

Extended bodies in General Relativity (applications to BH spacetimes)

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Collaborations

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Open problems (motivations for work)

Do we have a complete description of the motion of extended bodies in general relativity?

Is the description of motion the same for both macroscopic and microscopic particles?

Do we learn something fundamental from the study of the inner structure of an extended body?

For instance, from the fact that certain “couplings” between the parameters of the body and those of the background are preferred with respect to others?

Not enough data available → not a definite answer!

Introduction to the model and a brief review of results obtained

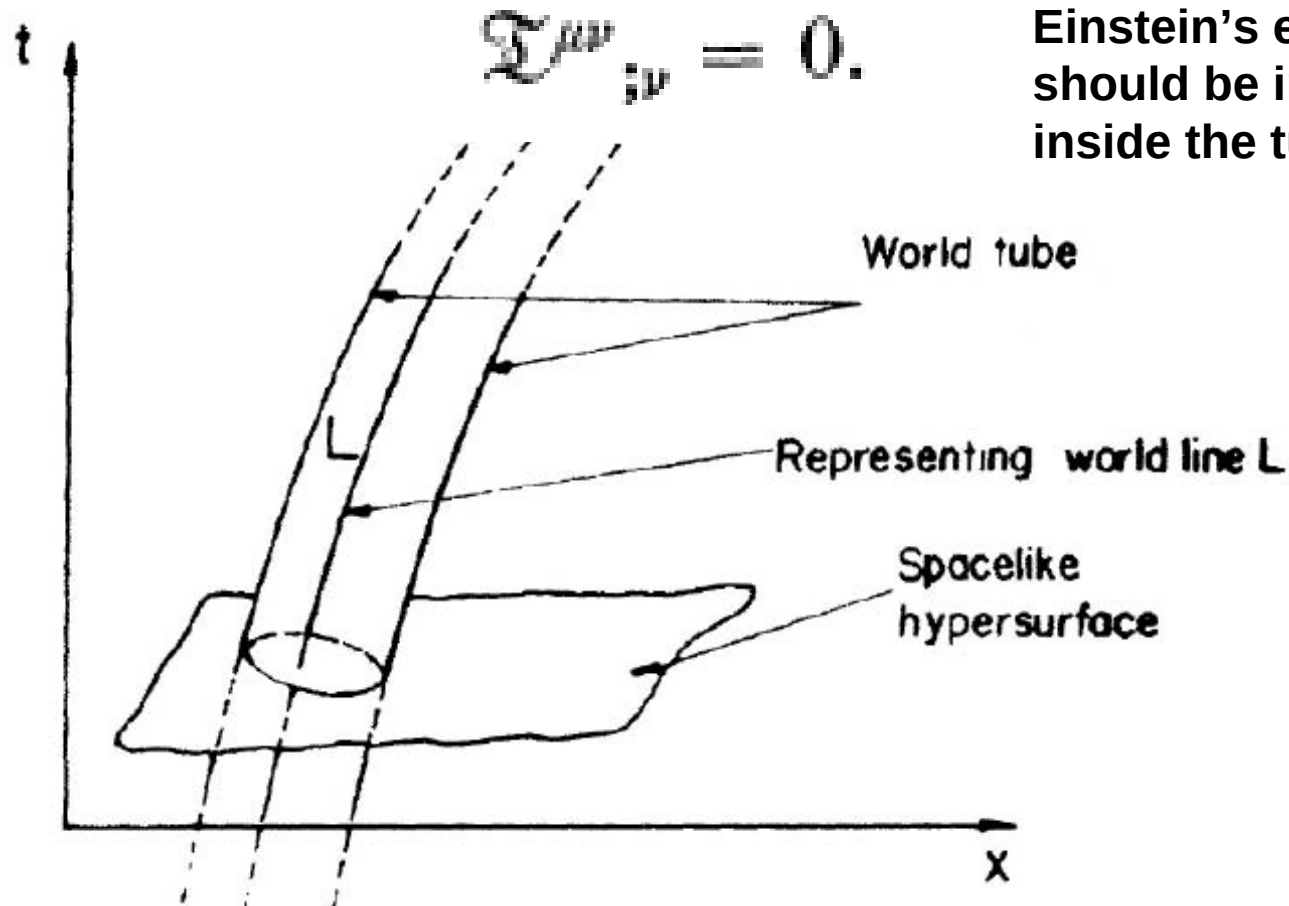
GR model for a test particle

A test particle in the spacetime is represented by a single geodesic world line, and a scalar quantity constant along it (a sort of “label” for the particle itself : the mass).

GR model for an extended test body

A test body in the spacetime is represented by a world line (roughly, the center of mass orbit) and a pair (or more) of tensors defined (and evolving) all along the CM world line (roughly, the way in which the body is allowed to move along the orbit due to its internal structure)

Extended body in the spacetime



$$\int T^{\mu\nu} dv, \quad \int \delta x^\rho T^{\mu\nu} dv, \quad \int \delta x^\rho \delta x^\sigma T^{\mu\nu} dv, \quad \dots,$$

Multipolar approximation

Competing models

Mathisson-Papapetrou model

Dixon model

Variations of both

Purely spinning particles (Mathisson-Papapetrou incomplete model)

$$\frac{DP^\mu}{d\tau_U} = -\frac{1}{2}R^\mu{}_{\nu\alpha\beta}U^\nu S^{\alpha\beta}$$

4 + 6 equations

$$\frac{DS^{\mu\nu}}{d\tau_U} = 2P^{[\mu}U^{\nu]}$$

U=timelike unit tangent vector
to the CM world line

P=generalized momentum of the
particle (evolving along U)

but

S=spin tensor (evolving along U)

4(P)+6(S)+3(U)=13 unknowns!

Something is missing here: model incomplete or incorrect?

Supplementary conditions: a mathematical tool

1. Corinaldesi–Papapetrou conditions (CP): $S^{tv} = 0$,
2. Pirani conditions (P): $S^{\mu\nu} U_\nu = 0$,
3. Tulczyjew conditions (T): $S^{\mu\nu} P_\nu = 0$.

Mathisson M 1937 *Acta Phys. Pol.* **6** 167

Papapetrou A 1951 *Proc. R. Soc.* **209** 248

Corinaldesi E and Papapetrou A 1951 *Proc. R. Soc.* **209** 259

Pirani F 1956 *Acta Phys. Pol.* **15** 389

Tulczyjew W 1959 *Acta Phys. Pol.* **18** 393

Physically correct supplementary conditions

Tulczyjew conditions (T): $S^{\mu\nu} P_\nu = 0$.

Particles with both dipolar and quadrupolar structure: Dixon's model

Fundamental equations:

$$\frac{DP^\mu}{d\tau_U} = -\frac{1}{2}R^\mu{}_{\nu\alpha\beta}U^\nu S^{\alpha\beta} - \frac{1}{6}J^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta};{}^\mu \equiv F^{(\text{spin})\mu} + F^{(\text{quad})\mu}$$

$$\frac{DS^{\mu\nu}}{d\tau_U} = 2P^{[\mu}U^{\nu]} - \frac{4}{3}J^{\alpha\beta\gamma[\mu}R^{\nu]}{}_{\alpha\beta\gamma} ,$$

$$P^\mu = mU_p^\mu \text{ (with } U_p \cdot U_p = -1)$$

U Unit timelike tangent vector to the center of mass line of the body

$J^{\alpha\beta\gamma\delta}$ Quadrupole moment tensor

The quadrupole moment tensor

The tensor J is the quadrupole moment of the stress-energy tensor of the body, and has the same algebraic symmetries as the Riemann tensor. A “1+3” splitting gives

$$J^{\alpha\beta\gamma\delta} = \Pi^{\alpha\beta\gamma\delta} - \bar{u}^{[\alpha} \pi^{\beta]\gamma\delta} - \bar{u}^{[\gamma} \pi^{\delta]\alpha\beta} - 3\bar{u}^{[\alpha} Q^{\beta][\gamma} \bar{u}^{\delta]}$$

$$\Pi^{\alpha\beta\gamma\delta} = \Pi^{[\alpha\beta][\gamma\delta]}$$

$$\pi^{\alpha\beta\gamma} = \pi^{\alpha[\beta\gamma]} \quad \pi^{[\alpha\beta\gamma]} = 0$$

$$Q^{\alpha\beta} = Q^{(\alpha\beta)}$$

Algebraic properties of J

The number of independent components of J is 20:

6 in $Q^{\alpha\beta}$

6 in $\Pi^{\alpha\beta\gamma\delta}$

and 8 in $\pi^{\alpha\beta\gamma}$

Physical meaning of the splitting fields

$$\Pi^{\alpha\beta\gamma\delta}$$

represents the stress of quadrupole moment

$$\pi^{\alpha\beta\gamma}$$

represents the linear momentum of quadrupole momentum

$$Q^{\alpha\beta}$$

represents the mass of quadrupole moment

Particles with both dipolar and quadrupolar structure: the Papapetrou model

$$\frac{DP^\mu}{d\tau_U} = -\frac{1}{2}R^\mu{}_{\nu\alpha\beta}U^\nu S^{\alpha\beta} \quad + \text{quadrupolar terms}$$

$$\frac{DS^{\mu\nu}}{d\tau_U} = 2P^{[\mu}U^{\nu]} \quad + \text{quadrupolar terms}$$

$$\frac{D}{d\tau_U} \text{quadrupolar terms} = \quad + \text{spin} + \text{quadrupolar terms}$$

Equations of motion for bodies with structure up to the quadrupole never written explicitly up to now!

Dixon's vs Papapetrou model

The fundamental difference between the two models is that Dixon's one does not include any evolution equation for the quadrupole tensor.

In Dixon's model the quadrupole moment of the body should be considered as given and representative of the material structure of the body itself.

Mathematical correctness of Dixon's model

In order Dixon's model to be mathematically correct the following additional conditions should be imposed to the spin tensor:

$$S^{\mu\nu}U_{p\nu} = 0$$

Similar conditions constrain also all the spatial tensor fields associated with the quadrupole tensor.

In any case, backreaction should always be negligible!

Ehlers approximations for the quadrupole tensor

The only contribution to the complete quadrupole moment J stems from the mass quadrupole moment Q , that is the only term which survives in the classical limit

$$\pi^{\alpha\beta\gamma} = 0 = \Pi^{\alpha\beta\gamma\delta}$$

$$J^{\alpha\beta\gamma\delta} = -3U_p^{[\alpha}Q^{\beta][\gamma}U_p^{\delta]} , \quad Q^{\alpha\beta}U_{p\beta} = 0$$

Spin tensor $\rightarrow$ spin vector

It is convenient to introduce the spin vector by spatial (with respect to U_p) duality:

$$S^\beta = \frac{1}{2} \eta_\alpha^{\beta\gamma\delta} U_p^\alpha S_{\gamma\delta}$$

$$\eta_{\alpha\beta\gamma\delta} = \sqrt{-g} \epsilon_{\alpha\beta\gamma\delta}$$

A typical length scale is represented then by the scalar

$$s^2 = \frac{1}{2} S_{\mu\nu} S^{\mu\nu}$$

Motion of extended bodies in Schwarzschild spacetime: an example

Consider the case of a Schwarzschild
spacetime

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

Introduce an orthonormal frame:

$$e_{\hat{t}} = (1 - 2M/r)^{-1/2} \partial_t, \quad e_{\hat{r}} = (1 - 2M/r)^{1/2} \partial_r, \quad e_{\hat{\theta}} = \frac{1}{r} \partial_\theta, \quad e_{\hat{\phi}} = \frac{1}{r \sin \theta} \partial_\phi$$

Simplifying assumptions

- Assume that U is tangent to a (timelike) spatially circular orbit.
- Limit considerations to equatorial plane motion.
- Assume that U_p also is tangent to a circular orbit.

More formally...but...
without the details

$$U = \Gamma[\partial_t + \zeta \partial_\phi] = \gamma[e_{\hat{t}} + \nu e_{\hat{\phi}}], \quad \gamma = (1 - \nu^2)^{-1/2}$$

$$\zeta = \sqrt{-\frac{g_{tt}}{g_{\phi\phi}}} \nu$$

$$U_p = \gamma_p [e_{\hat{t}} + \nu_p e_{\hat{\phi}}] , \quad \gamma_p = (1 - \nu_p^2)^{-1/2}$$

$$\theta = \pi/2$$

$$P = m U_p$$

Notation

(for a later convenience)

Introduce co/counter rotating geodesics:

$$U_{\pm} = \gamma_K [e_{\hat{t}} \pm \nu_K e_{\hat{\phi}}] , \quad \nu_K = \left[\frac{M}{r - 2M} \right]^{1/2}$$

$$\zeta_{\pm} \equiv \pm \zeta_K = \pm (M/r^3)^{1/2}$$

Introduce also the Lie relative curvature of a circular orbit

$$k_{(\text{lie})} = -\partial_{\hat{r}} \ln \sqrt{g_{\phi\phi}} = -\frac{1}{r} \left(1 - \frac{2M}{r} \right)^{1/2} = -\frac{\zeta_K}{\nu_K}$$

Adapted frame

Introduce a frame adapted to U_p

$$e_0 = U_p, \quad e_1 = e_{\hat{r}}, \quad e_2 = \gamma_p [\nu_p e_{\hat{t}} + e_{\hat{\phi}}], \quad e_3 = e_{\hat{z}}$$

Within this frame we have automatically

$$S = S^1 e_1 + S^2 e_2 + S^3 e_3$$

$$Q_{00} = Q_{01} = Q_{02} = Q_{03} = 0$$

Additional assumptions

- Let us assume that the spin of the body is aligned with the z axis and **constant** along the path:
$$S^1 = 0 = S^2$$

Quasi-rigidity of the body according to Ehlers-Rudolph definition: the surviving components of the mass quadrupole moment are all constant along the path.

Write down the equations...

From the spin equations we have

$$Q_{12} = Q_{13} = Q_{23} = 0$$

Let us introduce the notation:

$$Q_{11} = Q_{33} + f, \quad Q_{22} = Q_{33} + f'$$

f, f' are the **structure functions of the body**

The spin equations reduce to a single equation

$$0 = (\nu\nu_p - \nu_K^2)s + m\frac{\nu_K}{\zeta_K}(\nu - \nu_p) + 3\nu_K\zeta_K\frac{\gamma_p\nu_p}{\gamma}f$$

Momentum equations

After manipulations the momentum equations reduce to a single equation too:

$$0 = (\nu\nu_p - \nu_K^2)(-f' + 2\gamma_p^2 f) + \gamma_p^2 [2\nu\nu_p\nu_K^2 + \nu_p^2(\nu\nu_p + \nu_K^2) + 2\nu_K^2\nu_p^2]f - \frac{2}{3} \frac{m\gamma\gamma_p}{\zeta_K^2} (\nu^2\nu_p^2 + 2\nu_p^2\nu_K^2 - 3\nu\nu_p\nu_K^2 - \nu^2\nu_K^2 + \nu_K^4) ,$$

Note that as soon as the dipolar (s) and quadrupolar (f,f') structure of the body is known the previous two equations are enough to determine the motion: $\nu \quad \nu_p$

TF property of the quadrupole moment: f, f' determine Q

As for the classical case we can assume that the quadrupole moment of the body is trace-free; in this case Q is completely represented by f, f' :

$$Q_{11} = \frac{2}{3}f - \frac{1}{3}f', \quad Q_{22} = -\frac{1}{3}f + \frac{2}{3}f', \quad Q_{33} = -\frac{1}{3}(f + f').$$

Rescaled adimensional quantities

It is quite natural to introduce the following rescaled dimensionless angular and quadrupolar momentum quantities

$$\sigma = \frac{s}{m} \zeta_K , \quad F = \frac{f}{m} \zeta_K^2 , \quad F' = \frac{f'}{m} \zeta_K^2$$

due to the fact that along a circular orbit $r = \text{const.}$

$$\zeta_{\pm} \equiv \pm \zeta_K = \pm (M/r^3)^{1/2}$$

Smallness

The quantities σ , F and F' are necessarily small.

Although the quadrupolar terms f and f' are small only for a quasi-spherical body, the further rescaling by ζ_K makes indeed them small in any case.

In fact, the radius of the orbit is assumed to be large enough in comparison with certain natural length scales like $|s|/m$ (also known as the Møller radius of the body):

$$(|f|/m)^{1/2}, (|f'|/m)^{1/2}$$

Final set of equations

$$0 = (\nu\nu_p - \nu_K^2)\sigma + \nu_K(\nu - \nu_p) + 3\nu_K \frac{\gamma_p \nu_p}{\gamma} F$$

$$0 = (\nu\nu_p - \nu_K^2)(-F' + 2\gamma_p^2 F) + \gamma_p^2 [2\nu\nu_p \nu_K^2 + \nu_p^2(\nu\nu_p + \nu_K^2) + 2\nu_K^2 \nu_p^2] F \\ - \frac{2}{3} \gamma \gamma_p (\nu^2 \nu_p^2 + 2\nu_p^2 \nu_K^2 - 3\nu\nu_p \nu_K^2 - \nu^2 \nu_K^2 + \nu_K^4) .$$

The above equations can be solved to obtain the velocities in terms of spin and quadrupole parameters. All the various special cases like

$$(i) \quad \underline{\sigma = 0, F \neq 0, F' \neq 0.}$$

can be examined.

Limit of small spin and quadrupole

If the contribution of quadrupolar terms can be considered negligible with respect to the dipolar ones and comparable with second order terms in the spin itself:

$$\nu_{\pm} \simeq \pm \nu_K - \frac{3}{2} \nu_K \sigma \pm \frac{3}{8} (2F + \sigma^2) \nu_K$$

$$\nu_p^{(\pm)} \simeq \nu_{\pm} \pm 3(F - \sigma^2) \nu_K$$

where the signs $\pm$ correspond to co/counter rotating orbits.

In terms of angular velocities...

$$\frac{1}{\zeta_{\pm}} \simeq \pm \frac{1}{\zeta_K} + \frac{3}{2\zeta_K}\sigma \pm \frac{3}{8\zeta_K}(5\sigma^2 - 2F)$$

One can then evaluate the period of revolution around the central source which consists of three different terms

$$T = \frac{2\pi}{|\zeta_{\pm}|} = T_K |1 \pm \lambda_d + \lambda_Q|$$

$$T_K = \frac{2\pi}{\zeta_K} , \quad \lambda_d = \frac{3}{2}\sigma , \quad \lambda_Q = \frac{3}{8}(5\sigma^2 - 2F) .$$

Consequences

A direct measurement of T will then allow to estimate the quantity F determining the quadrupolar structure of the body, if its spin is known.

Note that the fraction due to the spin is different depending on whether the body is spinning up or down, whereas the term Q due to the quadrupole has a definite sign once the shape of the body is known (F cannot change its sign).

Applications and “numbers” (case of Schwarzschild)

In the case of the Earth:

$$\sigma \approx 2.3 \times 10^{-15} \text{ and } F = F' \approx -1.8 \times 10^{-20}, \text{ since } s/m_{\oplus} \approx 3.4 \times 10^2 \text{ cm}$$

$$f = f' = -J_2 m_{\oplus} r_{\oplus}^2, \text{ with } J_2 \approx 10^{-3}$$

$$r \approx 1.5 \times 10^{13} \text{ cm (distance Earth-Sun)}$$

$$T_K \approx 9.425 \times 10^{17} \text{ cm}$$

$$\lambda_d \approx 3.4 \times 10^{-15}$$

$$\lambda_Q \approx 1.3 \times 10^{-20}$$

Applications and “numbers” (case of Kerr)

An interesting opportunity to test the quadrupole effect of an extended body would arise, for instance, from the motion of ordinary or neutron stars around Sgr A*, the supermassive ($M \simeq 10^6 M_\odot$) rotating ($a \in [0.5, 1]M$) black hole located at the galactic center [10, 14, 19].

To illustrate the order of magnitude of the effect, we may consider a binary pulsar system similar to PSR J0737-3039 as orbiting Sgr A* at a distance of $r \simeq 10^9$ km. The PSR J0737-3039 system consists of two close neutron stars (their separation is only $d_{AB} \sim 8 \times 10^5$ km) of comparable masses $m_A \simeq 1.4M_\odot$, $m_B \simeq 1.2M_\odot$), but very different intrinsic spin period (23 ms of pulsar A versus 2.8 s of pulsar B) [20]. Its orbital period is about 2.4 h, the smallest yet known for such an object. Since the intrinsic rotations are negligible with respect to the orbital period, we can treat the binary system as a single object with reduced mass $\mu_{AB} \simeq 0.7M_\odot$ and intrinsic rotation equal to the orbital period. The spin parameter thus turns out to be equal

Table 1. The estimates of geodesic period T_+^g as well as the corrections λ_d^+ and λ_q^+ due to both the dipolar and quadrupolar structures of the PSR J0737-3039 binary system are listed for different values of Sgr A* rotational parameter a/M . Note that in order to resolve the dipolar and quadrupolar effects the measured period should be known with very high accuracy.

a/M	T_+^g (cm)	λ_d^+	λ_q^+
0.52	$1.622\,36 \times 10^{16}$	8.83×10^{-8}	-7.05×10^{-10}
0.75	$1.622\,38 \times 10^{16}$	8.75×10^{-8}	-6.99×10^{-10}
0.983	$1.622\,40 \times 10^{16}$	8.66×10^{-8}	-6.92×10^{-10}
0.996	$1.622\,41 \times 10^{16}$	8.66×10^{-8}	-6.92×10^{-10}
1	$1.622\,41 \times 10^{16}$	8.66×10^{-8}	-6.92×10^{-10}

to $\sigma \approx 6 \times 10^{-8}$, whereas the quadrupolar parameters are $F = F' \approx 9.6 \times 10^{-10}$, since we have taken $f = f' = \mu_{AB} d_{AB}^2$ as a rough estimate.

Since the rotation parameter of Sgr A* is not small we must use the exact expression (2.30). In the literature one finds different estimates of the spin parameter of the galactic center black hole, ranging from $a \simeq 0.52M$ [21] all the way up to $a \simeq 0.983M$ [22, 23] or even $a \simeq 0.996M$ [24, 25]. We list in table 1 the corresponding values of the geodesic period T_+^g of the PSR J0737-3039 binary system as well as the corrections $\lambda_d^\pm$ and $\lambda_q^\pm$ due to its dipolar and quadrupolar structures respectively.

It turns out that no relevant differences arise for the selected values of the black hole spin parameter. In particular, the results are not sensitive to the black hole being either fast rotating [23] or near extreme [25] or even extreme ($a = M$). In the latter case equations (2.30) reduce to

$$\begin{aligned} T_{\pm}^g &= 2\pi \left(M \pm \frac{1}{\zeta_K} \right), \\ \lambda_d^{\pm} &= \pm \frac{3}{2} \sigma \Lambda^{\pm}, \\ \lambda_q^{\pm} &= -\frac{3}{4} \Lambda^{\pm} \left[\pm F(1 - 6r^3 \zeta_K^3 \Lambda^{\pm}) + 2\sigma^2 \left(\pm 1 - \frac{3}{4} \Lambda^{\pm} (2 \pm 1 - r^3 \zeta_K^3) \right) \right], \end{aligned} \quad (3.1)$$

where

$$\Lambda^{\pm} = \frac{1 - r \zeta_K}{1 + r^3 \zeta_K^3} \frac{1 \pm r^3 \zeta_K^3}{1 + r^3 \zeta_K^3}. \quad (3.2)$$

The detection of pulsars in Sgr A* is difficult because of the intense scattering region located in front of Sgr A*. However, these pulsars may be detectable in the next decade by the SKA detector, which promises high frequency sensitivity and large collecting area [13].

Another possible application of our calculations could be the orbital motions of the so-called S-stars [21], i.e. the massive ($(30 \div 120)M_{\odot}$), young (<10 Myr) stars within the influence of the supermassive black hole. However, in this case the orbits are no longer circular and the problem of discriminating quadrupole effects would deserve further investigation.

Related works

Extensions of this work have been already considered for different spacetimes (Kerr, GW) and releasing some of the stringent assumptions adopted here.

Quadrupole effects on the motion of extended bodies in Kerr spacetime

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Abstract. The motion of a body endowed with dipolar as well as quadrupolar structure is investigated in the Kerr background according to the Dixon's model, extending a previous analysis done in the Schwarzschild background. The full set of evolution equations is solved under the simplifying assumptions of constant frame components for both the spin and the quadrupole tensors and that the center of mass moves along an equatorial circular orbit, the total four-momentum of the body being aligned with it. We find that the motion deviates from the geodesic one due to the internal structure of the body, leading to measurable effects. Corrections to the geodesic value of the orbital period of a close binary system orbiting the Galactic Center are discussed assuming that the Galactic Center is a Kerr supermassive black hole.

Dixon's extended bodies and weak gravitational waves

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Abstract General relativity considers Dixon's theory as the standard theory to deal with the motion of extended bodies in a given gravitational background. We discuss here the features of the "reaction" of an extended body to the passage of a weak gravitational wave. We find that the body acquires a dipolar moment induced by its quadrupole structure. Furthermore, we derive the "world function" for the weak field limit of a gravitational wave background and use it to estimate the deviation between geodesics and the world lines of structured bodies. Measuring such deviations, due to the existence of cumulative effects, should be favorite with respect to measuring the amplitude of the gravitational wave itself.

3 Discussion and general results

In the expressions (30) we can distinguish a harmonic part and a secular one; the latter is given by

$$\begin{aligned}\omega x_{\text{sec}} &= -\frac{A_{\times}}{2}(2\sigma_0^2\sigma_0^3 + q_0^{23})\omega\tau_U, \\ \omega y_{\text{sec}} &= \frac{1}{4}\left[2A_{\times}(\sigma_0^1\sigma_0^3 + q_0^{13}) + A_{+}\sigma_0^3(2 + f_0)\right]\omega\tau_U, \\ \omega z_{\text{sec}} &= \frac{1}{4}\left[2A_{\times}(\sigma_0^1\sigma_0^2 + q_0^{12}) + A_{+}\sigma_0^2(2 - f_0)\right]\omega\tau_U,\end{aligned}\tag{31}$$

and it is important in view of “cumulative” effects which it originates.

Dixon's extended bodies and impulsive gravitational waves

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Abstract

The "reaction" of an extended body (spinning and endowed with a quadrupolar structure) to the passage of an exact plane gravitational wave is discussed following Dixon's model under the condition that backreaction effects can be neglected. The analysis performed evidences several general features, e.g. even if initially absent, the body acquires a spin induced by the quadrupole structure, the center of mass moves from its initial position. Furthermore, special situations may exist in which certain spin components undergo "spin-flip" or "spin-glitch" effects which can be eventually observed.

“Glitches” observed in pulsars

A number of interesting results have been evidenced. For instance, in general
a) even if initially absent, the body acquires a dipolar moment induced by the quadrupole tensor; b) the center of mass moves from its initial position and the projection of the orbit on the wave front is a straight line, whose inclination depends on the initial spin of the body; c) special situations may occur in which certain spin components change their magnitude leading to effects (e.g. spin-flip) which can be eventually observed.

This interesting feature recalls the phenomenon of glitches observed in pulsars: a sudden increase in the rotation frequency, often accompanied by an increase in slow-down rate (see e.g. [11–13] and references therein). Currently, only multiple glitches of the Crab and Vela pulsars have been observed and studied extensively. Larger glitches in younger pulsars are usually followed by an exponential recovery or relaxation back toward the pre-glitch frequency,

while for older pulsars and small glitches the jump tends to be permanent. The physical mechanism triggering glitches is not well understood yet, even if these are commonly thought to be caused by internal processes.

If one models a pulsar by a Dixon's extended body, then the present analysis shows that a sort of glitch can be generated by the passage of a strong gravitational wave, due to the pulsar quadrupole structure. In fact, from Eq. (27) we see that the profile of a polarization function can be suitably selected in order to fit observed glitches and in particular to describe the post-glitch behavior.

Work in progress

Structured body of astrophysical interest (pulsars) modeled in terms of a similar analysis are still under consideration.

A constellation of satellites to be approximated as a single extended body with quadrupolar structure is under consideration too, in view of possible applications of all this formalism to GW detectors like LISA.

References for “spinning” papers:

Bini D., de Felice F., Geralico A., Lunari A.,
Spinning test particles in Weyl spacetimes,
J. Phys. A: Math. Gen. 38, 1163-1186, 2005.

Bini D., Cherubini C., Geralico A., Mashhoon B.,
Spinning particles in the vacuum C metric,
Classical and Quantum Gravity, vol. 22, 709-722, 2005.

Bini D., de Felice F., Geralico A.
Charged spinning particles on circular orbits in the Reissner-Nordström spacetime,
International Journal of Modern Physics D, vol. 14, 1793-1811, 2005.

Bini D., de Felice F., Geralico A. and Jantzen R.T.
Spin precession in the Schwarzschild spacetime: circular orbits,
Classical and Quantum Gravity, vol. 22, 2947-2970, 2005.

References for “spinning” papers:II

Bini D., Cherubini C., Geralico A., Jantzen R.T.

Massless spinning test particles in vacuum algebraically special spacetimes,

International Journal of Modern Physics D, vol. 15, 737-758, 2006.

Bini D., de Felice F., Geralico A., Jantzen R.T.

Spin precession along circular orbits in the Kerr spacetime: the Frenet-Serret description,

Classical and Quantum Gravity, vol. 23, 3287-3304, 2006.

Bini D., Lusanna L.

Spin-rotation couplings: spinning test particles and Dirac field

Gen. Rel. and Gravit., DOI 10.1007/s10714-007-0594-0, 2008.

“Quadrupole” papers

Bini D., Fortini, F., Geralico, A., Ortolan, A.

Quadrupole effects on the motion of extended bodies in Schwarzschild spacetime,

Class. and Quantum Gravit., vol. 25, 035005, 2008.

Bini D., Fortini, F., Geralico, A., Ortolan, A.

Quadrupole effects on the motion of extended bodies in Kerr spacetime,

Class. and Quantum Gravit., vol. 25, 125007, 2008.

Bini D., Cherubini, C., Geralico, A., Ortolan, A.

Dixon's extended bodies and weak gravitational waves,

GRG, to appear 2008.

Bini D., Fortini, F., Geralico, A., Ortolan, A.

Dixon's extended bodies and impulsive gravitational waves,

submitted, 2008.

Spinning test particles and clock effect in Schwarzschild spacetime

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spin

Spinning test particles and clock effect in Kerr spacetime

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spin

Spinning particles in the vacuum C metric

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spin

Spin precession in the Schwarzschild spacetime: circular orbits

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spin

Spin precession along circular orbits in the Kerr spacetime: the Frenet–Serret description

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spin

CHARGED SPINNING PARTICLES ON CIRCULAR ORBITS IN THE REISSNER–NORDSTRÖM SPACE–TIME

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spin and charge

Quadrupole effects on the motion of extended bodies in Schwarzschild spacetime

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Quadrupole