Gravitational Instability in Presence of Dissipative Effects

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Abstract: This talk focus on the analysis of gravitational instability in presence of dissipative effects. In particular, the standard Jeans Mechanism and the generalization in treating the Universe expansion are both analyzed when bulk viscosity affects the first order Newtonian dynamics. As results, the perturbations evolution is founded to be dumped by dissipative processes and the top-down mechanism of structure formation is suppressed. In such a scheme the Jeans mass remain unchanged also in presence of viscosity.

Outline:

1. Motion equations of a viscous fluid
2. Analysis of the Jeans Mechanism
3. Expanding Universe Generalization

Motion Equation of Viscous Fluids

Adiabatic ideal fluids are governed, in **Newtonian regime**, by the Eulerian set of eqs:

- Continuity Equation: $\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$ (energy conservation)
- Euler Equation: $\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla \phi$ (momentum conservation)
- Poisson Equation: $\nabla^2 \phi = 4\pi G \rho$ (gravitational field)
- Equation of State (EoS): $p = p\,(\rho, S)$ (pressure and energy density)

where ρ is the energy density, $\mathbf{v}$ in the local fluid velocity, p is the pressure, ϕ is the gravitational potential and S is the entropy of the adiabatic system ($\dot{S} = 0$)

- The sound speed is defined as: $v_s^2 = \frac{\delta p}{\delta \rho}$

Motion equation of a viscous fluid

Let us introduce the effects of the **energy dissipation during the fluid motion**: thermodynamical non-reversibility and internal friction (we neglect thermal conductivity).

Additional terms in the motion eqs:

$\rightarrow$ continuity eq remain unchanged (energy conservation)

$\rightarrow$ Euler eq (without gravitational field) writes ($\alpha = 1, 2, 3$):

$$\frac{\partial}{\partial t}(\rho v_\alpha) = -\frac{\partial \Pi_{\alpha\beta}}{\partial x_\beta}$$

$\Pi_{\alpha\beta}$: momentum flux energy tensor (ideal fluid $\Pi_{\alpha\beta} = p\delta_{\alpha\beta} + \rho v_\alpha v_\beta$)

$\rightarrow$ Viscosity: irreversible transfer of momentum

$$\Pi_{\alpha\beta} = p\delta_{\alpha\beta} + \rho v_\alpha v_\beta - \sigma'_{\alpha\beta} = -\sigma_{\alpha\beta} + \rho v_\alpha v_\beta \qquad \sigma_{\alpha\beta} = -p\delta_{\alpha\beta} + \sigma'_{\alpha\beta}$$

$\sigma_{\alpha\beta}$ is the *stress tensor* and $\sigma'_{\alpha\beta}$ is called the *viscous stress tensor*.

Motion equation of a viscous fluid

The general form of $\sigma'_{\alpha\beta}$ is [L.D. Landau and E.M. Lifshitz, *Fluid Mechanics*]

$$\sigma'_{\alpha\beta} = \eta \left(\frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} - \frac{2}{3}\,\delta_{\alpha\beta}\,\frac{\partial v_\gamma}{\partial x_\gamma} \right) + \zeta\,\delta_{\alpha\beta}\,\frac{\partial v_\gamma}{\partial x_\gamma}$$

where the coefficients η e ζ are not dependent of velocity (fluid isotropy needs only scalar quantities)

$\rightarrow$ The term proportional to η coefficient vanish for α and β contraction

$\rightarrow$ $\eta > 0$: *shear viscosity* $\zeta > 0$: *bulk viscosity*

Using the continuity eq, ideal Euler eq rewrites:

$$\rho \left(\frac{\partial v_\alpha}{\partial t} + v_\beta\,\frac{\partial v_\alpha}{\partial x_\beta} \right) = -\frac{\partial p}{\partial x_\alpha}$$

The motion eq of a viscous fluids can now be obtained by adding the expression $\partial\sigma'_{\alpha\beta}/\partial x_\beta$ to the right hand side of the eq above

Thus we obtain

$$\rho\left(\frac{\partial v_\alpha}{\partial t}+v_\beta\,\frac{\partial v_\alpha}{\partial x_\beta}\right)=-\frac{\partial p}{\partial x_\alpha}+\frac{\partial}{\partial x_\beta}\left[\eta\left(\frac{\partial v_\alpha}{\partial x_\beta}+\frac{\partial v_\beta}{\partial x_\alpha}-\frac{2}{3}\,\delta_{\alpha\beta}\frac{\partial v_\gamma}{\partial x_\gamma}\right)\right]+\frac{\partial}{\partial x_\alpha}\left(\zeta\,\frac{\partial v_\gamma}{\partial x_\gamma}\right)$$

- Homogeneous model: no internal friction between different fluid layers
  $\longrightarrow$ *no shear viscosity*
- Bulk Viscosity ζ can be expressed in terms of *thermodynamical parameters* of the fluid. In the homogeneous models, this quantity ***depends only on time***, and therefore we may consider it as a function of the Universe energy density ρ:

$$\boxed{\zeta=\zeta_o\,\rho^{\,s}}\qquad\qquad s=const$$

ζ_o parameter defines the intensity of viscous effects

[V.A. Belinskii and I.M. Khalatnikov, *Sov. Phys. JETP*, **42** (1976) - **45** (1977)]

With these considerations, the Euler eq assumes the following form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{\nabla p}{\rho} - \frac{\zeta}{\rho} \nabla(\nabla \cdot \mathbf{v}) = -\nabla \phi$$

which is the well-known Navier-Stokes Equation in presence of an external gravitational potential ϕ.

Analysis of the Jeans Mechanism

Universe fragmentation on small scales: gravitational instability

Density perturbation $\longrightarrow$ gravitational contraction of volume $\longrightarrow$ instability.

- Pressure forces: contrast the gravitational contraction
  $\longrightarrow$ maintain uniform the energy density

What are the conditions for which perturbations become unstable?

Jeans Model: small fluctuation on a static homogeneous and isotropic fluid.
Newtonian approach (no Universe expansion)
$\longrightarrow$ Density perturbations: exponential collapse or pure oscillation regime

"Jeans swindle": a uniform and static solution of the system is supposed

$$\mathbf{v} = 0 \qquad \rho = cost \qquad p = cost \qquad \phi = cost$$

- We impose that **bulk viscosity** does not affect the zeroth order dynamics

$$\rho,\, p,\, \phi\, = cost \qquad \mathbf{v} = 0$$

Homogeneous matter: viscous fluid motion equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{\nabla p}{\rho} + \nabla \phi - \frac{\zeta}{\rho} \nabla(\nabla \cdot \mathbf{v}) = 0$$

$$\nabla^2 \phi = 4\pi G \rho$$

→ This system is the starting point to analyze the evolution of the density perturbations and the gravitational instability

Perturbation theory adding small fluctuations

$\rightarrow \quad \rho + \delta\rho \quad p + \delta p \quad \phi + \delta\phi \quad \mathbf{v} + \delta\mathbf{v}$

- Bulk viscosity perturbations $\zeta = \zeta_o \, \rho^{\,s}$:

$$\zeta \rightarrow \zeta + \delta\zeta : \qquad \zeta = \zeta(\rho) = const. \qquad \delta\zeta = \delta\rho\,(\partial\zeta/\partial\rho) + ... = \zeta_o\, s\, \rho^{\,s-1}\,\delta\rho\, + ...$$

Motion eqs for first-order perturbations:

$$\frac{\partial\delta\rho}{\partial t} + \rho\nabla\cdot\delta\mathbf{v} = 0$$

$$\frac{\partial\delta\mathbf{v}}{\partial t} + \frac{v_s^2}{\rho}\nabla\delta\rho + \nabla\delta\phi - \frac{\zeta}{\rho}\nabla(\nabla\cdot\delta\mathbf{v}) = 0$$

$$\nabla^2\delta\phi = 4\pi G\delta\rho$$

where the adiabatic sound speed is defined as $\quad v_s^2 = \frac{\delta p}{\delta\rho}$

With some little algebra we obtain an unique eq for **density perturbations**

$$\frac{\partial^2}{\partial t^2}\,\delta\rho - v_s^2\nabla^2\,\delta\rho - \frac{\zeta}{\rho}\frac{\partial}{\partial t}\nabla^2\,\delta\rho = 4\pi G\rho\,\delta\rho$$

- Plane waves solutions: linearity of eq and decomposition in Fourier expansion

$$\delta\rho\,(\mathbf{r},t) = Ae^{i\omega t - i\mathbf{k}\cdot\mathbf{r}}$$

Substituting this expression in the eq above we obtain the dispersion relation for the angular frequency ω and the wave number $k = \mid \boldsymbol{k} \mid$

$$\boxed{\omega^2 - i\,\frac{\zeta\,k^2}{\rho}\,\omega + (4\pi G\rho - v_s^2 k^2) = 0}$$

The nature of ω is responsible of two regimes for $\delta\rho$:

- exponential $\longrightarrow$ collapse
- oscillatory $\longrightarrow$ no structure formation

The dispersion relation has the solution

$$\omega = i\,\frac{\zeta k^2}{2\,\rho} \pm \sqrt{-\frac{k^4\zeta^2}{4\rho^2} + v_s^2 k^2 - 4\pi G\rho}$$

Behavior of ω

$$\bar{\omega} \;=\; -\frac{k^4\zeta^2}{4\rho^2} + v_s^2 k^2 - 4\pi G\rho \qquad \begin{array}{lll} \bar{\omega} \leqslant 0 & \omega = iz & \Rightarrow\; \delta\rho \sim e^{-zt} \\ \bar{\omega} > 0 & \omega = x + iy & \Rightarrow\; \delta\rho \sim e^{-yt}\,\cos x \end{array}$$

- The pure oscillatory regime of the ideal fluid Jeans Model is lost

The solutions of the eq $\bar{\omega} = 0$ are

$$K_1 = \sqrt{2}\left(1 - \sqrt{1 - K_J^2\,\bar{\zeta}^2}\right)^{\frac{1}{2}} / \,\bar{\zeta} \qquad K_2 = \sqrt{2}\left(1 + \sqrt{1 - K_J^2\,\bar{\zeta}^2}\right)^{\frac{1}{2}} / \,\bar{\zeta}$$

$$\text{where :} \quad \bar{\zeta} = \zeta \,/\, \rho v_s \qquad K_J = \sqrt{\frac{4\pi G\rho}{v_s^2}}$$

- The relations $K_1 > 0,\ K_2 > 0,\ K_1 < K_2$ holds.
- *Constraint on the viscosity coefficient*: $(1 - K_J^2\lambda^2) \geqslant 0 \ \rightarrow \ \zeta \leqslant \sqrt{\frac{v_s^4\rho}{4\pi G}} = \frac{\rho v_s}{K_J} = \zeta_c$

  Estimation in the recombination era, after decoupling

$$p = \frac{c^2}{\rho_0^{\gamma-1}}\,\rho^\gamma \qquad M_J \sim 10^6 M_\odot \qquad \gamma = 5/3 \qquad \rho_c = 1.879\,h^2 \cdot 10^{-29}\ g\,cm^{-3} \qquad h = 0.7$$

$$z = 10^3 \quad \rho = \rho_c\ z^3 = 0.92 \cdot 10^{-20}\ g\,cm^{-3} \quad \rho_0 = 9.034 \cdot 10^{-7}\ g\,cm^{-3} \quad v_s = 8.39 \cdot 10^5\ cm\,s^{-1}$$

$$\zeta_c = 7.38 \cdot 10^4\ g\,cm^{-1}\,s^{-1}$$

confronting this threshold value with usual viscosity (*e.g.* $Hydr. = 8.4 \cdot 10^{-7} g\,cm^{-1}\,s^{-1}$) we can conclude that the range $\zeta \leqslant \zeta_c$ is the only of physical interest

Finally we obtain: $\bar{\omega} \leqslant 0: \quad k \leqslant K_1 \;\; \mathrm{e} \;\; K_2 \leqslant k \qquad \bar{\omega} > 0: \quad K_1 < k < K_2$

We study now the $\delta\rho$ exponential solution for $\bar{\omega} \leqslant 0$

$$\delta\rho = A e^{-ik\cdot x}\, e^{w\, t} \qquad\qquad w = -\frac{\zeta k^2}{2\rho} \mp \sqrt{\frac{k^4\zeta^2}{4\rho^2} - v_s^2 k^2 + 4\pi G\rho}$$

Structure formation: exponential collapse

$$w > 0 \qquad \text{iff} \qquad \boxed{k < K_J = \sqrt{\frac{4\pi G\rho}{v_s^2}}} \qquad\qquad K_J < K_1 < K_2$$

As result we show how the structure formation occurs if

$$M > M_J = \lambda_J^3 \rho = \left(\frac{2\pi}{K_J}\right)^3 \rho = \pi^{\frac{3}{2}} \frac{v_s^3}{\sqrt{G^3 \rho}}$$

$\longrightarrow$ <u>the viscous effects do not alter the threshold value of the Jeans Mass</u> but change the behavior of the perturbation: no pure oscillatory regime

We can reassume the behavior of the density perturbations:

$k < K_J$	$\delta\rho \sim e^{wt}$
$K_J < k < K_1$	$\delta\rho \sim e^{-wt}$
$K_1 < k < K_2$	$\delta\rho \sim e^{-\frac{\zeta k^2}{2\rho} t} a(\cos w_0 t + \alpha)$
$K_2 < k$	$\delta\rho \sim e^{-wt}$

where $\quad w > 0, \quad w_0 = \pm\sqrt{-\frac{k^4 \zeta^2}{4\rho} + v_s^2 k^2 - 4\pi G\rho}$

Pure Jeans Model : non viscous limit $\zeta = 0$.
In this picture we have $\bar{\zeta} \to 0$: $\quad K_1,\ K_2 \to \infty$ and

$$w \to \sqrt{-v_s^2 k^2 + 4\pi G\rho}$$

which results to be real in correspondence of the Jeans range $k < K_J$

- If $k > K_J$, $\delta\rho$ behave like two progressive sound waves, of constant amplitude, propagate in the directions $\pm\mathbf{k}$ with velocity $c_s = v_s\sqrt{1-(\lambda/\lambda_J)^2}$

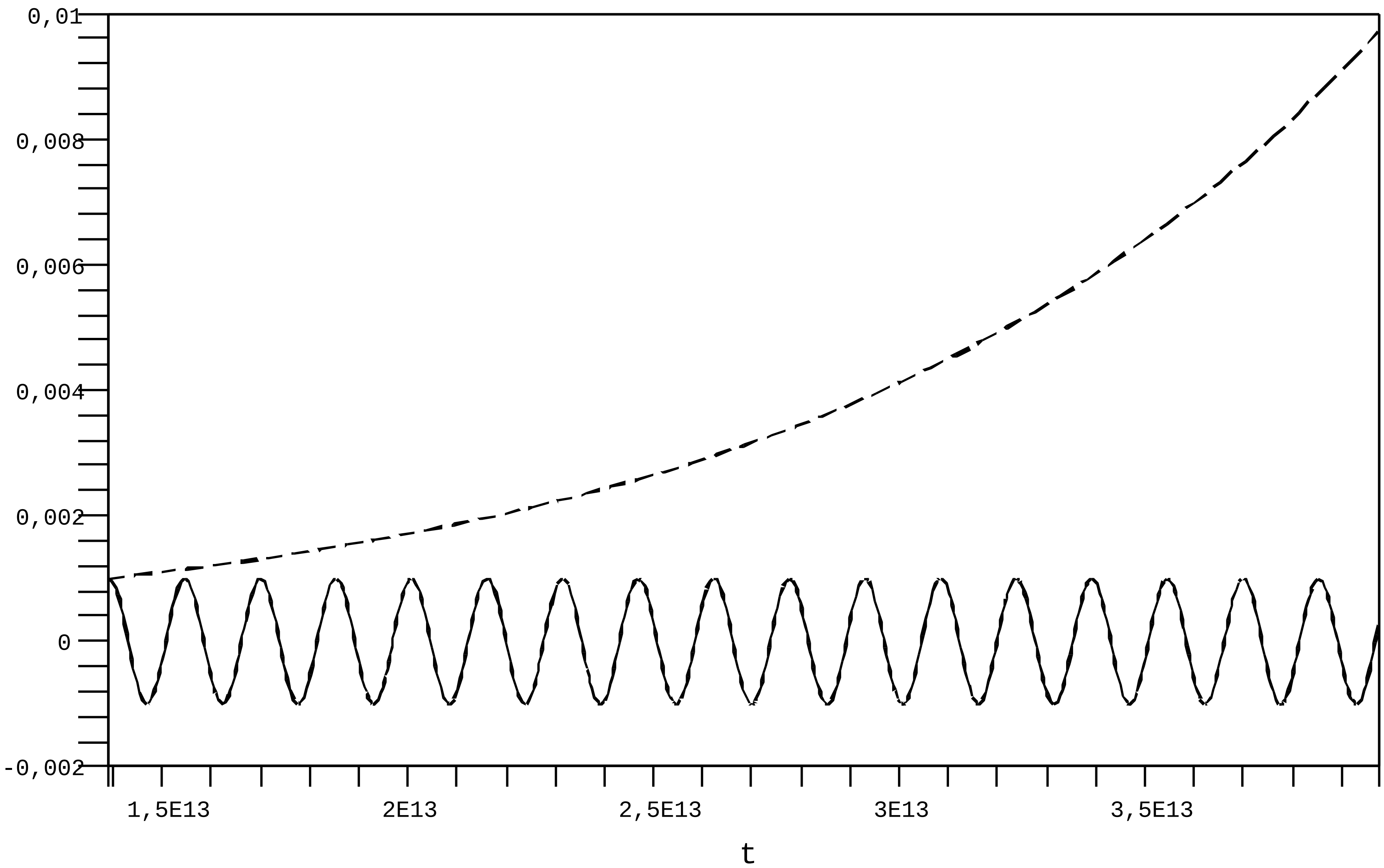
$\rightarrow$ $\lambda \to 0$: $\quad c_s \sim v_s \quad$ (pure sound waves)
$\rightarrow$ $\lambda \to \lambda_J \quad c_s \sim 0 \quad$ (stationary waves)

No pure oscillatory regime $\rightarrow$ **decreasing exponential or dumped oscillation**

Qualitative analysis of the ***top-down scheme***: comparison between two evolutions.

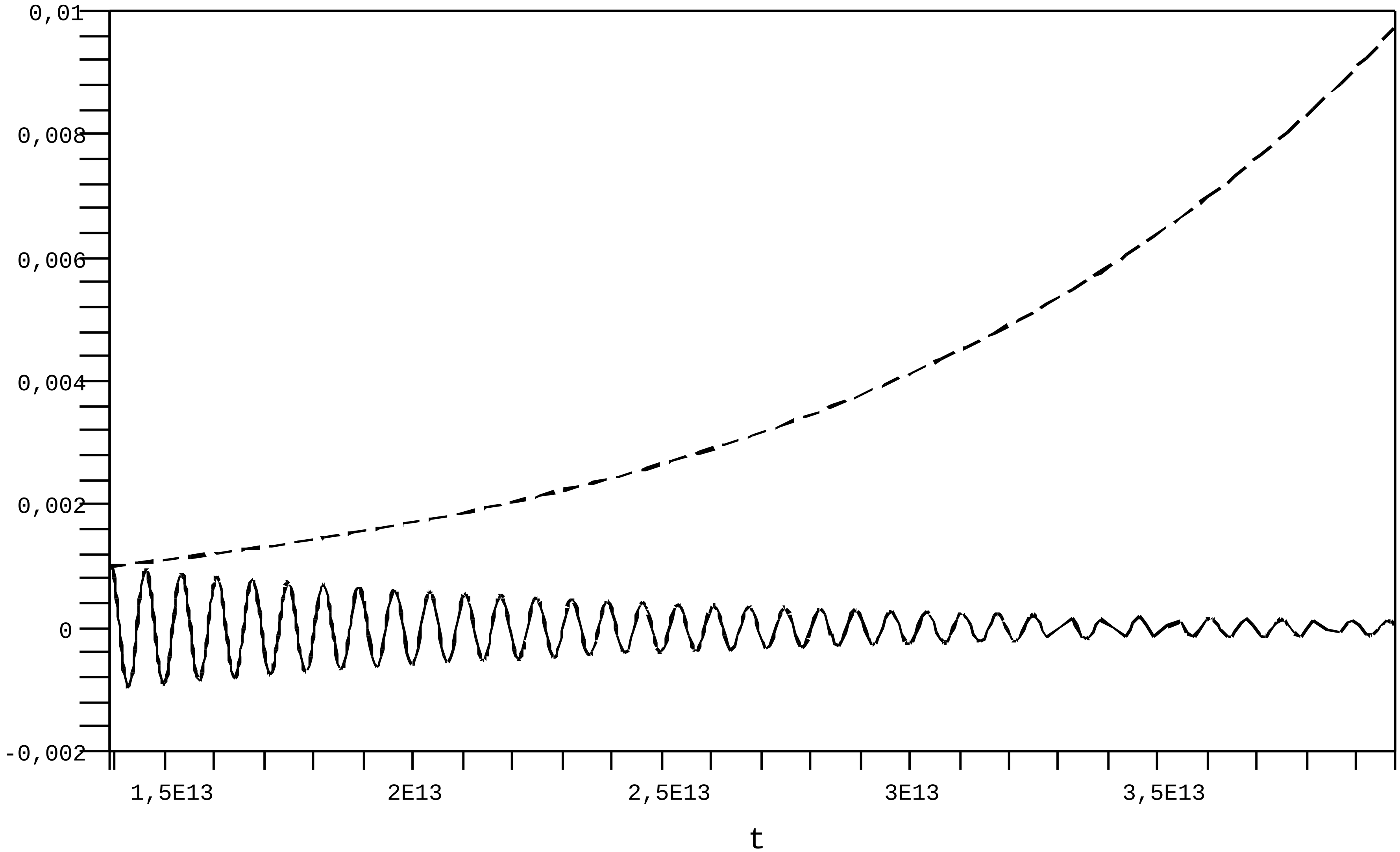
$\rightarrow$ collapsing agglomerate with $M \gg M_J$
$\rightarrow$ internal sub structure with $M < M_J$

Perturbation validity limit: $\delta\rho/\rho \sim 0.1$ (recombination era parameters, no expansion: $\rho = const$)



Galaxy δ_G $\rightarrow$ $M_G = 10^{12}\, M_\odot$ (dashed line) $\zeta = 0.00001\, g\, cm^{-1}\, s^{-1}$

Star δ_S $\rightarrow$ $M_S = 10\, M_\odot$ (normal line) : it does not collapse but it survives



Galaxy $\delta_G \rightarrow \quad M_G = 10^{12}\, M_{\odot}$ (dashed line) $\qquad \zeta = 14\, g\, cm^{-1}\, s^{-1}$

Star $\quad \delta_S \rightarrow \quad M_S = 1\, M_{\odot}$ (normal line) : it vanish **Top-down mech. suppressed**

Expanding Universe Generalization

We here calculate the behavior of small fluctuations, using Newtonian eqs, but now taking into account the Universe expansion.

No "Jeans swindle" : zeroth-order solutions are derived by the motion eqs of the isotropic and homogeneous Universe - no static and constant solution

- Matter-dominated era: $p \ll \rho$
  very small energy density $\rightarrow$ no bulk viscosity in the unperturbed dynamics

$$\zeta = \zeta_o \, \rho^s \qquad\qquad s > 0$$

$\rightarrow$ We can safely employ Newtonian mechanic (perturbation theory) to deal with astronomical problems in which the energy density is dominated by non-relativistic particles and in which the *linear scales involved are small compared with the characteristic scale of the Universe*

Zeroth-order dynamics: **Friedmann Eqs** for homogeneous and isotropic Universe

$$3\ddot{a} = -4\pi G(\rho + 3p)\, a \qquad a\ddot{a} + 2\dot{a}^2 + 2\mathcal{K} = 4\pi G(\rho - p)\, a^2 \qquad \boxed{\dot{a}^2 + \mathcal{K} = \frac{8\pi G}{3}\rho\, a^2}$$

where $a(t)$ is the *scale factor* and $\mathcal{K}$ is the curvature constant

$\longrightarrow$ Matter-dominated era EoS: $p \ll \rho$

$\longrightarrow$ Background solutions:

$$\rho = \rho_0\left(\frac{a_0^3}{a^3}\right) \qquad \mathbf{v} = \mathbf{r}\,\frac{\dot{a}}{a} \qquad \nabla\phi = \mathbf{r}\,\frac{4\pi G\rho}{3}$$

(The Euler eq is verified since the Friedmann eqs hold)

We recall the Newtonian motion eqs: the starting point for the perturbation theory

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{\nabla p}{\rho} + \nabla \phi - \frac{\zeta}{\rho} \nabla(\nabla \cdot \mathbf{v}) = 0$$

$$\nabla^2 \phi = 4\pi G \rho$$

We add small fluctuations

$$\rightarrow \quad \rho + \delta\rho \quad p + \delta p \quad \phi + \delta\phi \quad \mathbf{v} + \delta\mathbf{v}$$

and with the following relations

$$\dot{\rho} = -\dot{\rho}\, 3\, \frac{\dot{a}}{a} \qquad \nabla \cdot \mathbf{v} = 3 \frac{\dot{a}}{a} \qquad \nabla^2 \mathbf{v} = 0 \qquad \dot{\mathbf{v}} = \frac{\mathbf{r}}{a^2}(a\, \ddot{a} - \dot{a}^2)$$

we get the perturbed system - *neglecting second order terms* $(\delta p = v_s^2\ \delta\rho)$

$$\frac{\partial}{\partial t}\,\delta\rho + 3\,\frac{\dot a}{a}\,\delta\rho + \frac{\dot a}{a}\,(\mathbf{r}\cdot\nabla)\delta\rho + \rho\,\nabla\cdot\delta\mathbf{v} = 0$$

$$\frac{\partial}{\partial t}\,\delta\mathbf{v} + \frac{\dot a}{a}\,\delta\mathbf{v} + \frac{\dot a}{a}\,(\mathbf{r}\cdot\nabla)\delta\mathbf{v} + \frac{v_s^2}{\rho}\nabla\,\delta\rho + \nabla\,\delta\phi - \frac{\zeta}{\rho}\nabla\,(\nabla\cdot\delta\mathbf{v}) = 0$$

$$\nabla^2\delta\phi = 4\pi G\delta\rho$$

- The eqs above are spatially homogeneous
  $\rightarrow$ we expect to find plane waves solutions: $\delta\rho(\mathbf{r},t) \rightarrow \delta\rho(t)\,e^{\frac{i\,\mathbf{r}\cdot\mathbf{q}}{a(t)}}$
  (likewise for $\delta\mathbf{v}$ and $\delta\phi$)

$$\frac{\partial}{\partial t}\,\delta\rho + 3\,\frac{\dot a}{a}\,\delta\rho + \frac{i\,\rho}{a}\,(\mathbf{q}\cdot\delta\mathbf{v}) = 0 \qquad \text{where} \quad r \ll a\,,\ r/a \sim 0$$

$$\frac{\partial}{\partial t}\,\delta\mathbf{v} + \frac{\dot a}{a}\,\delta\mathbf{v} = -\frac{i\,v_s^2}{a\,\rho}\,\mathbf{q}\,\delta\rho + 4\pi i\,Ga\,\delta\rho\,\frac{\mathbf{q}}{q^2} - \frac{\zeta}{a^2\,\rho}\,\mathbf{q}\,(\mathbf{q}\cdot\delta\mathbf{v})$$

It is now convenient to decompose $\delta\mathbf{v}$ into parts normal and parallel to $\mathbf{q}$

$$\delta\mathbf{v} = \delta\mathbf{v}_\perp \; + \; i\mathbf{q}\,\epsilon \qquad \text{with} \qquad \mathbf{q}\cdot\delta\mathbf{v}_\perp = 0 \qquad \epsilon = -\tfrac{i}{q^2}(\mathbf{q}\cdot\delta\mathbf{v})$$

We finally get (setting $\delta\rho = \rho(t)\,\delta$)

$$\frac{\partial}{\partial t}\,\delta\mathbf{v}_\perp \; + \frac{\dot a}{a}\,\delta\mathbf{v}_\perp \; = 0 \qquad \dot\epsilon + \left(\frac{\dot a}{a} + \frac{\zeta\, q^2}{\rho\, a^2}\right)\epsilon = \left(\frac{4\pi G\rho a}{q^2} - \frac{v_s^2}{a}\right)\delta \qquad \dot\delta = \frac{q^2}{a}\,\epsilon$$

Rotational modes: governed by the first of the eqs above.

$\longrightarrow$ They are not affected by the presence of viscosity

$$\delta\mathbf{v}_\perp(t) \sim \; a^{-1}(t)$$

velocity perturbations normal to $\mathbf{q}$ decay as $1/a$ during the Universe expansion

Compressional Modes: governed by the equation

$$\ddot{\delta} + \left(2\frac{\dot{a}}{a} + \frac{\zeta q^2}{\rho\, a^2}\right)\dot{\delta} + \left(\frac{v_s^2\, q^2}{a^2} - 4\pi G\rho\right)\delta = 0$$

- Physical wave vector: $\mathbf{k} = \mathbf{q}/a$
- Assumption: $a(t) \ll a_0 \quad (\dot{a}^2\,, 8\pi\rho a^2/3 \gg 1)$ $\longrightarrow$ $\mathcal{K} = 0$ zero curvature solution

$$a \sim t^{\frac{2}{3}} \qquad \rho = \frac{1}{6\pi G t^2} \qquad p \sim \rho^\gamma \quad v_s = \left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}} \qquad \Rightarrow \qquad v_s \sim t^{1-\gamma}$$

- Using the relation $\zeta = \zeta_o \rho^s$ we obtain

$$\zeta = \bar{\zeta}_o\, t^{-2s} \qquad \bar{\zeta}_o = \zeta_o/(6\pi G)^s$$

The main equation rewrites now

$$\ddot{\delta} + \left[\frac{4}{3\,t} + \frac{\chi}{t^{2(s-1/3)}}\right]\dot{\delta} + \left[\frac{\Lambda^2}{t^{2\gamma-2/3}} - \frac{2}{3\,t^2}\right]\delta = 0$$

$\rightarrow$ where χ and Λ are two constants:

$$\chi = \frac{t^{2(s-1/3)}\,\zeta q^2}{\rho\,a^2} \qquad\qquad \Lambda^2 = \frac{t^{2\gamma-2/3}\,v_s^2 q^2}{a^2}$$

This eq can not be analytically solved in general $\rightarrow$ we set $s = 5/6$

$$\ddot{\delta} + \left[\frac{4}{3} + \chi\right]\frac{\dot{\delta}}{t} + \left[\frac{\Lambda^2}{t^{2\gamma-2/3}} - \frac{2}{3\,t^2}\right]\delta = 0$$

The solutions are:

$$\delta(t) = t^{-\frac{1}{6}-\frac{\chi}{2}} \left[C_1\, J_n\!\left(\frac{\Lambda t^{-\bar{\gamma}}}{\bar{\gamma}}\right) + C_2\, Y_n\!\left(\frac{\Lambda t^{-\bar{\gamma}}}{\bar{\gamma}}\right) \right]$$

where J e Y denotes the Bessel functions of first and second species respectively

$$n = -\sqrt{25 + 6\chi + 9\chi^2}\,/\,(6\,\bar{\gamma}) \qquad \bar{\gamma} = \gamma - \tfrac{4}{3}$$

- Bessel functions are power-laws or oscillate in the asymptotic limits:

$$J, Y_n(x) \sim {}^{\cos}_{\sin}(x) \quad x \gg 1 \qquad\qquad J, Y_n(x) \sim x^{\pm n} \quad x \ll 1$$

Threshold value of the argument: it change regime of the density contrast evolution

$$\frac{\Lambda\, t^{-\bar{\gamma}}}{\bar{\gamma}} < 1 \qquad \Rightarrow \qquad t < \frac{\Lambda^{1/\bar{\gamma}}}{\bar{\gamma}^{1/\bar{\gamma}}}$$

Expressing the parameters for an adiabatic Universe $\rightarrow$ $\gamma > 4/3$ we get

$$k < \bar{K}_J = \sqrt{\frac{6\pi G\rho}{\bar{\gamma}^2\, v_s^2}}$$

which is substantially the same as the **Jeans condition**: $K_J = \sqrt{\frac{4\pi G\rho}{v_s^2}}$

Solution will apply for a matter-dominated Universe after recombination $\rightarrow 4/3 < \gamma < 5/3$

Pure adiabatic case $\gamma = 5/3$ $n = \frac{1}{2}\sqrt{25 + 6\chi + 9\chi^2}$ $\bar{\gamma} = 1/3$

If $k < \bar{K}_J$ we get an exponential behavior of the Bessel functions:

$$\delta \sim t^{-1/6 - \chi/2 \mp n/3}$$

and the condition $n/3 > 1/6 + \chi/2$ is satisfied $\forall \chi \rightarrow$ **gravitational collapse**

- Non-viscous limit: $\delta \sim t^{2/3}$ $\chi = 0$ for $k < \bar{K}_J$

The viscous effects are summarized by the constant χ:

$\rightarrow$ **Damping of the density contrast evolution**

$\rightarrow$ The threshold Jeans mass can be addressed also in presence of viscosity

Conclusions

- The effects that viscosity induced on the motion equation in Newtonian approximation are briefly summarized. In particular the Navier-Stokes equation is derived.
- A generalization of the Jeans mechanism for the structure formation is addressed in presence of bulk viscosity. The threshold value of the Jeans mass is still obtained the viscosity is found to affect the behavior of the density contrast. In particular, dissipative effects induce a *dumping* of the perturbation growth suppressing the top-down structure formation as far as they increase.
- If the expansion of the Universe is taken into account similar results are obtained and the concept of the Jeans length still remains valid.