_\mu \psi - \frac{i}{2}\,e^\mu{}_a \partial_\mu \bar{\psi}\gamma^a \psi \,-\, m\,\bar{\psi}\psi$$

$$\mathcal{L}_{int} = \frac{1}{8}\,e^\mu{}_c\,\overline{\psi}\{\gamma^c, \Sigma_{ab}\}\,A_\mu{}^{ab}\,\psi = -S^\mu{}_{ab}\,A_\mu{}^{ab}$$

where the curl brackets indicate the *anti-commutator*

$$\{\gamma^c, \Sigma_{ab}\} = 2\,\epsilon^c{}_{abd}\,\gamma_5\,\gamma^d \qquad S_\mu{}^{ab} = -\frac{1}{4}\,\epsilon^{ab}{}_{cd} e_\mu{}^c j^d_A$$

- $j^d_{(A)} = \overline{\psi}\gamma_5\gamma^d\psi$: *spinor axial current* interacting with the gauge field A_μ

Explicit form of the interaction Lagrangian density:

$$\mathcal{L}_{int} = \frac{1}{4}\,\overline{\psi}\,\epsilon^{c}_{abd}\,\gamma_5\,\gamma^{d}\,A^{ab}_{c}\,\psi$$

$a = \{0, \alpha\}$: split of the gauge field $\rightarrow$ $A^{0\alpha}_{0}\,,\;A^{\alpha\beta}_{0}\,,\;A^{0\alpha}_{\gamma}\,,\;A^{\alpha\beta}_{\gamma}$

- We impose the *time-gauge* associated to this picture: $A^{\alpha\beta}_{0} = 0$
- $A^{0\alpha}_{0}$ is saturated on the completely anti-symmetric symbol $\epsilon^{0}_{0\alpha d} \equiv 0$

Now we get

$$\mathcal{L}_{int} = \frac{1}{4}\,\overline{\psi}\,(\epsilon^{\gamma}_{0\alpha\delta}\,\gamma_5\,\gamma^{\delta}\,A^{0\alpha}_{\gamma} + \epsilon^{\gamma}_{\alpha\beta 0}\,\gamma_5\,\gamma^{0}\,A^{\alpha\beta}_{\gamma})\,\psi$$

The total Lagrangian density rewrites as

$$\mathcal{L} = \tfrac{i}{2}(\,\psi^\dagger\,\gamma^0\gamma^a\partial_a\psi\,-\,\partial_a\psi^\dagger\gamma^0\gamma^a\psi) - m\,\gamma^0\,\psi^\dagger\psi\,+$$
$$+\,\psi^\dagger\,C_0\,\gamma^0\gamma_5\gamma^0\psi\,+\,\psi^\dagger\,C_\alpha\,\gamma^0\gamma_5\gamma^\alpha\psi$$

with the identifications: $C_0 = \tfrac{1}{4}\epsilon^{\gamma}_{\alpha\beta0}A^{\alpha\beta}_{\gamma}$ $\quad C_\alpha = \tfrac{1}{4}\epsilon^{\gamma}_{0\beta\alpha}A^{0\beta}_{\gamma}$

Finally, from $\delta S = 0$ variation wrt $\psi^\dagger$ leads to the modified Dirac eq

$$(i\,\gamma^0\gamma^0\partial_0\,+\,C_\alpha\,\gamma^0\gamma_5\gamma^\alpha\,+\,i\,\gamma^0\gamma^\alpha\partial_\alpha\,+\,C_0\,\gamma^0\gamma_5\gamma^0)\,\psi\;=\;m\,\gamma^0\,\psi$$

We look now for ***stationary solution*** of the Dirac equation: $\psi(\mathbf{x},t) \to \psi(\mathbf{x})\, e^{-i\mathcal{E}t}$

Using the Standard Representation of Dirac matrices:

$$\psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix} \qquad \psi^\dagger = (\, \chi^\dagger \,,\, \phi^\dagger \,)$$

$$\gamma^\alpha = \begin{pmatrix} 0 & \sigma_\alpha \\ -\sigma_\alpha & 0 \end{pmatrix} \qquad \gamma^0 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$$

the 2-component spinors χ and ϕ are found to satisfy the two coupled eqs

$$(\mathcal{E} - \sigma_\alpha C^\alpha)\,\chi \; - \; (\sigma^\alpha p_\alpha + C_0)\,\phi \; = \; m\,\chi$$

$$(\mathcal{E} - \sigma_\alpha C^\alpha)\,\phi \; - \; (\sigma^\alpha p_\alpha + C_0)\,\chi \; = - \, m\,\phi$$

In order to investigate the **non-relativistic limit** $\longrightarrow$ $\mathcal{E} = E + m$
The coupled equations rewrite now

$$(E - \sigma_\alpha C^\alpha)\,\chi \;=\; (\sigma^\alpha p_\alpha + C_0)\,\phi$$

$$(E - \sigma_\alpha C^\alpha + m)\,\phi \;=\; (\sigma^\alpha p_\alpha + C_0)\,\chi \;-\; m\,\phi$$

- In the non-relativistic limit both $|E|$ and $|\sigma_\alpha C^\alpha|$ terms *are small* in comparison wrt the mass term m:

$$\phi \;=\; \frac{1}{2m}(\sigma^\alpha p_\alpha + C_0)\,\chi$$

ϕ is smaller than χ by a factor of order p/m (*i.e.* v/c): ***small components***

Using the Pauli matrix relation $(\boldsymbol{\sigma} \cdot \mathbf{A})(\boldsymbol{\sigma} \cdot \mathbf{B}) = \mathbf{A} \cdot \mathbf{B} + i\boldsymbol{\sigma} \cdot (\mathbf{A} \times \mathbf{B})$ we can combine the two eqs in the following expression

$$\boxed{E\,\chi(\mathbf{x}) \;=\; \frac{1}{2m}\left[\,p^2 \;+\; C_0^2 \;+\; 2C_0\,(\sigma^\alpha p_\alpha) \;+\; \sigma_\alpha C^\alpha\,\right]\chi(\mathbf{x})}$$

Strong analogies with the electro-magnetic case: **Pauli Equation**

$$E\,\chi(\mathbf{x}) \;=\; \frac{1}{2m}\left[(\mathbf{p}+\mathbf{A})^2 \;+\; \mu_B\,\boldsymbol{\sigma}\cdot\mathbf{B} \;+\; \Phi\right]\chi(\mathbf{x})$$

where $\mu_B = e/2m$ is the Bohr magneton and **A** is the vector potential.

→ It is worth noting the presence of a term related to the *helicity* of the *2*-spinor: this coupling is controlled by the rotation-like component associated to C_0.

→ A Zeeman-like coupling associated to the boost-like component C_α is also present.

Fermion dynamics in the non-relativistic limit: a generalized Pauli Equation

Let us now neglect the term C_0^2 and implement the symmetry

$\partial_\mu \to \partial_\mu + A_\mu^{U(1)} + A_\mu{}^{ab}\Sigma_{ab}$ with $\mathbf{A} = 0$

$\to$ we introduce a Coulomb central potential $V(r)$: $E \to E - V(r)$

$$H_0 = \frac{p^2}{2m} - \frac{Ze^2}{(4\pi\bar{\epsilon}_0)r}$$

$$H' = \frac{1}{m}\left[\, 2C_0\,(S_\alpha\, p^{\,\alpha}) + S_\alpha\, C^{\alpha}\,\right]$$

where **S** is the spin operator: $\sigma_\alpha = 2S_\alpha$.

These Hamiltonian characterize the electron dynamics in a hydrogen-like atom in presence of a gauge field of the Lorentz Group.

Generalization to curved space-time and the role of the torsion field

First Order Approach: in presence of **torsion** $\mathcal{T}^{\rho}_{\mu\nu}$ (Riemann-Cartan space U^4), the II Cartan Structure eq writes

$$\partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a - \widetilde{\omega}_\mu{}^{ab} e_{\nu b} + \widetilde{\omega}_\nu{}^{ab} e_{\mu b} = e_\rho{}^a \mathcal{T}^{\rho}_{\mu\nu} = \mathcal{T}^{a}_{\mu\nu}$$

- The connections, solutions of the Cartan eq, are

$$\widetilde{\omega}_\mu{}^{ab} = \omega_\mu{}^{ab} + \mathcal{K}_\mu{}^{ab}$$

here $\mathcal{K}_\mu{}^{ab}$ is the contortion field $\quad \mathcal{K}^{\mu}_{\nu\rho} = -\frac{1}{2}(\mathcal{T}^{\mu}_{\nu\rho} - \mathcal{T}^{\mu}_{\rho\nu} + \mathcal{T}^{\mu}_{\nu\rho})$
and $\omega_\mu{}^{ab}$ are the usual Reimannian spin connections.

$\rightarrow$ These connections do not describe any physical field:
$\mathcal{K}_\mu{}^{ab}$ appear only in a non-dynamical term: in presence of fermionic matter, substituting $\widetilde{\omega}_\mu{}^{ab}$ in the HE action, $\mathcal{K}_\mu{}^{ab}$ become proportional to the spin density (Einstein-Cartan model).

Total connections can be rewritten as: $C_\mu{}^{ab} = \widetilde{\omega}_\mu{}^{ab} + A_\mu{}^{ab}$

- $A_\mu{}^{ab}$: Lorentz gauge connections connected with the appearance of torsion

Interaction term between the spin connections ω and the fields A:
(writes in a more compact formalism)

$$\mathcal{S}_{int} = 2\int \epsilon_{abcd}\, e^a \wedge e^b \wedge \omega_g^{[c} \wedge A^{gd]}$$

$$S\left(e, \omega, A, \psi, \overline{\psi}\right) = \frac{1}{4}\int \epsilon_{abcd}\, e^a \wedge e^b \wedge R^{cd} +$$
$$-\frac{1}{32}\int tr \star F \wedge F - \frac{1}{4}\int \epsilon_{abcd}\, e^a \wedge e^b \wedge \omega^{[c}{}_f \wedge A^{fd]} +$$
$$+\frac{1}{2}\int \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \left[i\overline{\psi}\gamma^d \left(d - \frac{i}{4}\left(\omega + A\right)\right)\psi - i\left(d + \frac{i}{4}\left(\omega + A\right)\right)\overline{\psi}\gamma^d\psi\right]$$

Variations:

- No fermion matter:

$$d^{(\widetilde{\omega})} e^a = A^a{}_b \wedge e^b \qquad\qquad \widetilde{\omega}_\mu{}^{ab} = \omega_\mu{}^{ab} + A_\mu{}^{ab}$$

$\rightarrow$ $A_\mu{}^{ab}$ *can be identified with the contortion field* $\mathcal{K}_\mu{}^{ab}$ (II Cartan eq)

- With fermion matter: $\widetilde{\omega}_\mu{}^{ab} = \omega_\mu{}^{ab} + A_\mu{}^{ab} + \frac{1}{4}\,\epsilon^{a}_{bcd} e_\mu{}^c \jmath^{d}_{(A)}$

  $\rightarrow$ spin density results to be the source term of the Yang-Mills eq for the new Lorentz connections.

Conclusions

- A gauge theory of the Lorentz group is developed starting from the ambiguity in dealing with isometric diffeomorphism-induced Lorentz transformations. The not clear behaviors under local transformations of fermion field and spin connections allows to introduce new connections for the model.
- In order to restore the invariance under diffeo-induced local Lorentz *trs* of a spinor lagrangian in a flat scape-time, we need to introduce a new gauge field behaving like a Yang-Mill field. *Spin connections are assumed to behave like coordinate vectors and are not gauge fields*.
- The analysis of the spinor interaction lagrangian in presence of the new gauge field, in the non-relativistic limit, leads to a Pauli-like equation describing the behavior of the large components of a 4-spinor.
- The generalization in curved space-time allows to identify the Lorentz gauge field with the tetradic projection of the contortion field arising from the Second Cartan Structure equation.