Plasminos –
collective excitations of relativistic fermions

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Stückelberg Workshop

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H-QM Helmholtz Research School Quark Matter Studies

QCD – Quantum Chromodynamics

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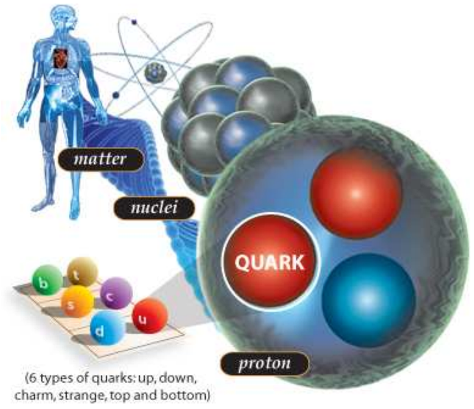
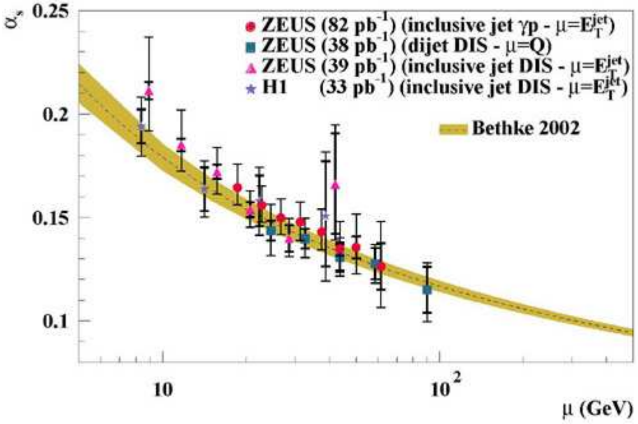
- ’ordinary’ nuclear matter
  - 3 (light) quarks (fermions)
  - interact via gluons (bosons)
  - carry color charge



Jefferson Lab, December 2007

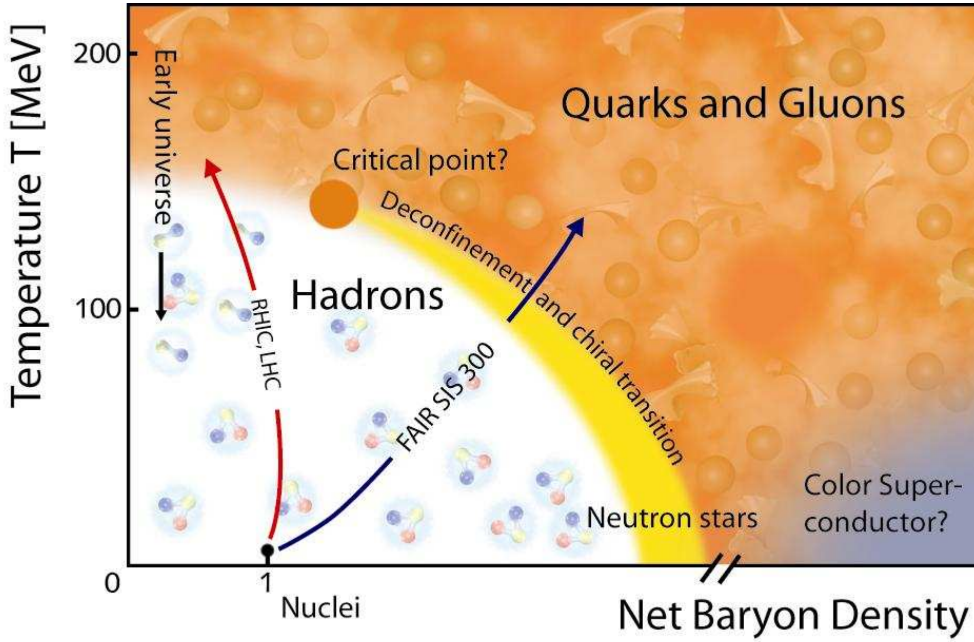
- main aspects
  - confinement: isolated quarks are never observed
  - asymptotic freedom: interaction becomes arbitrarily weak at shorter distances



HERA experiment, Hamburg, 2004

Phase diagram of QCD

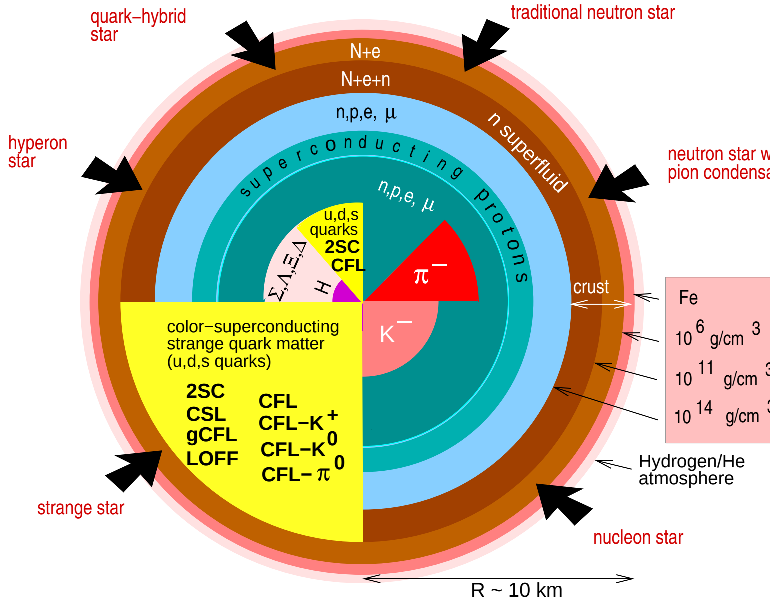
- at high temperatures and/or densities
  - free quarks and gluons (QGP)
- at small temperatures and densities
  - bound quarks and gluons (hadrons)



- at large densities and low temperatures
  - attractive interacton between quarks
  - new ground state of matter color superconductivity

Compact star

- Atmosphere: ionised atoms, nuclei and electrons
- Outer crust: nuclei and free electrons
- Inner crust: nucleons
- Outer core: composed of n, p, e^- and muons
- Inner core: densities above saturation, exotic matter

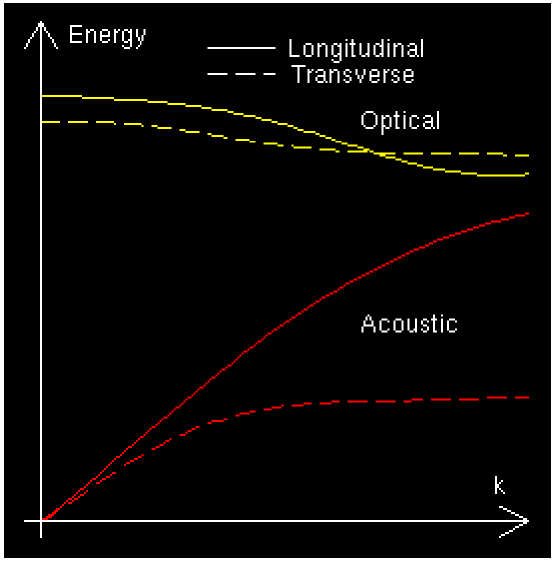


F. Weber, astro-ph/0407115

[Irina Sagert, Palaver University Frankfurt, June 2008]

Quasiparticles

- Quasiparticles:
  - excitations in a system of interacting particles
- Quasiparticle models (QPM) for QCD
  - useful phenomenological parametrization for matter (above T_c)
  - describes QGP as assembly of non-interacting excitations
- Quasiparticles are characterized by their dispersion relations
- dispersion relations: poles of the propagator (real part)



phonon dispersion relation, Hans–Erik Nielsson, Harvard University

Plasminos I

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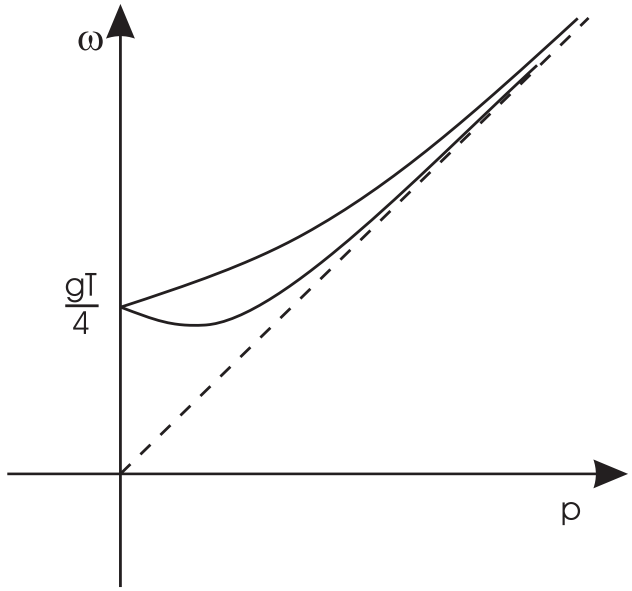
- relativistic fermionic systems
  - quarks, antiquarks and gluons (QCD)
  - electrons, positrons and photons (QED)
- high temperature/ density

  $\longrightarrow$ 2 types of excitations



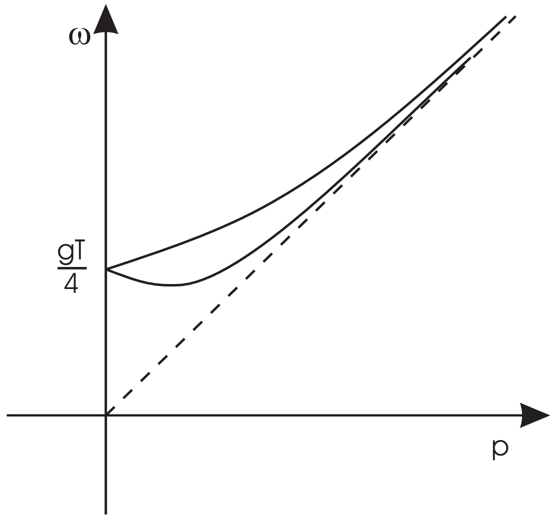
G. Baym, J.-P. Blaizot, B. Svetitsky,

Phys. Rev. D 46 (1992) 4043

- ◇ particles and antiparticles
- ◇ additional collective excitations: $\Rightarrow$ plasmino and anti-plasmino

Plasminos II

- fermionic analogon to plasma oscillation
  - plasmon is a quantum of a plasma oscillation
  - 'plasmino': fermionic excitation, 'plasmon': bosonic excitation
- significant characteristics of dispersion relation:
  - minimum at certain $p^* \neq 0$
  - residue vanishes for large momenta
  - energy gap at $p = 0$
  - meets particle branch at $p = 0$
  - plasmino branch has opposite chirality and helicity



G. Baym, J.-P. Blaizot, B. Svetitsky, Phys. Rev. D 46 (1992) 4043

Chirality

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- chirality: an object differs from its mirror image
- helicity: projection of the spin onto the momentum direction

  high–energy limit: helicity = chirality
- massive particle P: moves with $v < c$

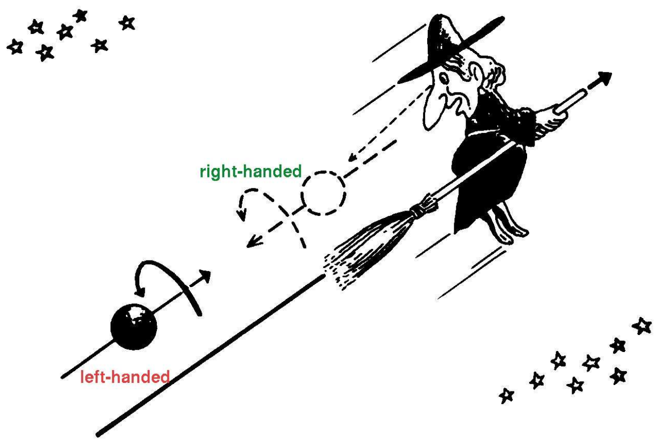
  if P is left–handed in the laboratory system,

  P is right–handed in a rest frame moving faster than P

  $\Rightarrow$ chirality not conserved

- massless system:

  chirality is conserved



Chirality of Plasminos I

Using the chirality and helicity projectors

$$\mathcal{P}_{r,l} \equiv \frac{1 \pm \gamma_5}{2} \qquad \mathcal{P}_{\pm}(\vec{p}) \equiv \frac{1 \pm \gamma_5 \gamma_0 \vec{\gamma} \cdot \hat{p}}{2}$$

and the energy projectors are

$$\Lambda^{+}_{\vec{p}} = \mathcal{P}^{+}_{r+} + \mathcal{P}^{+}_{l-} = \frac{1 + \gamma_0 \vec{\gamma} \cdot \hat{p}}{2}$$

$$\Lambda^{-}_{\vec{p}} = \mathcal{P}^{-}_{r-} + \mathcal{P}^{-}_{l+} = \frac{1 - \gamma_0 \vec{\gamma} \cdot \hat{p}}{2}$$

$\Rightarrow$ $\Lambda^{+}_{\vec{p}}$: product of projectors with the same chirality and helicity ($++$ and $--$)

$\Rightarrow$ $\Lambda^{-}_{\vec{p}}$: product of projectors with the opposite chirality and helicity ($+-$ and $-+$)

Chirality of Plasminos II

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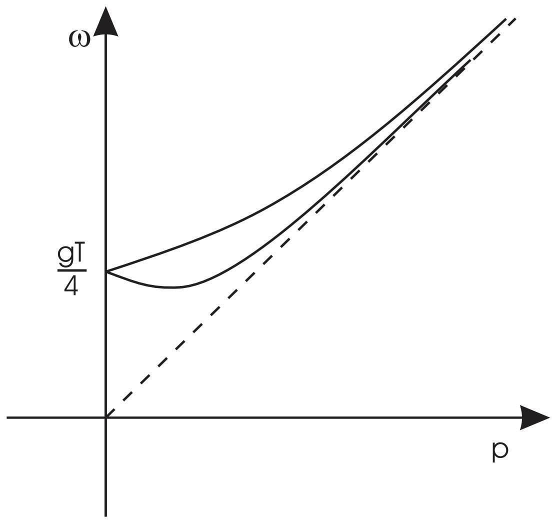
- ◦ for each energy projector there are two solutions:
- ∗ ψ^+: particles and antiplasminos
- ∗ ψ^-: antiparticles and plasminos

$\Rightarrow$ plasmino branch has opposite chirality and helicity



G. Baym, J.-P. Blaizot, B. Svetitsky, Phys. Rev. D 46 (1992) 4043

Plasminos – what is known?

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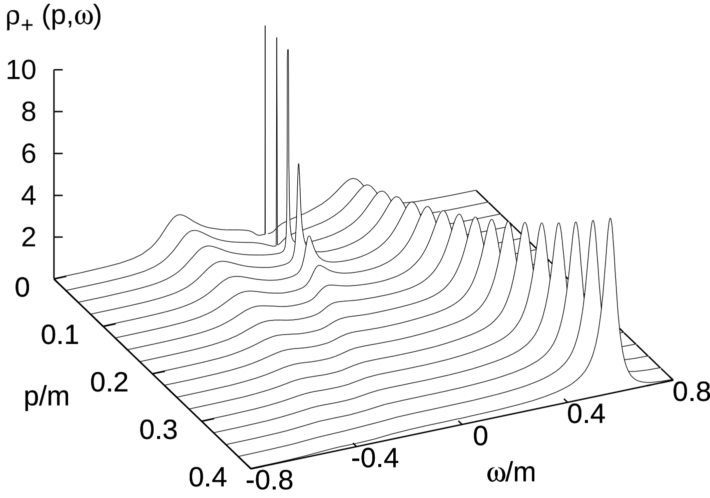
- for high temperatures,
  - all quarks, charged leptons and neutrinos have a thermal mass and a plasmino branch
- for high temperatures and chiral symmetry:
  - plasminos exist independent of kind of interaction
- included into the calculation of dilepton and strangeness production [J. Letessier, J. Rafelski and A. Tomsi, Phys, Lett. B 323 (393) 1990]
- neutrino emissivity of a weak interacting plasma (small T) is reduced due to the annihilation $e^- \tilde{e}^+ \rightarrow \nu\bar{\nu}$

  [E. Braaten, ApJ 392 (70) 1992]
- so far, no unambiguous experimental signal for plasminos could be found

Plasminos at High Temperatures

- First investigations by H. Weldon [Phys, Rev. D 26 (2789) 1982]
- Kitazawa et al. (see below): $T > T_c$ of chiral transition
- Yukawa model for massive scalar
- three–peaked structure due to
  - mass of exchange boson
  - mixture of processes leading to upper and lower excitation (resonant scattering)
- $T \gg m$ two–peaks



spectral density for $T/m = 1.2$
arXiv:0706.2697 [hep-ph]

Plasminos and QPM I

Quasiparticle Models (QPM) are used to

- calculate thermodynamic properties $\Rightarrow$ EoS:
  - $*$ effective action Γ
  - $\Rightarrow$ grand canonical ensemble $\Omega = -T\Gamma$
  - $*$ self–energies
  - $\Rightarrow$ are functional derivaties of the effective action
  - $*$ entropy: $s = -V^{-1}\frac{\partial\Omega}{\partial T}|_\mu$
  - $\Rightarrow$ pressure

Plasminos and QPM II

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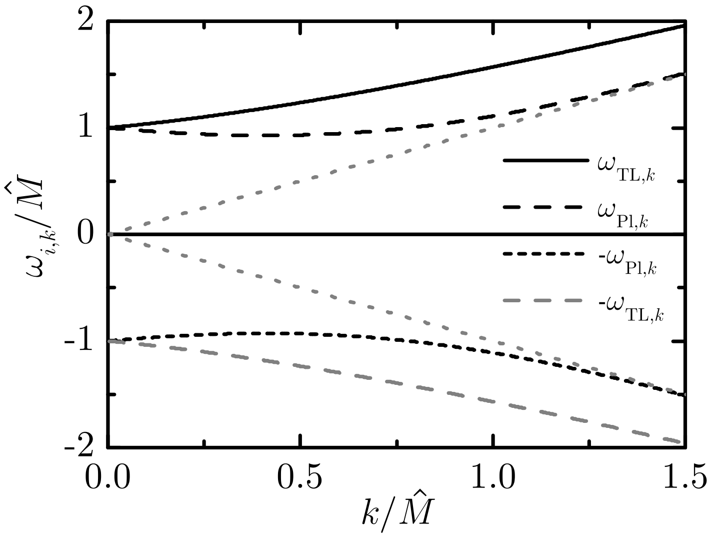
- using a QPM in the HTL limit

  (R. Schulze, M. Bluhm, B. Kämpfer [Eur.Phys.J.ST 155:177-190,2008])

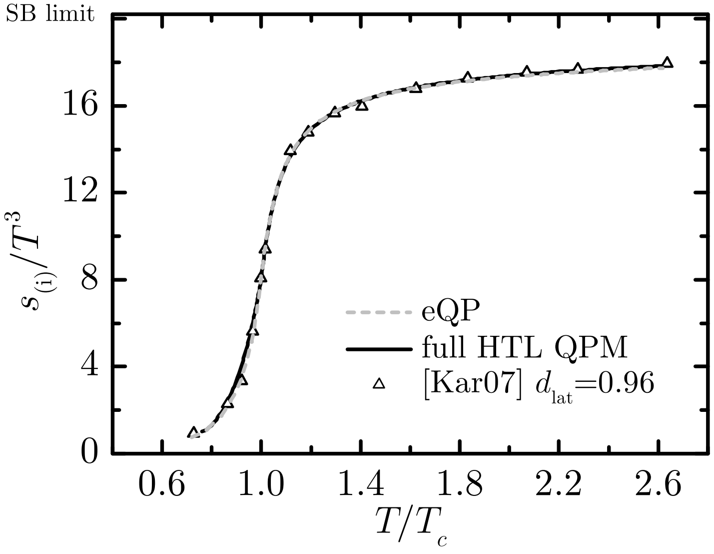
  - describes translational invariant QGP in equilibrium without spontaneous symmetry breaking

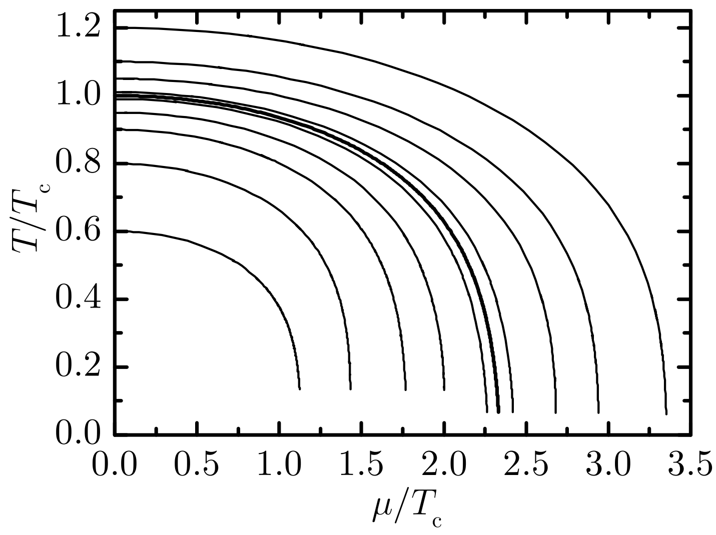


HTL (Hard Thermal Loop) limit: $T \sim \omega, p$

Plasminos and QPM III

- $\mu = 0$: matches lattice results
- extension to $\mu \neq 0$: for unambiguousness include collective modes





Eur.Phys.J.ST 155:177-190,2008

Plasminos – How to calculate

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- dispersion relation: $G^{-1}(P) = G_0^{-1}(P) + \Sigma(P)$

  spectral density: $\rho(P) = -\frac{1}{\pi}\,\text{Im}\, G(P)$

- self–energy $\Sigma = \frac{\delta \ln Z_I}{\delta \mathcal{G}_0}$:

  Yukawa interaction: $\mathcal{L}_I = g\bar{\psi}\psi\varphi$

  action: $S_I = \int_0^\beta d\tau \int d^3\vec{x}\mathcal{L}_I(\vec{x},\tau)$

  partition function: $Z = N' \int [d\varphi] e^S$

  $$\ln Z = \ln Z_0 + \ln Z_I$$

  B. Betz and D. Rischke, Phys.Rev.D75:065022,2007

  J.-P. Blaizot and J.-Y. Ollitrault, Phys. Rev. D 48, 1390, 1993

Plasminos – Lowest Order Correction

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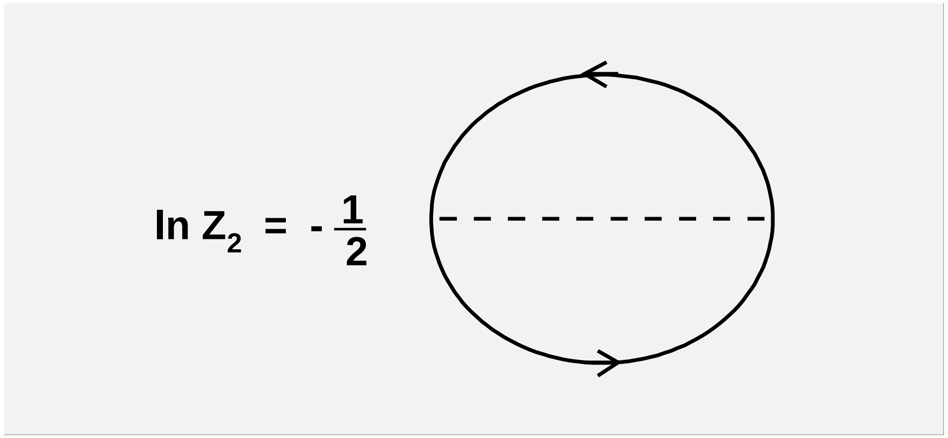
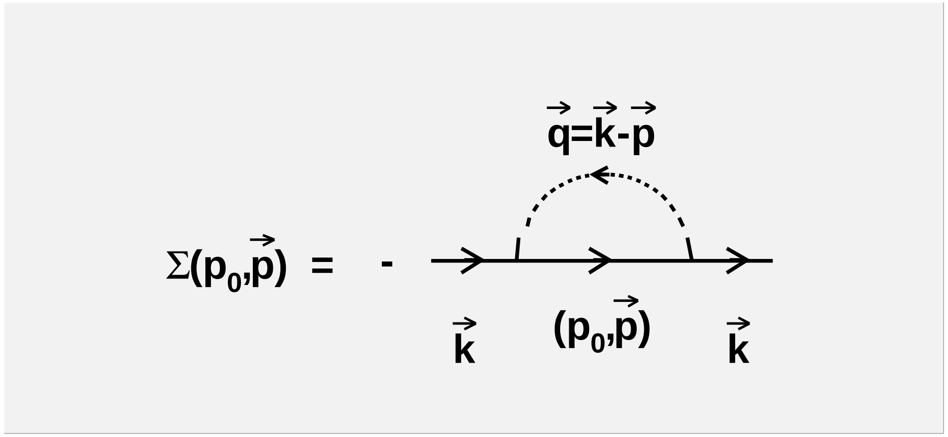
- The lowest order correction to ln Z_0 is



- The fermion self-energy is given by $\Sigma = \frac{\delta \ln Z_I}{\delta \mathcal{G}_0}$:



Plasminos – Self–Energy I

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- $\Sigma(P) = -g^2\, T \sum_n \int \frac{d^3\vec{k}}{(2\pi)^3}\, \mathcal{D}_0(K-P)\, G_0(K)$
  - free–particle fermion propagator: $G_0^{-1}(K) = \not{K} + \mu\gamma_0$
  - free–particle boson propagator: $\mathcal{D}_0(Q) = -1/Q^2$
  - using the mixed representation:

$$\mathcal{D}_0(q_0,\vec{q}) = \int_0^{\frac{1}{T}} d\tau e^{q_0\tau}\mathcal{D}_0(\tau,\vec{q}) \quad \mathcal{D}_0(\tau,\vec{q}) = T\sum_n e^{-q_0\tau}\mathcal{D}_0(q_0,\vec{q})$$

$$\mathcal{G}_0(p_0,\vec{p}) = \int_0^{\frac{1}{T}} d\tilde{\tau} e^{p_0\tilde{\tau}}\mathcal{G}_0(\tilde{\tau},\vec{p}) \quad \mathcal{G}_0(\tilde{\tau},\vec{p}) = T\sum_n e^{-p_0\tilde{\tau}}\mathcal{G}_0(p_0,\vec{p})$$

- performing the sums over the Matsubara frequencies, one obtains ...

Plasminos – Self–Energy II

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$$\Sigma(P) = -g^2 \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{2E_b} \sum_{e=\pm} \Lambda^e_{\vec{k}} \gamma_0 \left[\frac{1 - N^e_F(k) + N_B(E_b)}{p_0 + \mu - e(k + E_b)} + \frac{N^e_F(k) + N_B(E_b)}{p_0 + \mu - e(k - E_b)} \right]$$

- energy projectors $\Lambda^{\pm}_{\vec{p}} = \frac{1}{2}(1 \pm \gamma_0 \vec{\gamma} \cdot \hat{p})$
- thermal distribution functions of
  - fermions/antifermions $N^{\pm}_F(E) = [e^{(E \mp \mu)/T} + 1]^{-1}$
  - bosons $N_B(E) = (e^{E/T} - 1)^{-1}$
- energy of the exchanged boson $E_b = |\vec{k} - \vec{p}|$

Plasminos – Self–Energy III

- projection on positive/negative–energy solutions

  $$\Sigma_\pm(P) \equiv \frac{1}{2}\,\mathrm{Tr}\left[\Lambda^\pm_{\vec{p}}\,\gamma_0\,\Sigma(P)\right]$$

- and perform an analytic continuation

  $$p_0 + \mu \longrightarrow \omega + i\eta$$

* imaginary part $\quad \mathrm{Im}\,\Sigma_\pm(\omega,p) = \mathrm{Im}\,a(\omega,p) \pm p\,\mathrm{Im}\,b(\omega,p)$
* real part $\quad \mathrm{Re}\,\Sigma_\pm(\omega,p) = \mathrm{Re}\,a(\omega,p) \pm p\,\mathrm{Re}\,b(\omega,p)$

Plasminos – Im $\Sigma_{\pm}(\omega, \mathrm{p})$

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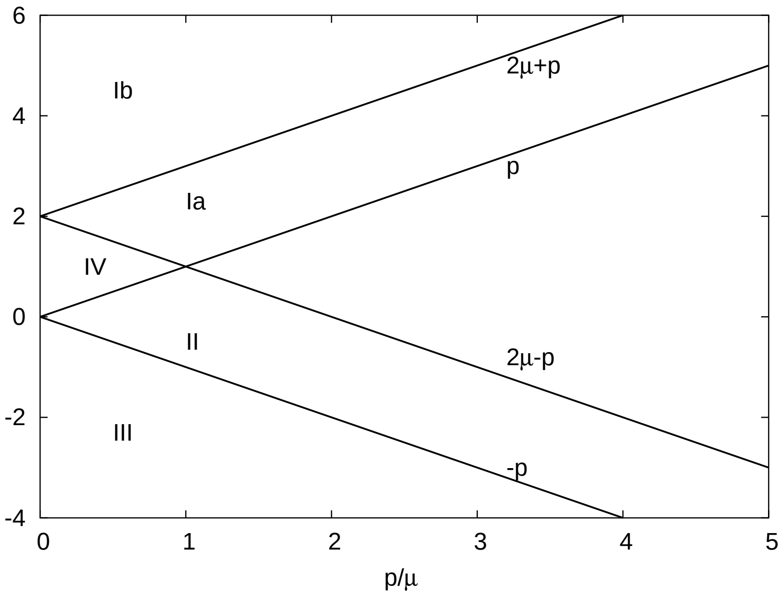
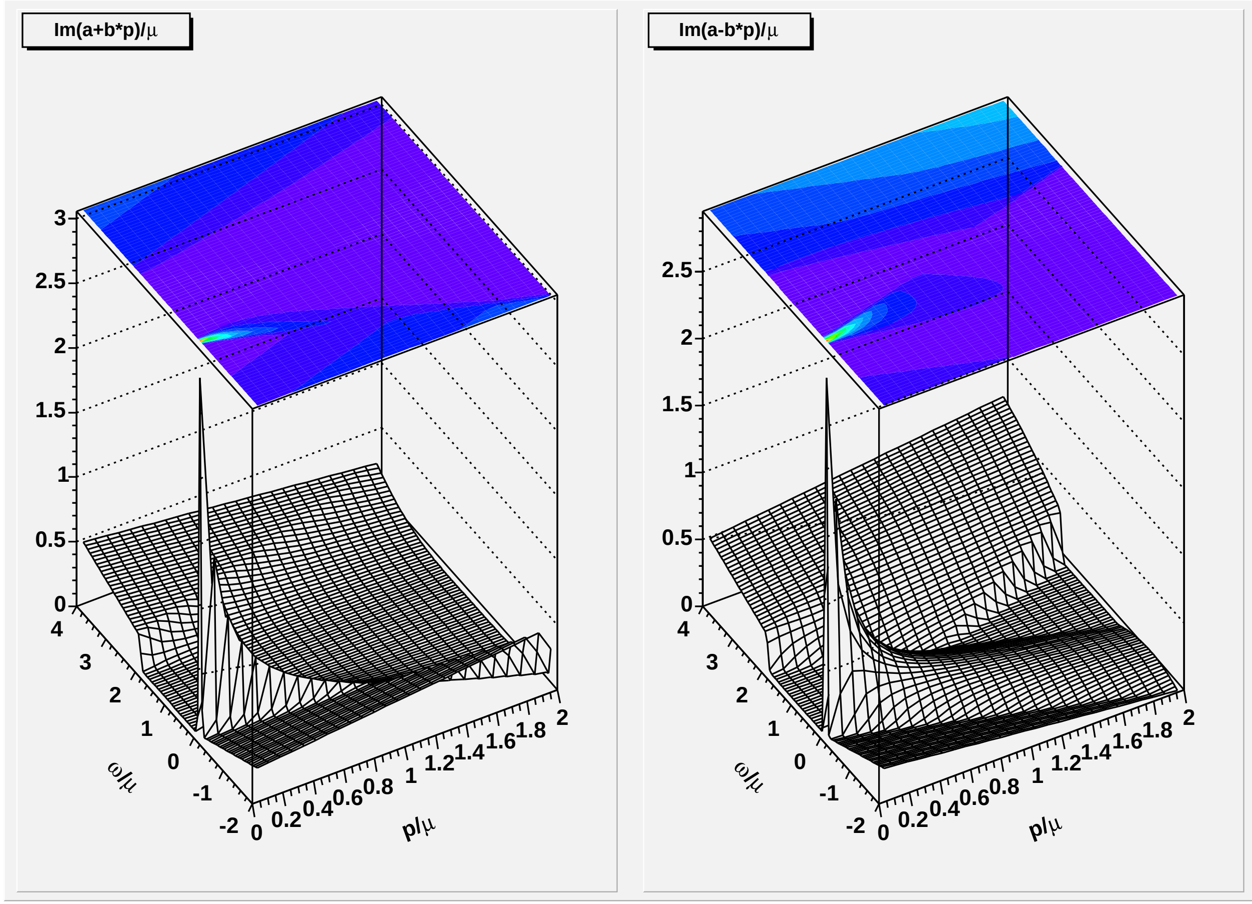
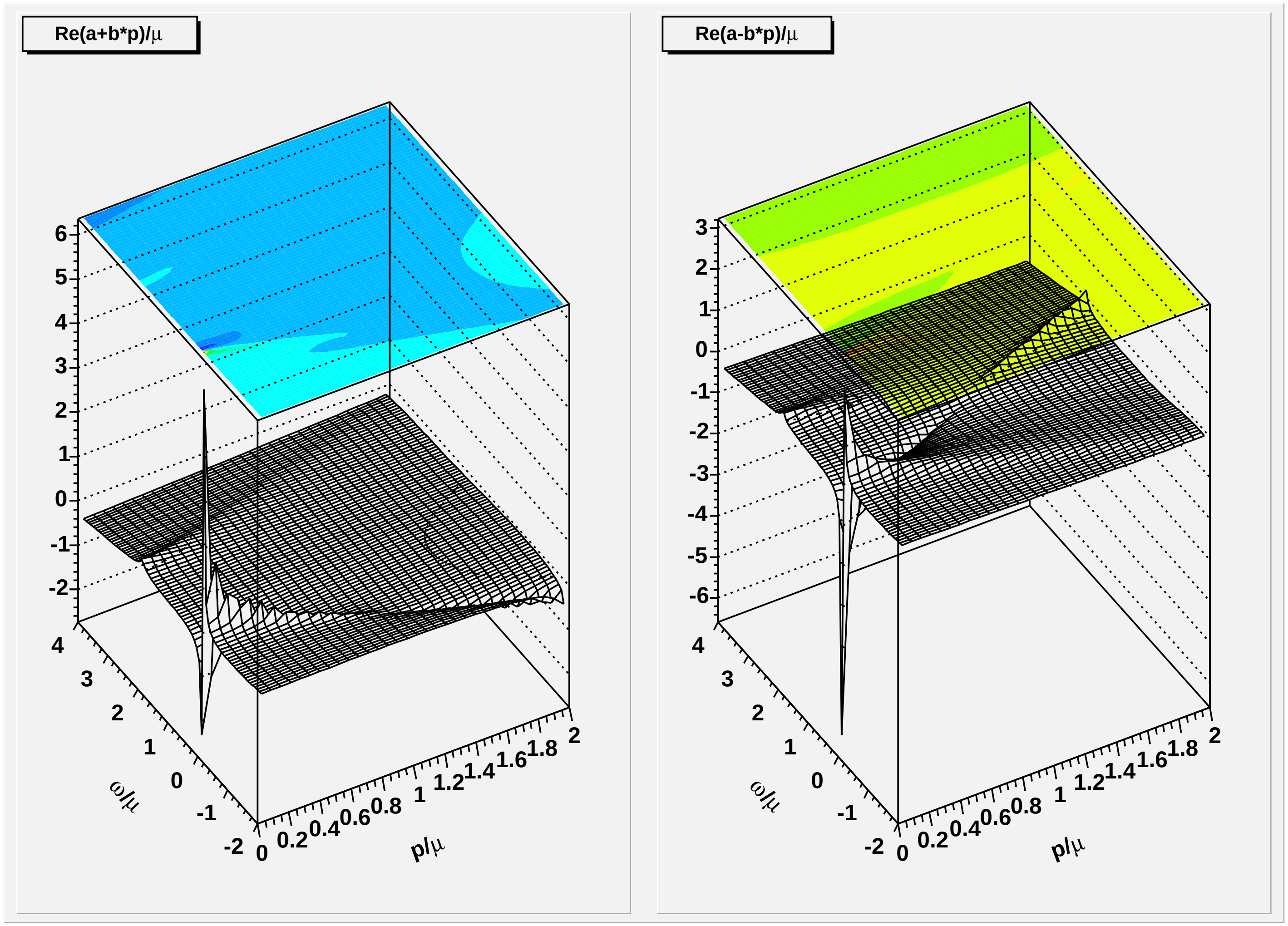


Different domains in the energy–momentum plane



massless fermions (left panel) and antifermions (right panel) for $g^2/4\pi = 1$, at $T = 0$

Plasminos – Re $\Sigma_{\pm}(\omega, p)$ 3d Plots



massless fermions (left panel) and antifermions (right panel)

for $g^2/4\pi = 1$, at $T = 0$

Plasminos – Dispersion Relation

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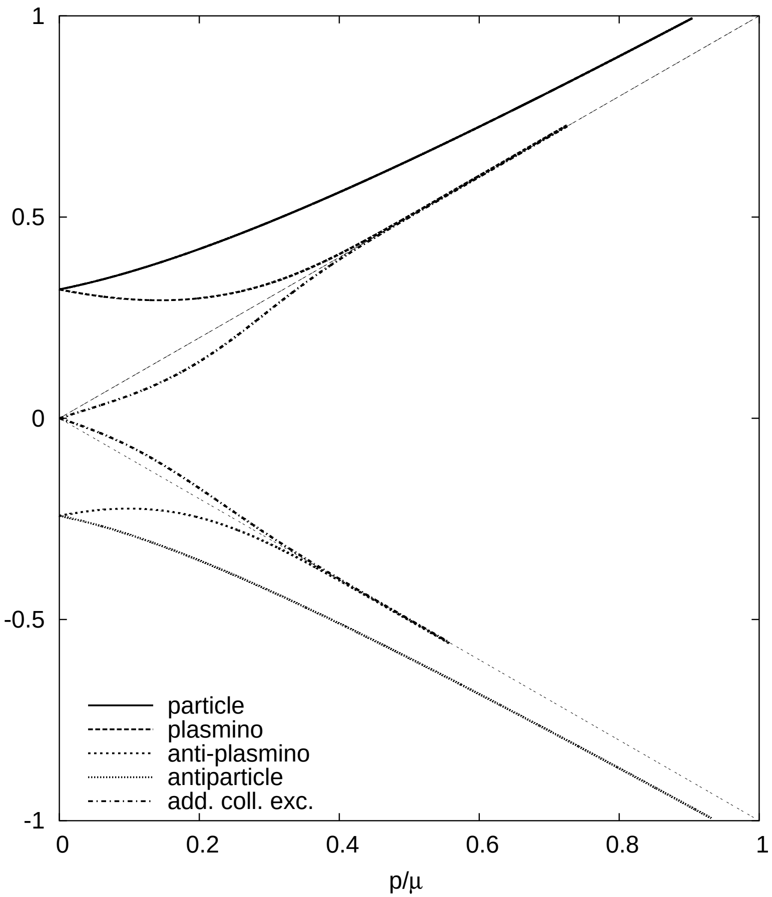
- is given by the roots of the full inverse propagator
  $G_{\pm}^{-1}(P) \equiv G_{0,\pm}^{-1}(P) + \Sigma_{\pm}(P)$



- $*$ $G_0^{-1}(P) = \not{P} + \mu\gamma_0 \equiv \gamma_0 \sum_{e=\pm} G_{0,e}^{-1}(P)\,\Lambda_{\vec{p}}^{e}$
  $G_{0,\pm}^{-1}(P) \equiv p_0 + \mu \mp p$
- $*$ plasmino solutions come from $G_{-}^{-1} = 0$ (opposite chirality)
- $*$ two solutions in space–like region
- $*$ coupling constant determines size of momentum region with plasminos
- $*$ decreasing coupling $\rightarrow$ region shrinks

Plasminos – Spectral Density

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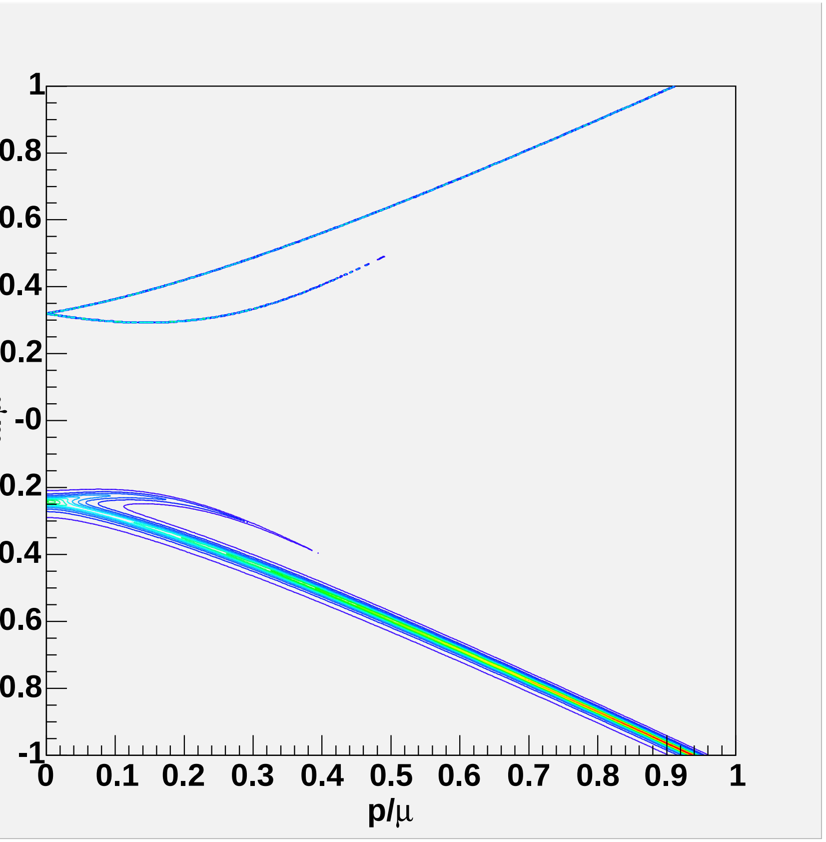
- is determined from $\rho_\pm(\omega, p) = -\frac{1}{\pi}\,\mathrm{Im}\,G_\pm(\omega, p)$



- $Im\,\Sigma_\pm(\omega, p) = 0 \rightarrow$ spectral density proportional to δ–function
- particle and plasmino branch: infinite lifetime
- finite lifetime for antiparticle and antiplasmino

Superconducting fermions – Self-energy

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- self-energy: $\Sigma(P) = -g^2\, T \sum_n \int \frac{d^3\vec{k}}{(2\pi)^3} \mathcal{D}_0(K-P)\, \mathcal{G}_0(K)$
  - quasifermion propagator $\mathcal{G}_0$

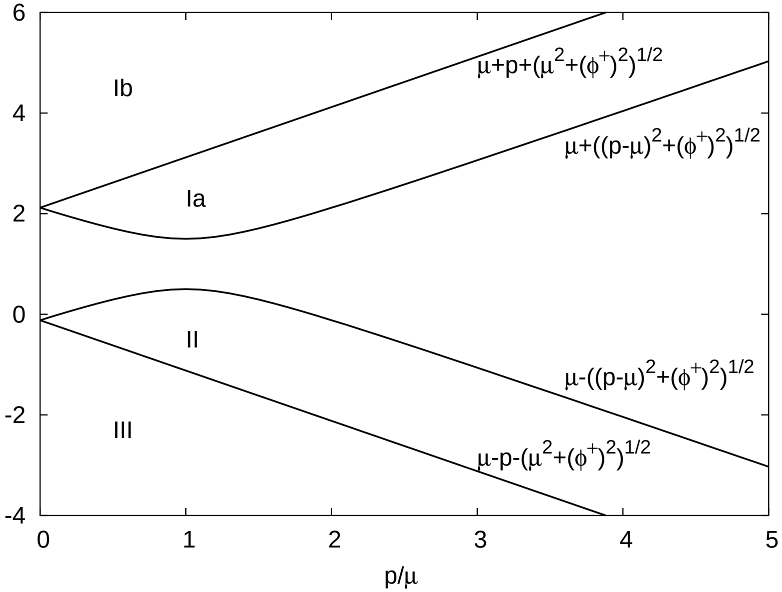
    $$\mathcal{G}_0(P) = \sum_{e=\pm} \frac{p_0-(\mu-ep)}{p_0^2-(\mu-ep)^2-|\phi_e(P)|^2}\, \Lambda^e_{\vec{p}} \gamma_0$$

- performing Matsubara sums, with mixed representation:

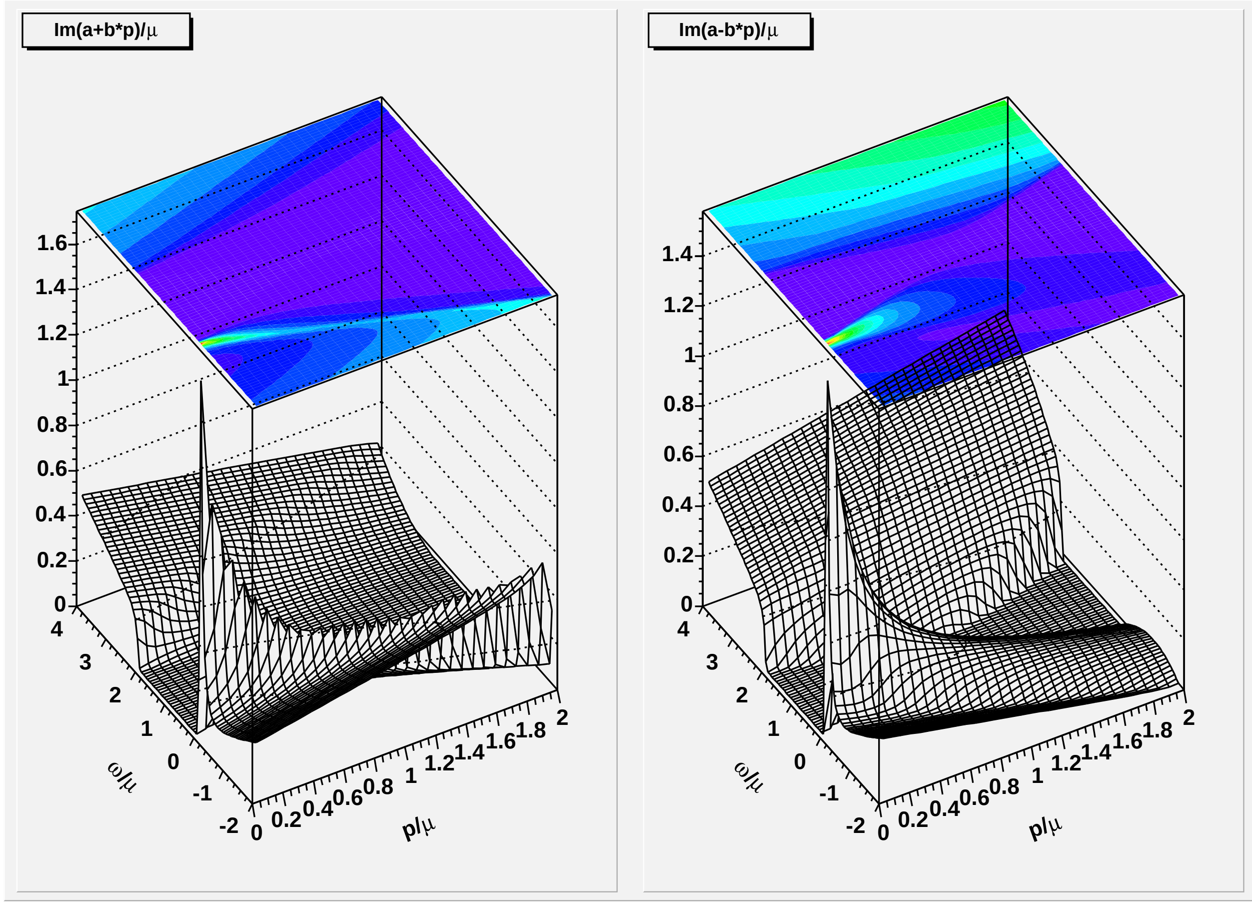
$$\Sigma(P) = -g^2 \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{4E_b} \sum_{e=\pm} \Lambda^e_{\vec{k}} \gamma_0 \left[\left(1 - \frac{\mu-ek}{\epsilon_e}\right) \frac{1}{p_0-\epsilon_e-E_b} + \left(1 + \frac{\mu-ek}{\epsilon_e}\right) \frac{1}{p_0+\epsilon_e+E_b} \right]$$

$\Rightarrow$ calculate imaginary and real part of the self-energy

Superconducting fermions – $\text{Im}\,\Sigma_{\pm}(\omega, \text{p})$



Different domains in the energy–momentum plane



massless superconducting fermions (left panel) and antifermions (right panel) for $g^2/4\pi = 1$, at $T = 0$

Superconducting fermions – $\mathrm{Re}\,\Sigma_{\pm}(\omega, \mathrm{p})$

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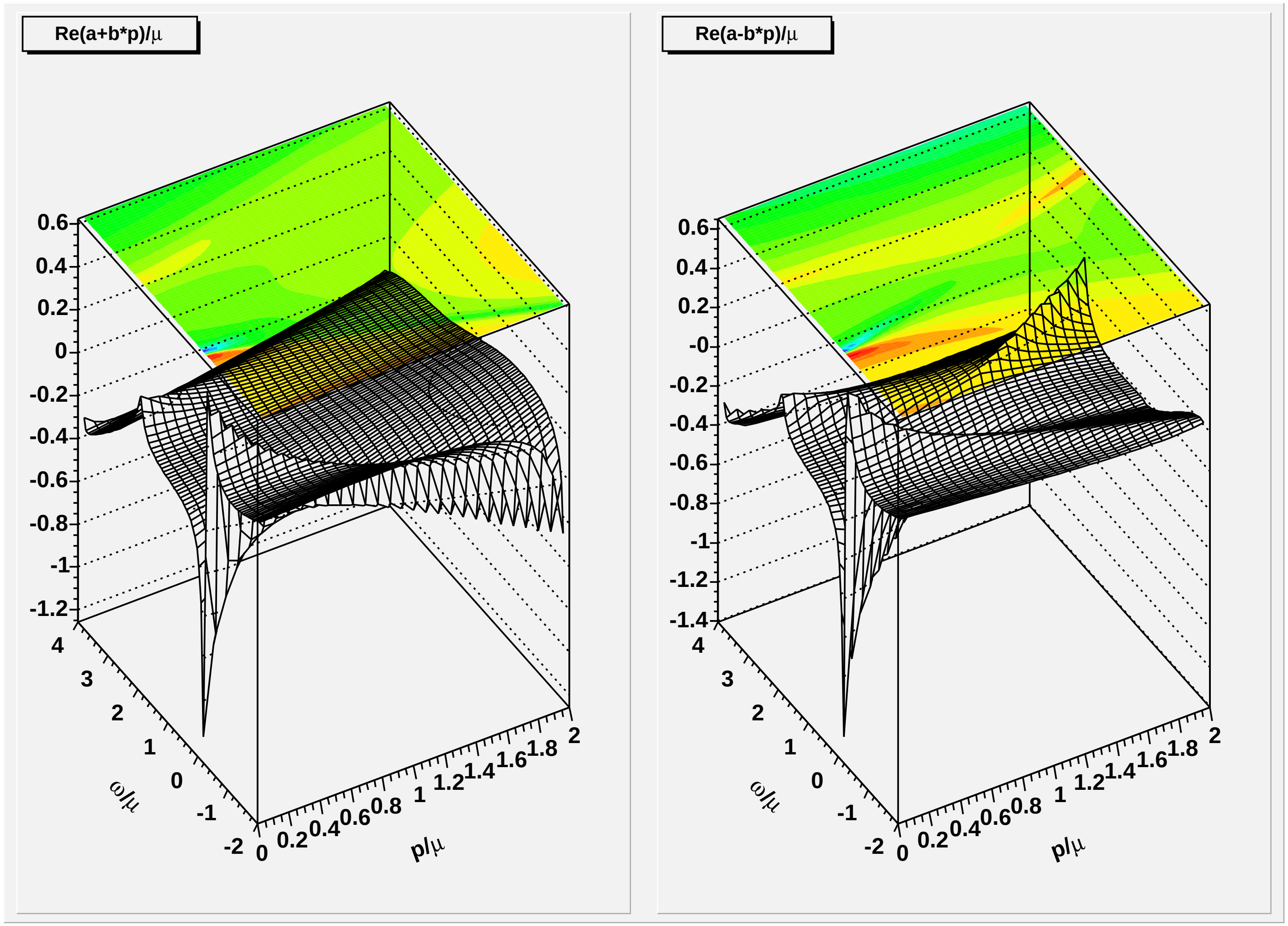

massless superconducting fermions (left panel) and antifermions (right panel) for $g^2/4\pi = 1$, at $T = 0$

Superconducting fermions – Disp. rel. I

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- is given by the poles of the propagator

$$\mathcal{G}^{\pm} = \left([G_0^{\pm}]^{-1} + \Sigma^{\pm} - \Phi^{\mp} \left\{ [G_0^{\mp}]^{-1} + \Sigma^{\mp} \right\}^{-1} \Phi^{\pm} \right)^{-1}$$

  * free propagator for particles and charge–conjugate particles $[G_0^{\pm}]^{-1}(P) = \gamma_0 \sum_{e=\pm} [p_0 \pm (\mu - ep)] \Lambda_{\vec{p}}^{\pm e}$
  * Σ^+ was computed above
  * $\Sigma^-(P) \equiv C \left[\Sigma^+(-P) \right]^T C^{-1}$ is self-energy for charge-conjugate particles
  * charge conjugation matrix $C = i\gamma^2\gamma_0$
  * Φ^+ is order parameter for condensation $\Phi^+(P) = \sum_{e=\pm} \phi_e(P) \, \Lambda_{\vec{p}}^{e} \gamma_5$

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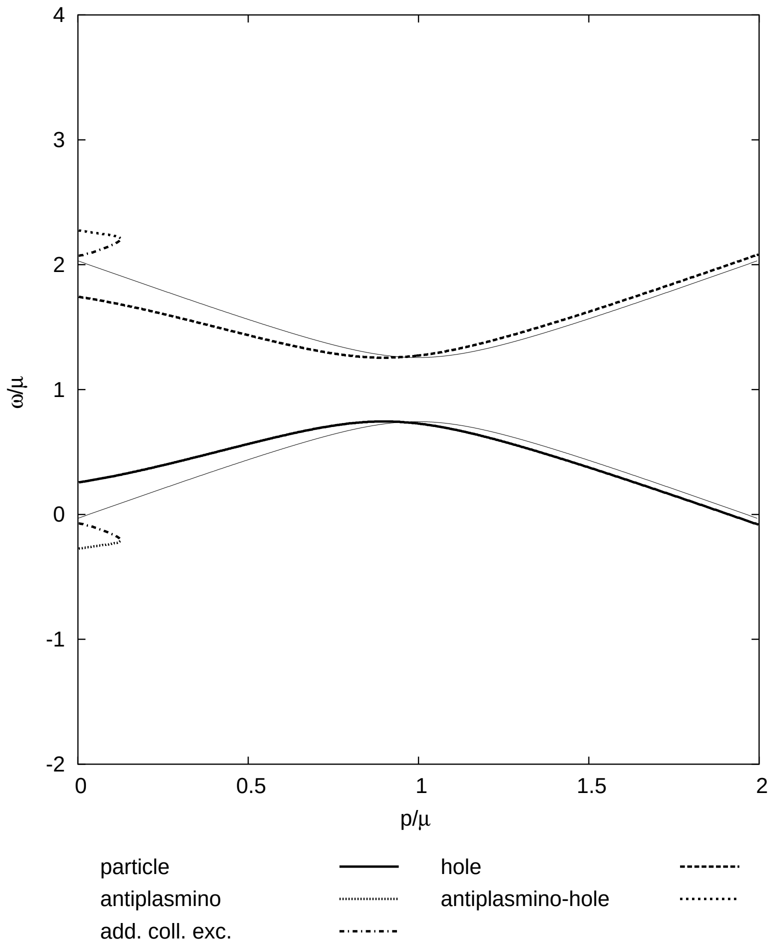
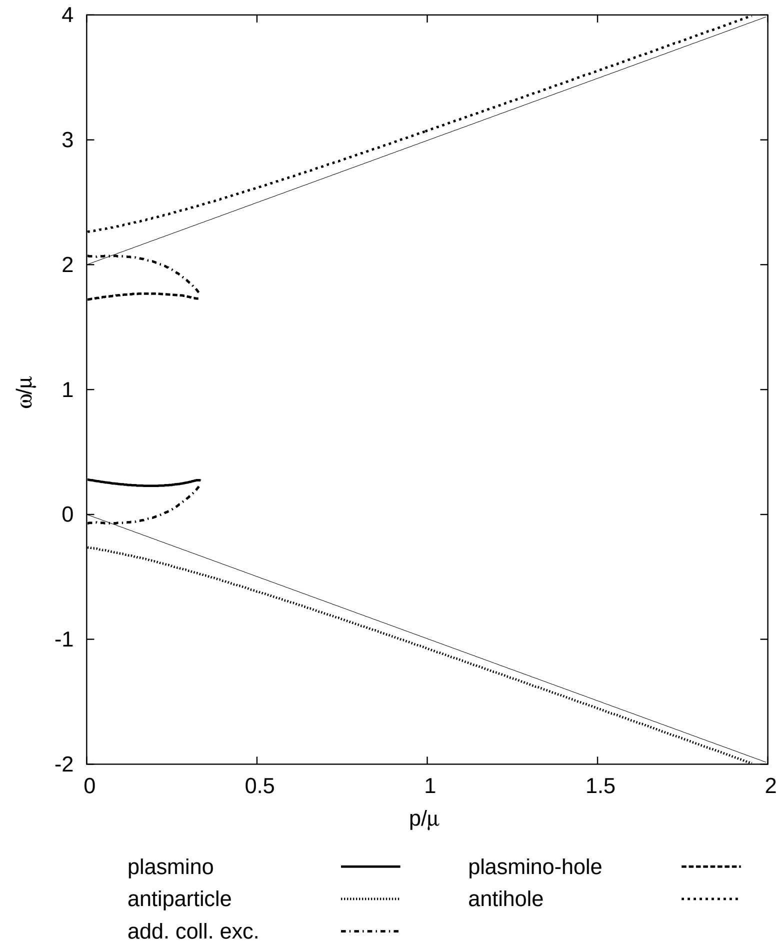


positive energy dispersion relation, ($\mathcal{G}_+^+$ and $\mathcal{G}_-^-$), $g^2/(4\pi) = 1$, $T = 0$

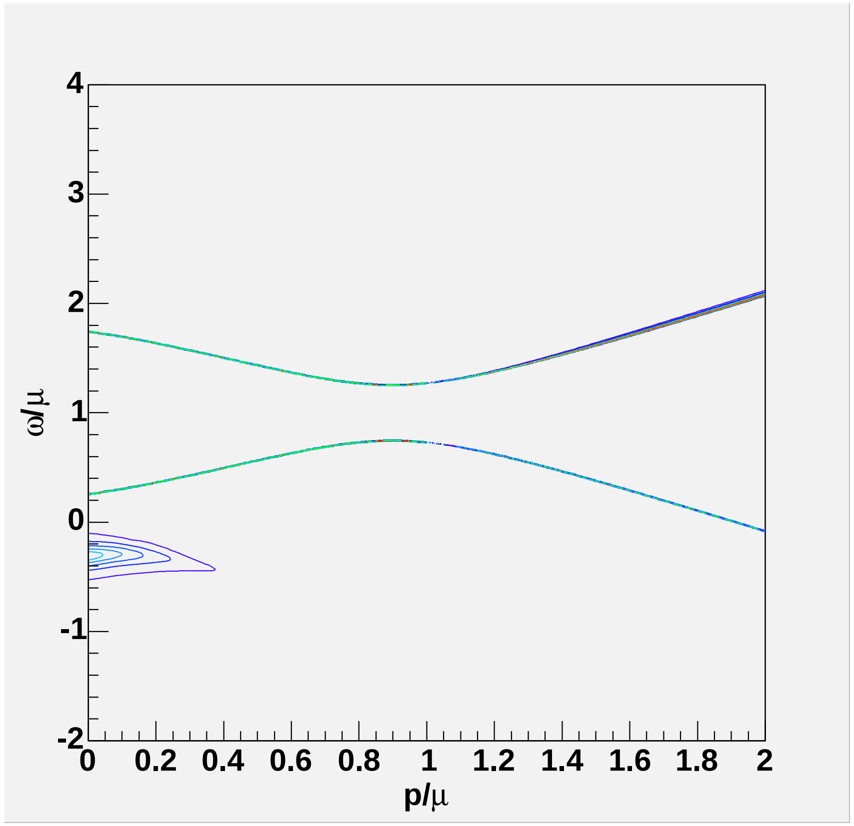


negative energy dispersion relation, ($\mathcal{G}_+^-$ and $\mathcal{G}_-^+$), $g^2/(4\pi) = 1$, $T = 0$

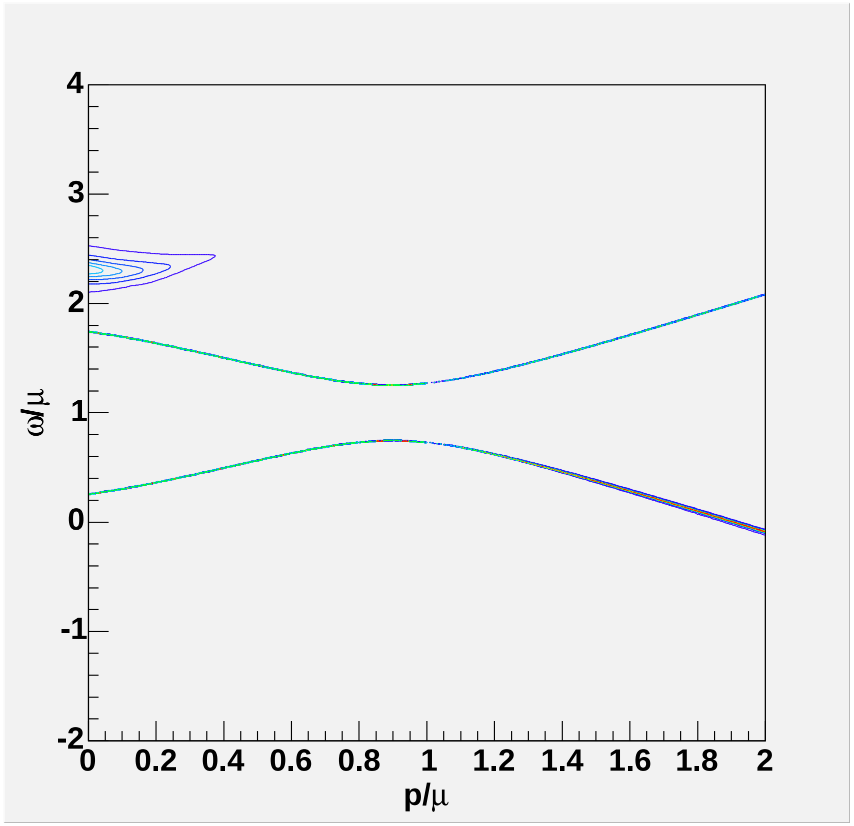
$$\mathcal{G}_e^{\pm}(P) \equiv \frac{1}{2}\,\mathrm{Tr}\left[\mathcal{G}^{\pm}(P)\,\gamma_0 \Lambda_{\vec{p}}^{e}\right]$$

Superconducting fermions – Spec. Dens. I

- $\rho_e^{\pm}(\omega, p) = -\frac{1}{\pi} \operatorname{Im} \mathcal{G}_e^{\pm}(\omega, p)$

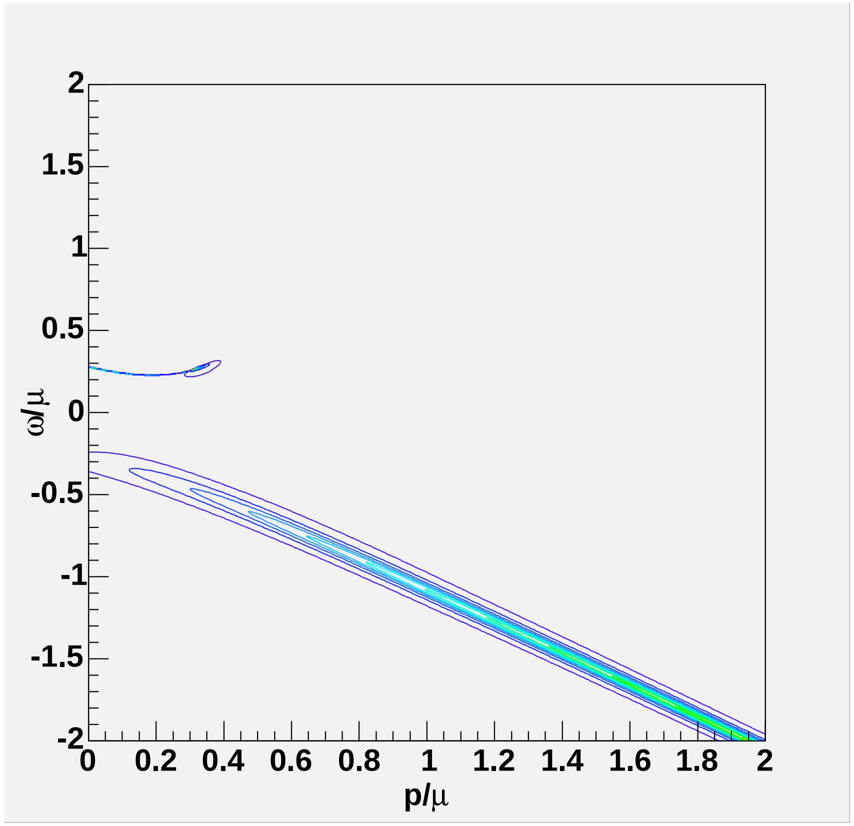


spectral density ρ_+^+ for $g^2/(4\pi) = 1$, $T = 0$

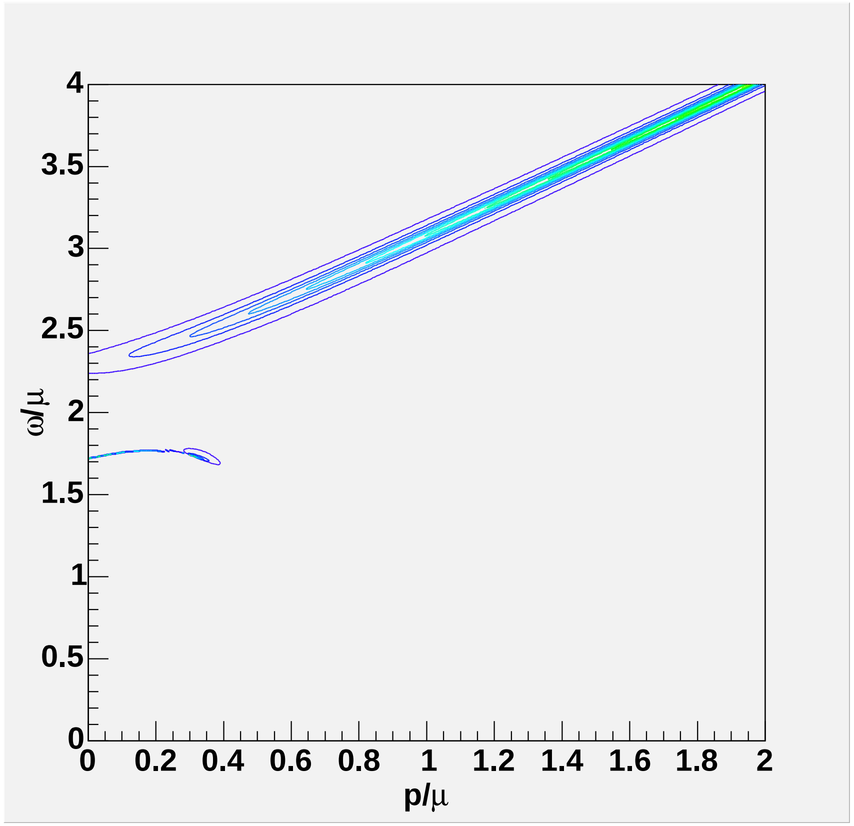


spectral density ρ_-^- for $g^2/(4\pi) = 1$, $T = 0$

- small momenta: particle and hole excitations are undamped ($Im\,\Sigma_+(\omega, p) = 0$)



spectral density ρ_-^+ for $g^2/(4\pi) = 1$, $T = 0$



spectral density ρ_+^- for $g^2/(4\pi) = 1$, $T = 0$

- antiparticle and antihole branches are strongly damped

Summary

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We studied the fermionic excitation spectrum

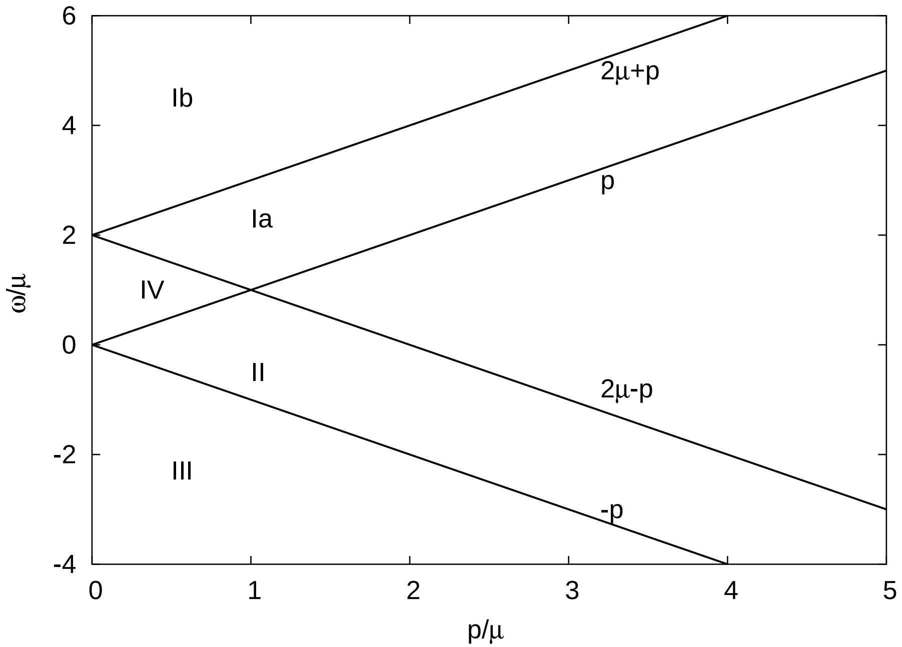
* used eg. in Quasiparticle Models (QPM),
* for normal- and superconducting systems,
* for massive/massless fermions interacting with massive/massless bosons,
* for vanishing and non–vanishing temperature and finite chemical potential.

We discussed

* the calculation of the dispersion relations,
* the occurring excitations,
* and their relevance (for a special QPM).

Back-up Slides

Plasminos – Im $\Sigma_\pm(\omega, \mathbf{p})$



Ib: $\text{Im}\, a = -\frac{g^2}{32\pi}\frac{1}{4p}(2\mu - \omega - p)\times (2\mu + \omega + p)$

$\text{Im}\, b = -\frac{g^2}{32\pi}\frac{1}{2p^3}(2\mu - \omega - p)\times \left[\omega^2 - p^2 - \frac{\omega}{2}(2\mu + \omega + p)\right]$

Ia: $\text{Im}\, a = \frac{g^2}{32\pi}\omega$, $\quad \text{Im}\, b = -\frac{g^2}{32\pi}$

II: $\quad \text{Im}\, a = \frac{g^2}{32\pi}\frac{1}{4p}(2\mu - \omega - p)(2\mu + \omega + p)$

$\text{Im}\, b = \frac{g^2}{32\pi}\frac{1}{2p^3}(2\mu - \omega - p)\left[\omega^2 - p^2 - \frac{\omega}{2}(2\mu + \omega + p)\right]$

III: $\quad \text{Im}\, a = -\frac{g^2}{32\pi}\omega$, $\text{Im}\, b = \frac{g^2}{32\pi}$

IV: $\quad \text{Im}\, a = \text{Im}\, b = 0$

Plasminos – Re $\Sigma_\pm(\omega, \mathbf{p})$

- Re $\Sigma_\pm(\omega,p) =$ Re $a(\omega,p) \pm p$ Re $b(\omega,p)$

$$\text{Re}\, a(\omega,\vec{p}) = -\frac{g^2}{32\pi^2}\Bigg\{\mu + \frac{(2\mu-\omega-p)(2\mu+\omega+p)}{4p}\ln\left|\frac{\omega+p}{\omega+p-2\mu}\right| - \frac{(2\mu-\omega+p)(2\mu+\omega-p)}{4p}\ln\left|\frac{\omega-p}{\omega-p-2\mu}\right| + \omega\ln\left|\frac{\omega^2-p^2}{\mu^2}\right|\Bigg\}$$

$$\text{Re}\, b(\omega,\vec{p}) = -\frac{g^2}{16\pi^2}\Bigg\{\frac{\mu^2}{p^2} - \frac{\omega\mu}{2p^2} + \frac{(2\mu-\omega-p)}{4p^3}\left[\omega^2 - p^2 - \frac{\omega}{2}(2\mu+\omega+p)\right]\ln\left|\frac{\omega+p}{\omega+p-2\mu}\right| - \frac{(2\mu-\omega+p)}{4p^3}\left[\omega^2 - p^2 - \frac{\omega}{2}(2\mu+\omega-p)\right]\ln\left|\frac{\omega-p}{\omega-p-2\mu}\right| - \frac{1}{2}\ln\left|\frac{\omega^2-p^2}{\mu^2}\right|\Bigg\}$$

Superconducting fermions – Disp. rel. II

- To distinguish the solutions, we use

$$\mathcal{G}_e^{\pm}(P) \equiv \frac{1}{2}\,\mathrm{Tr}\left[\mathcal{G}^{\pm}(P)\,\gamma_0 \Lambda_{\vec{p}}^{e}\right]$$

$$\Rightarrow \mathcal{G}_+^{+}(P) = \frac{p_0-\mu+p-\Sigma_+(-P)}{[p_0+\mu-p+\Sigma_+(P)][p_0-\mu+p-\Sigma_+(-P)]-|\phi_+(P)|^2}$$

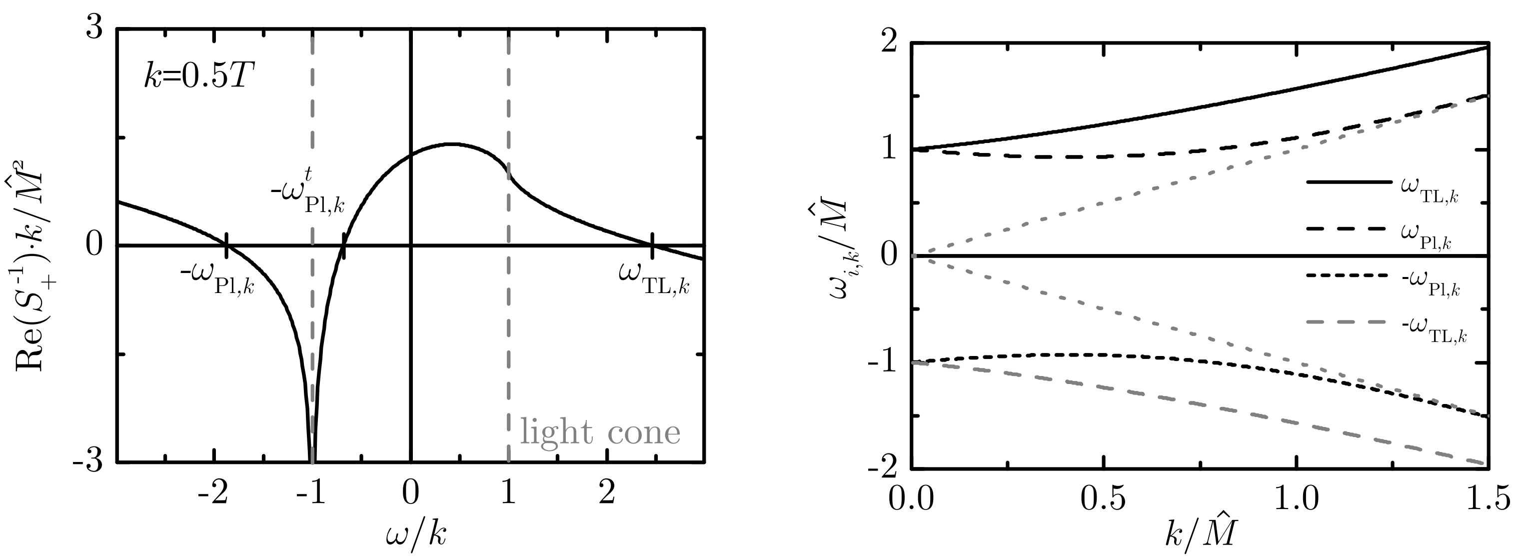
$$\mathcal{G}_-^{+}(P) = \frac{1}{p_0+\mu+p+\Sigma_-(P)}$$

$$\mathcal{G}_+^{-}(P) = \frac{1}{p_0-\mu-p-\Sigma_-(-P)}$$

$$\mathcal{G}_-^{-}(P) = \frac{p_0+\mu-p+\Sigma_+(P)}{[p_0+\mu-p+\Sigma_+(P)][p_0-\mu+p-\Sigma_+(-P)]-|\phi_+(P)|^2}$$

Plasminos and QPM II

- using a QPM in the HTL limit (R. Schulze, M. Bluhm, B. Kämpfer, see below)
  - describes translational invariant QGP in equilibrium without spontaneous symmetry breaking



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Landau damping: collective effect caused by energy transfer between gauge field and plasma particles with $v \sim c$ (resonant particles)