GWs from neutron star oscillations: comparisons between linear and nonlinear evolutions

Pescara, July 18th 2008

S.Bernuzzi

Parma University and INFN

L.Baiotti (Tokyo), G.Corvino (Parma), R. De Pietri (Parma) and A.Nagar (IHES)

Istituto Nazionale di Fisica Nucleare

Outline of the work

GWs from even-parity oscillation of a perturbed TOV star

Compare the results obtained from 3D FGR simulations with perturbative ones (1D,linear)

- ✓ Zerilli extraction
- ✓ Ψ_4 extraction
- ✓ Quadrupole formulas
- ✓ Non-linear effects (as a function of the amplitude of the initial perturbation)

Motivation

GWs from NS oscillations

- excited e.g. after Supernova Core Collapse
- non-linear oscillations !

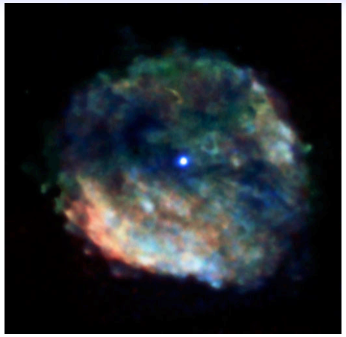
- test-bed for 3D wave extraction methods (in non-vacuum spacetimes) and for analysis methods

Why a linear time-domain code ?

- Perturbative methods: quasi equilibrium systems
- 1D: computationally less expensive then 3D
- Accurate results (more resolution)

- Check 3D extraction methods
- Basis for non-linear analysis

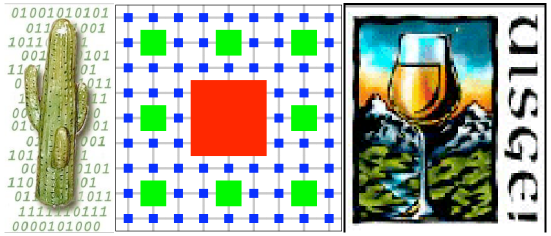
Startegy: the double approach



$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

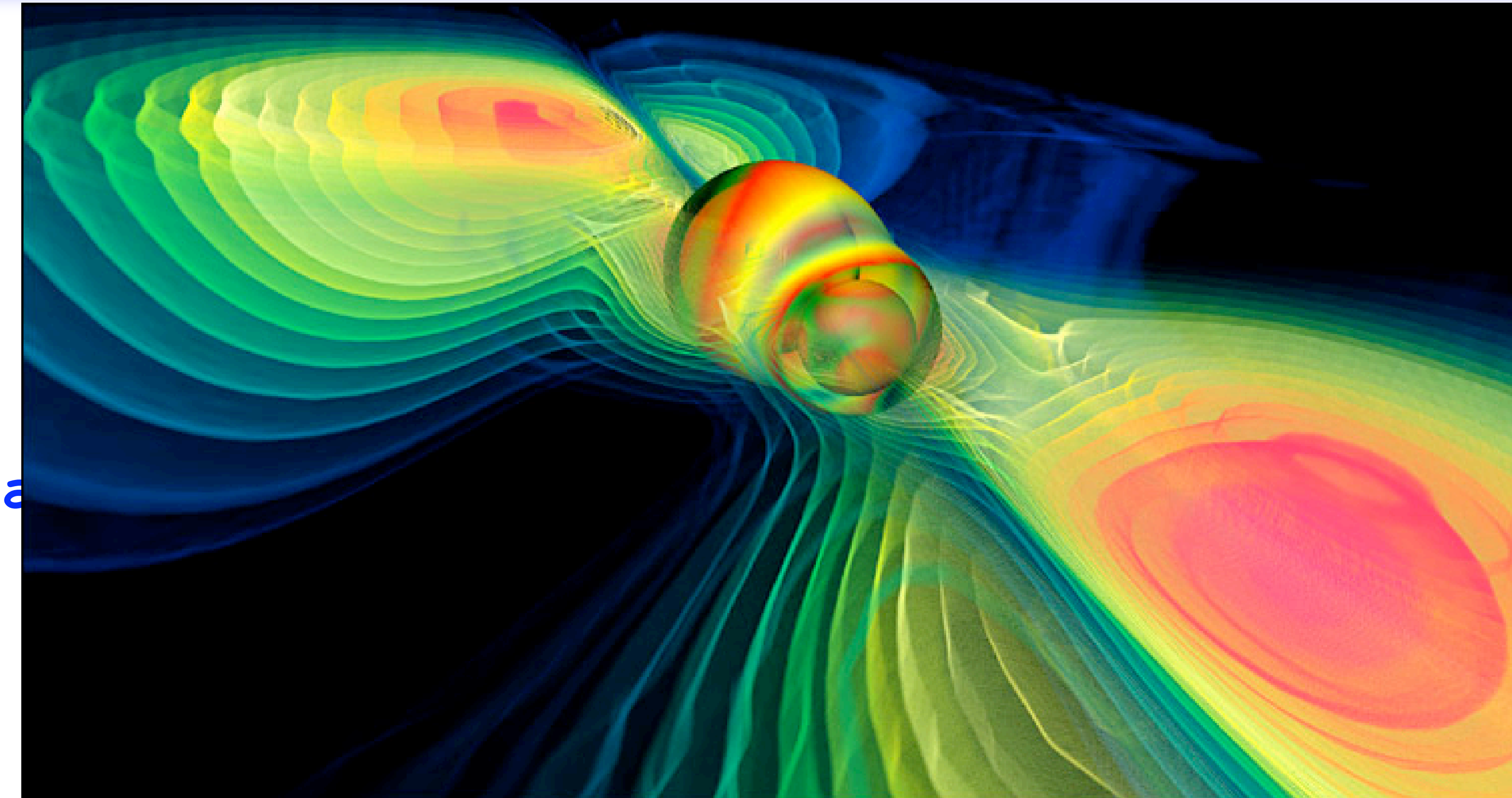
Cactus-Carpet-Whisky



PerBaCCo

Startegy: the double approach

Ca



Co

1D time-domain code: PerBaCCo

PerturBative Constrained Code

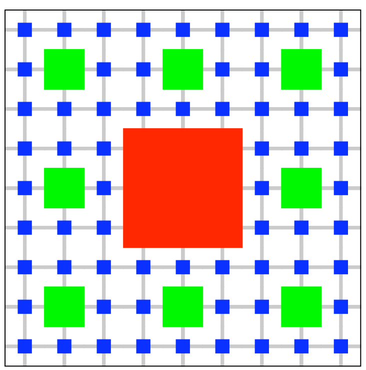
- All kind of TOV perturbations (RW gauge, spherical coord.)
- Radial, Axial and Polar perturbations: (constrained) wave Eqs
- Standard II order FD schemes
  - ✓ Even-parity: constrained algorithm
- Use tabulated equations of state (EOS) for nuclear matter
- Zerilli-Moncrief (even-parity) and Regge-Wheeler (odd-parity) gauge invariant functions

$$h_+ - \imath h_\times = \frac{1}{r}\sum_{l=2}^{\infty}\sum_{m=-\ell}^{\ell} N_\ell \left(\Psi^{(e)}_{\ell,m} + \imath\Psi^{(o)}_{\ell,m}\right) {}_{-2}Y_{\ell,m}(\theta,\phi)$$

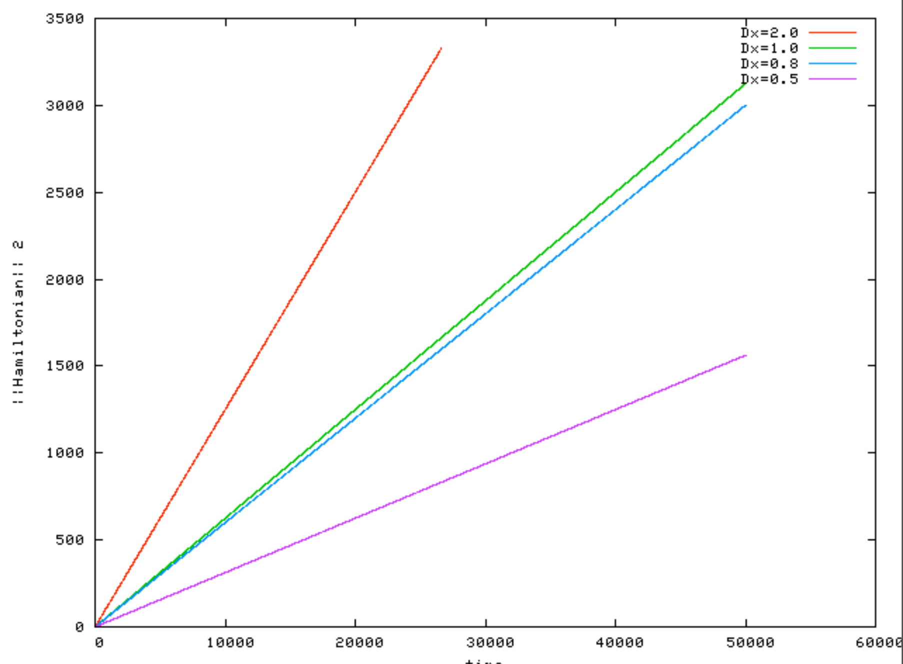
REFs: [Nagar 2004 / gr-.qc/0408041 2004 / Nagar et al. 2004 / Bernuzzi et al 2008 / Bernuzzi & Nagar 2008]

Cactus-Carpet-Whisky: *setup*

- *Metric/Matter evolution:*
  - ✓ (ADM) NOK-BSSN + GRHD Cons Form
  - ✓ gauge: “1+log” + Gamma Driver
  - ✓ MoL: ICN
  - ✓ HRSC: Marquina + PPM



- *Grid:*
  - ✓ 3 cubic boxes, Dx=0.5
  - ✓ Octant Sym
  - ✓ CFL = 0.25



Developed mainly @ **AEI, LSU**

Computer Cluster in Parma

★ ALBERT2

INFN Istituto Nazionale di Fisica Nucleare

✳ ALBERT100

- ★ 16 nodes: bi-processor opteron 2 GHz
- ★ 4 GB RAM
- ★ 3 TB RAID 5 storage
- ★ infiniband

- ✳ 32 nodes: bi-processor Pentium III - 1.5 GB RAM
- ✳ 100BaseT fast ethernet
- ✳ Peak : 100 Gflops

Initial Data: Whisky_PerturbTOV

- TOV eqs (Whisky_TOVSolverC)
- Perturbation (Whisky_PerturbTOV):
  - ✓ add pressure perturbation
  - ✓ solve (perturbative) constraints for each multipoles
  - ✓ construct perturbed metric
    - Fix a specific multipole (l constraint eq)
    - Axisymmetric pressure perturbation
    - Metric perturbation:

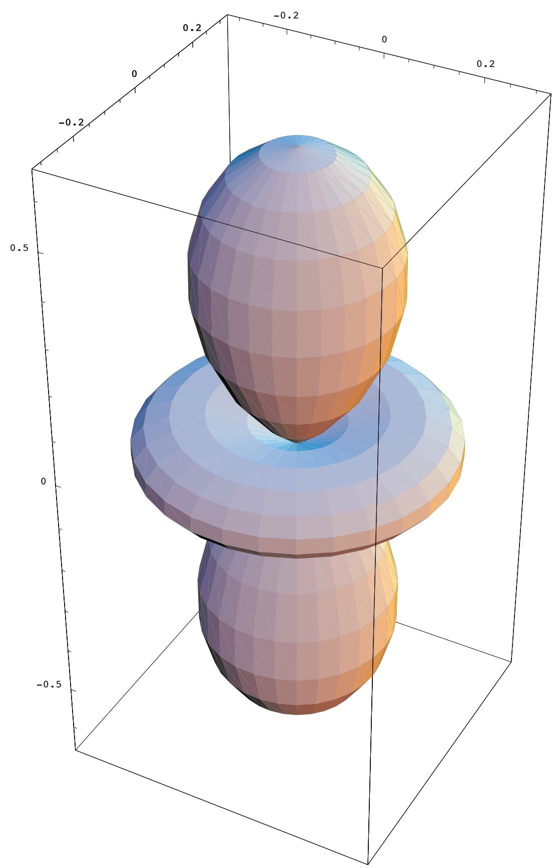
$$\delta s^2_{\ell 0} = (\chi_{\ell 0} + k_{\ell 0}) e^{2a} dt^2 - 2\psi_{\ell 0} e^{a+b} dt d\bar{r} + e^{2b} \left[(\chi_{\ell 0} + k_{\ell 0}) d\bar{r}^2 + \bar{r}^2 k_{\ell 0} d\Omega \right] Y_{\ell 0}$$

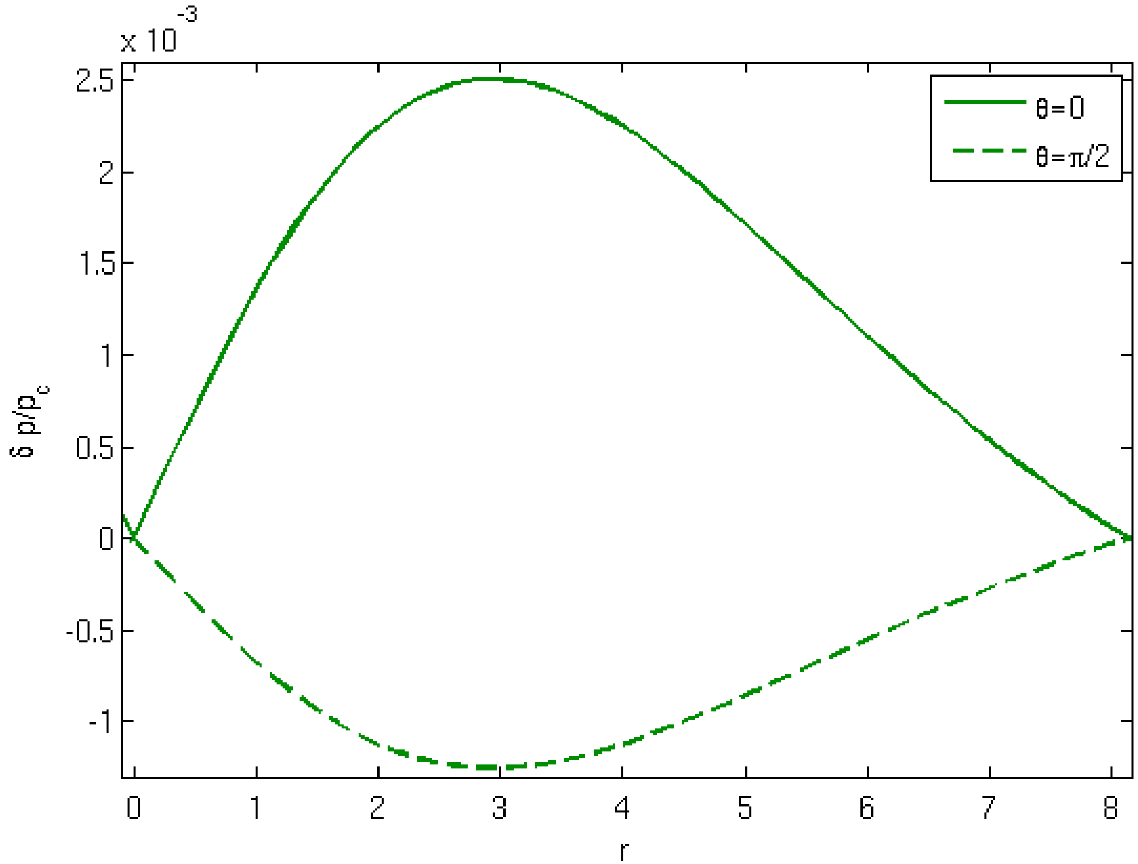
Matter perturbation

Perturbed pressure: $\delta p(r,\theta) \equiv (p+\mu) H_{\ell 0}(r) Y_{\ell 0}(\theta)$

Enthalpy profile:

$$H = h \sin\left[\frac{(n+1)\pi r}{2R}\right]$$

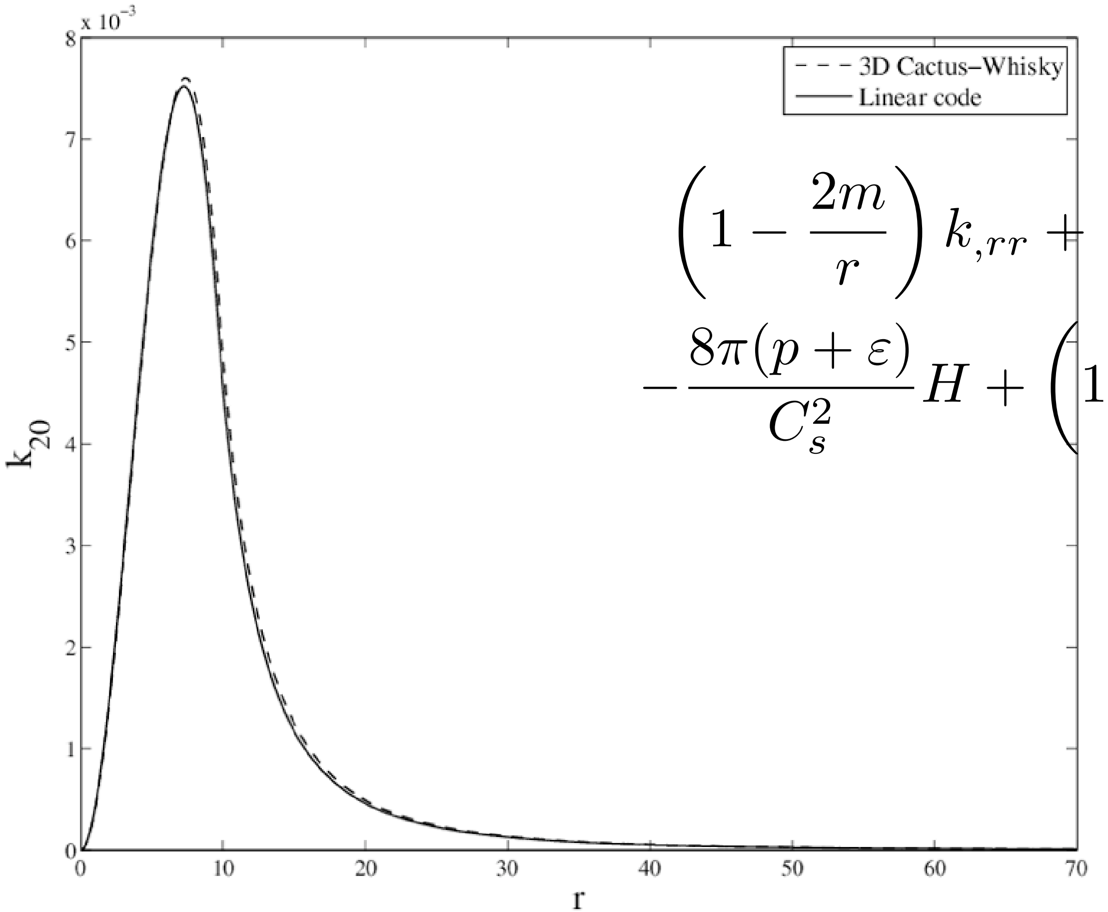




Quadrupolar mode:

$$\ell = 2$$

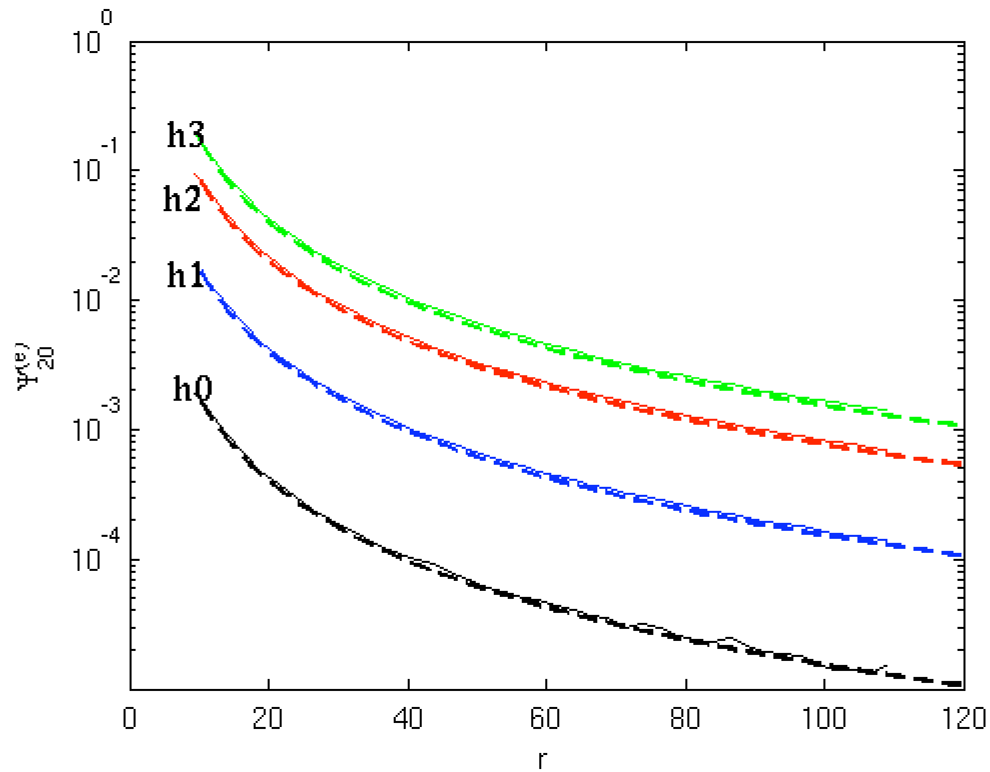
(Linearised) Hamiltonian constraint solution



$$\left(1-\frac{2m}{r}\right)k_{,rr}+\left[\frac{2}{r}-\frac{3m}{r^2}-4\pi\varepsilon r\right]k_{,r}-\left[\frac{\Lambda}{r^2}-8\pi\varepsilon\right]k=$$
$$-\frac{8\pi(p+\varepsilon)}{C_s^2}H+\left(1-\frac{2m}{r}\right)\chi_{,r}+\left[\frac{2}{r}-\frac{2m}{r^2}+\frac{\Lambda}{2r}-8\pi\varepsilon r\right]\chi$$

Conformally Flat ID (fluid modes):

$$\chi_{\ell 0}=0$$



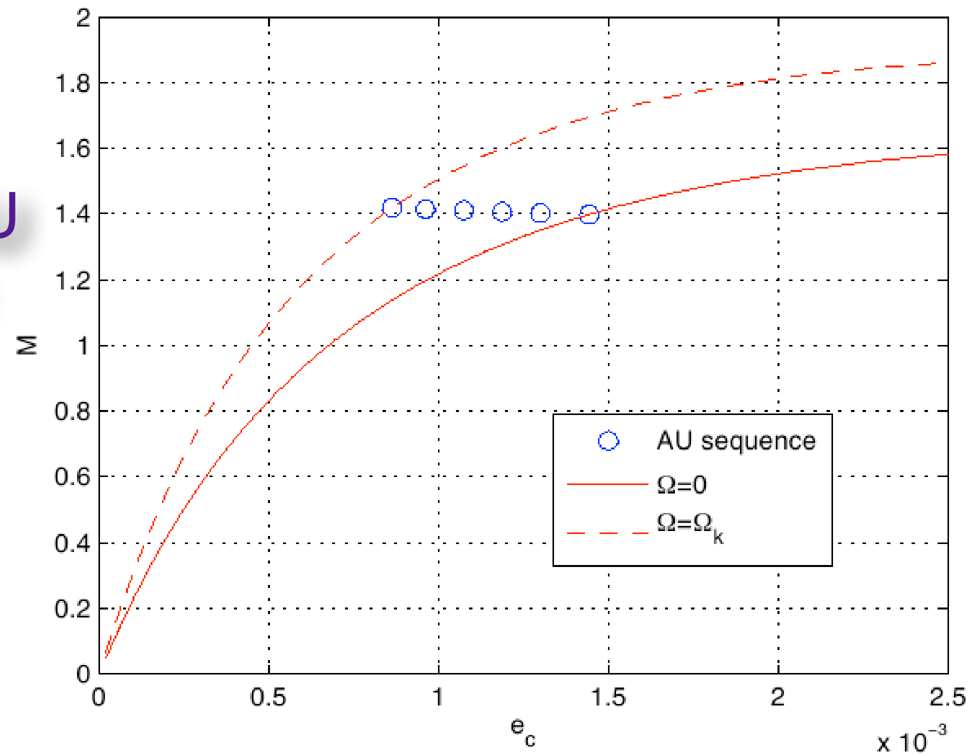
Equilibrium model and radial modes

- Perfect fluid, Polytropic Model A0 $M = 1.4M_\odot \quad \rho_c = 1.28 \times 10^{-3} \quad R = 9.57$
- Stable Evolution unperturbed model (Radial Modes)

n	Pert.[Hz]	3D [Hz]	Diff. [%]
0	1462	1466	0.3
1	3938	3935	0.1
2	5928	5978	0.8

- Stable Evolution of the sequence AU (uniformly rotating models and fixed mass)

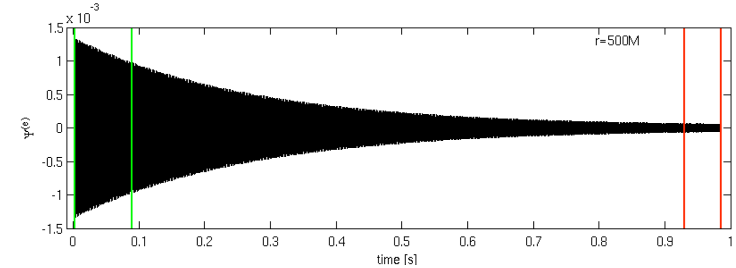
MODEL	F [Hz]	F(CF) [Hz]
AU0	1466	1458
AU1	1369	1398
AU2	1329	1345
AU3	1265	1283
AU4	1166	1196
AU5	1093	1107

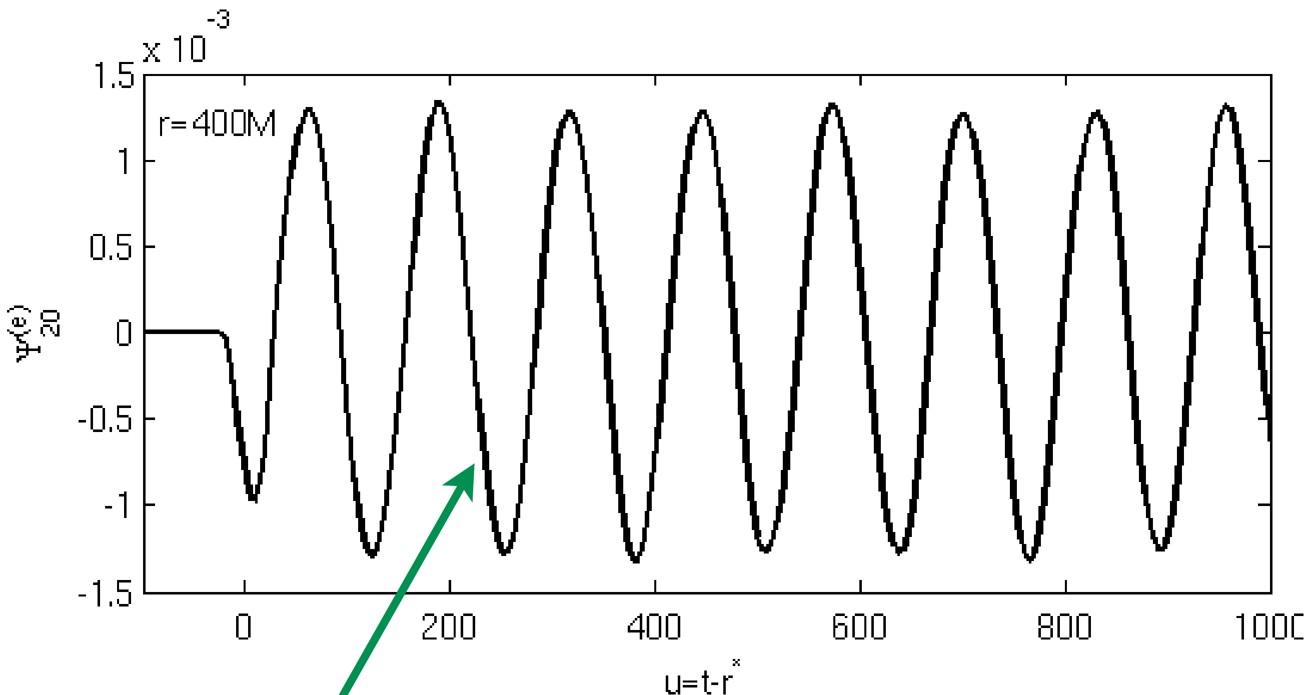


[Dimmelmeier et al 2007]

Even-parity perturbative waves: identikit

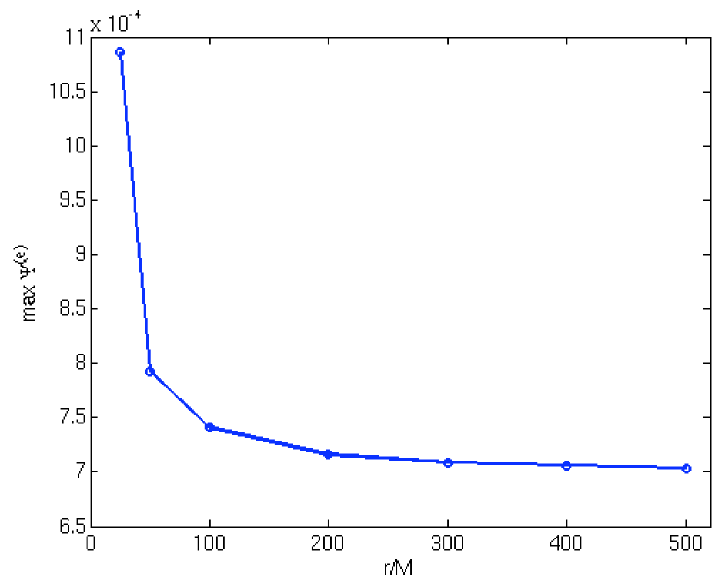
- Radial grid with 300pts inside the star
- Long evolution (about 1 sec)

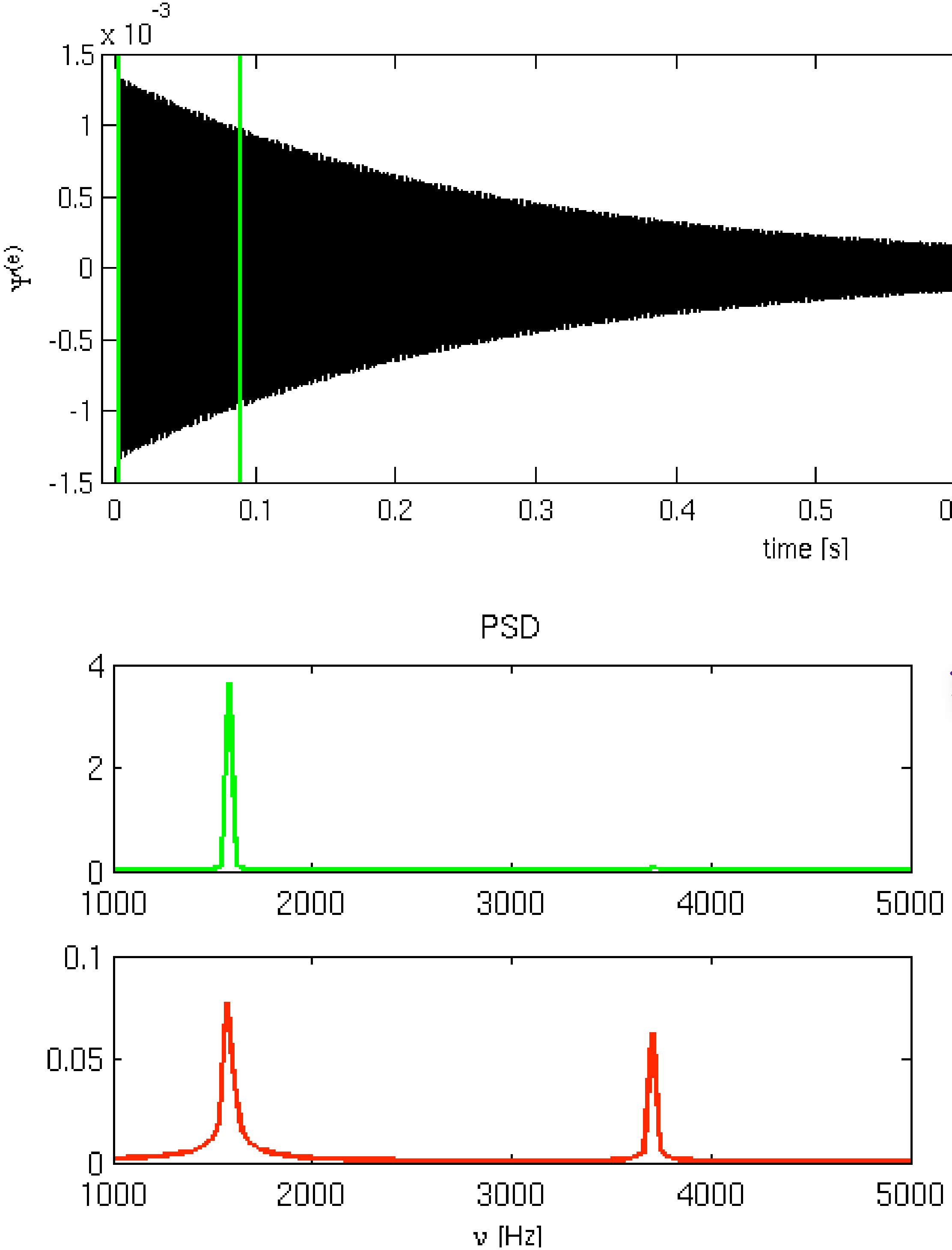




f-mode

1. Fourier anaysis
2. Fit analysis
3. Finite extraction effects



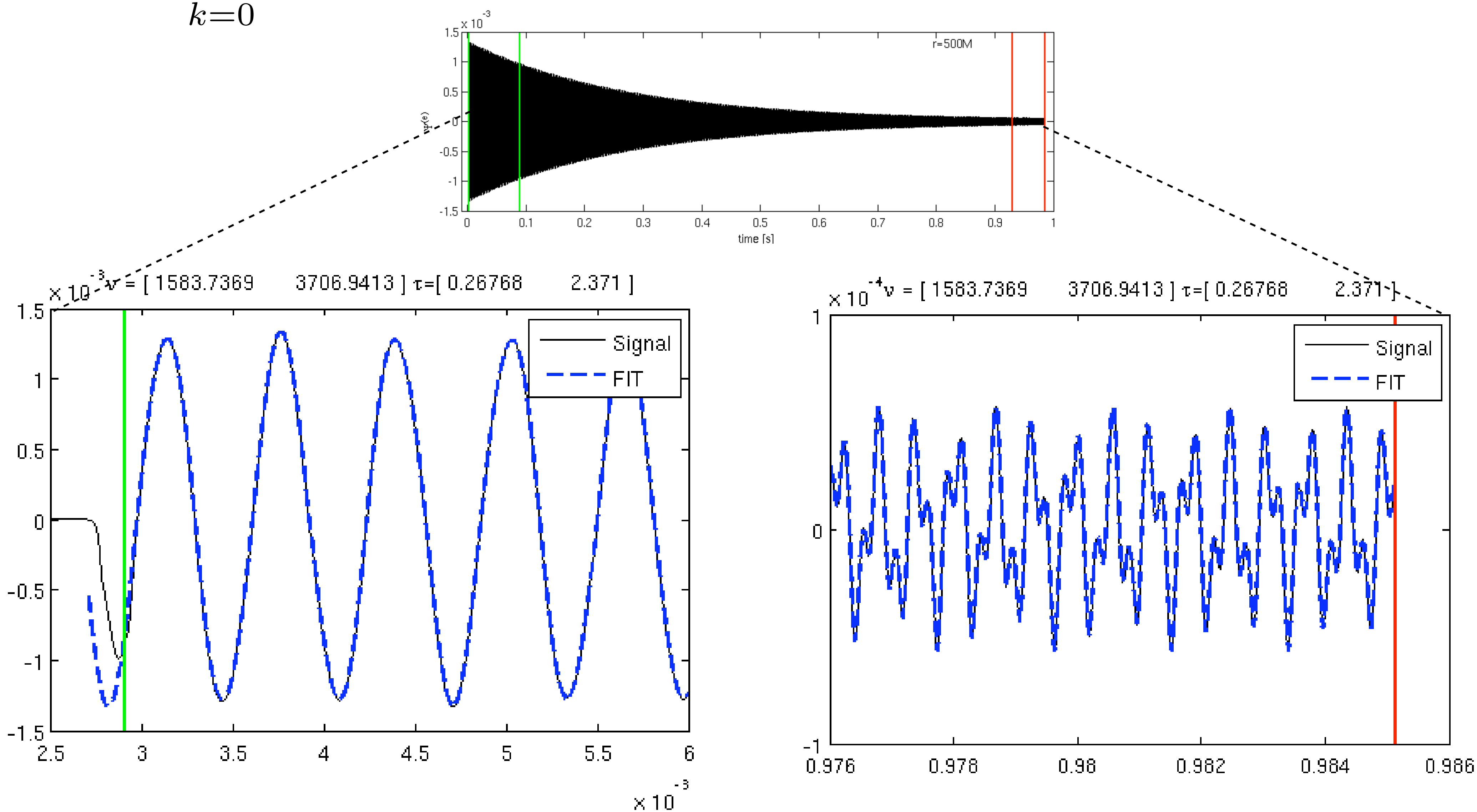


1. Fourier analysis:

ν_f	1581 Hz
ν_{p_1}	3724 Hz

2. Fit analysis - QNMs template:

$$\Psi_{20}^{(e)} \sim \sum_{k=0}^{N} A_{2k} \cos(2\pi\nu_{2k}t + \phi_{2k}) \exp(-\alpha_{2k}t) \qquad N = 2$$



2. Fit analysis - results:

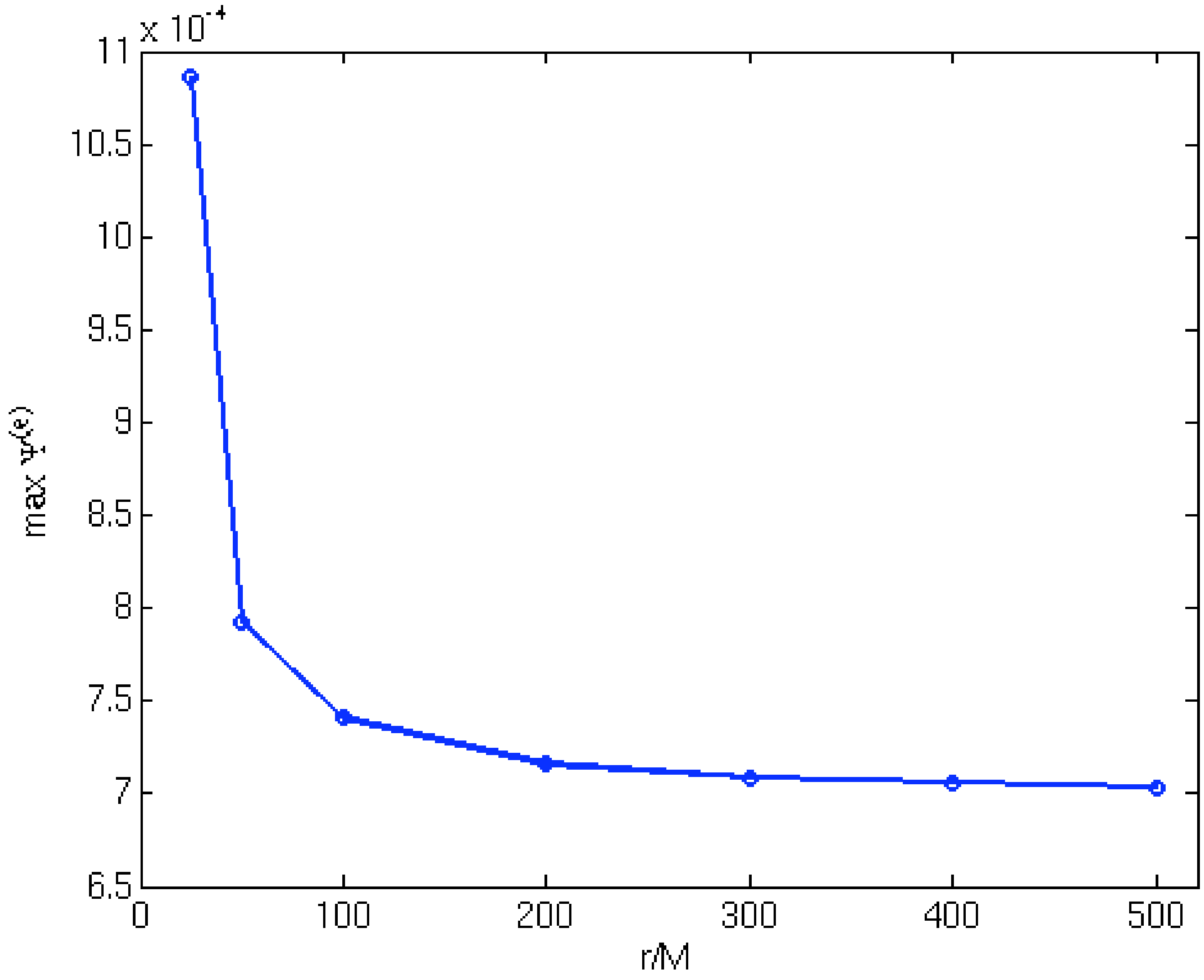
Parameter	Value	Conf-	Conf+
ν_{20}	1.5837369e+03	1.5837368e+03	1.583737e+03
ν_{21}	3.7069413e+03	3.7069401e+03	3.7069424e+03
α_{20}	3.7358	3.7349	3.7367
α_{21}	4.22e-01	4.15e-01	4.29e-01
A_{20}	1.31452e-03	1.31430e-03	1.31475e-03
A_{21}	3.52e-05	3.50e-05	3.53e-05
ϕ_{20}	2.809e-01	2.807e-01	2.811e-01
ϕ_{21}	3.965e-01	3.929e-01	4.002e-01

Damping Times:

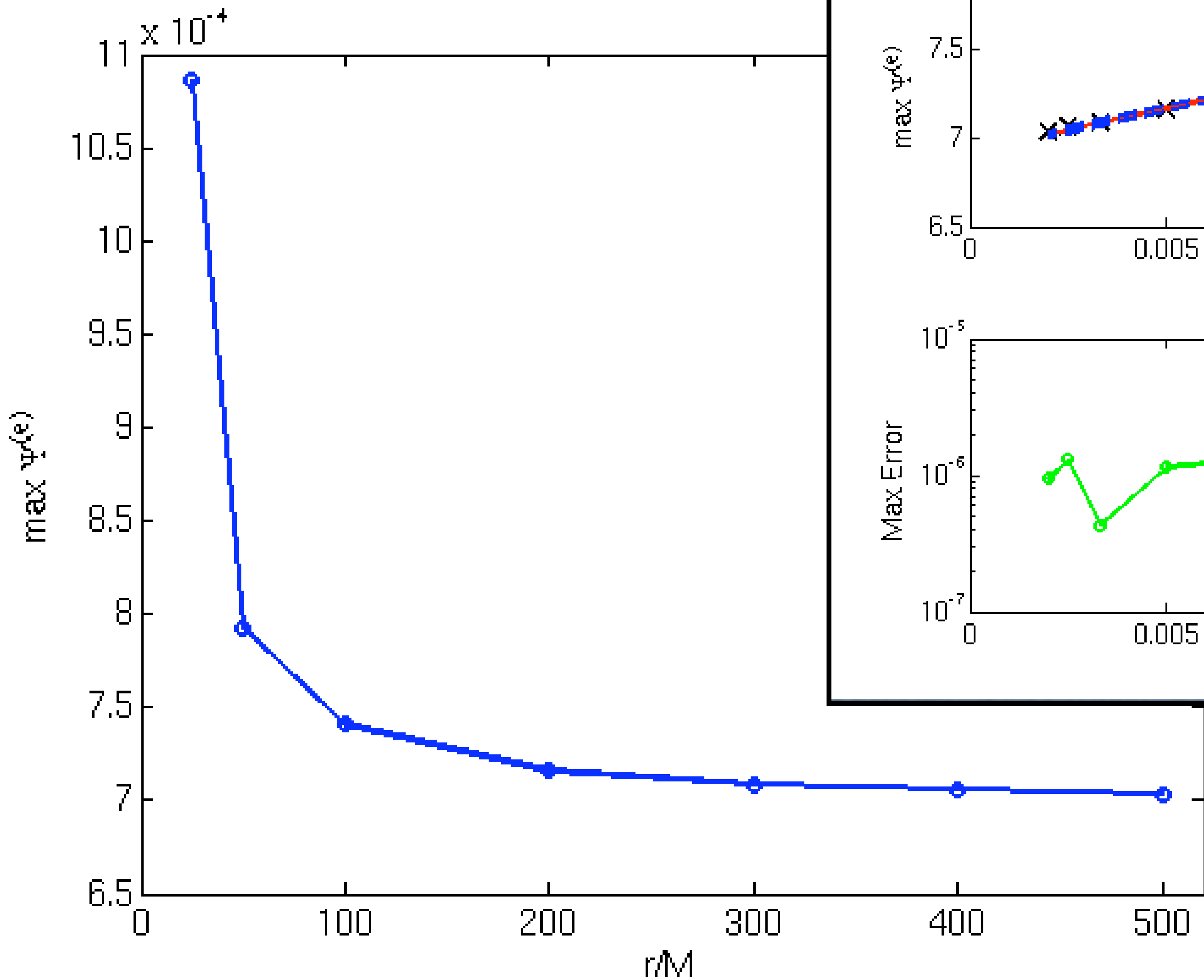
$$\tau_f = 0.268 \ \text{ sec } \ (0.1\%)$$

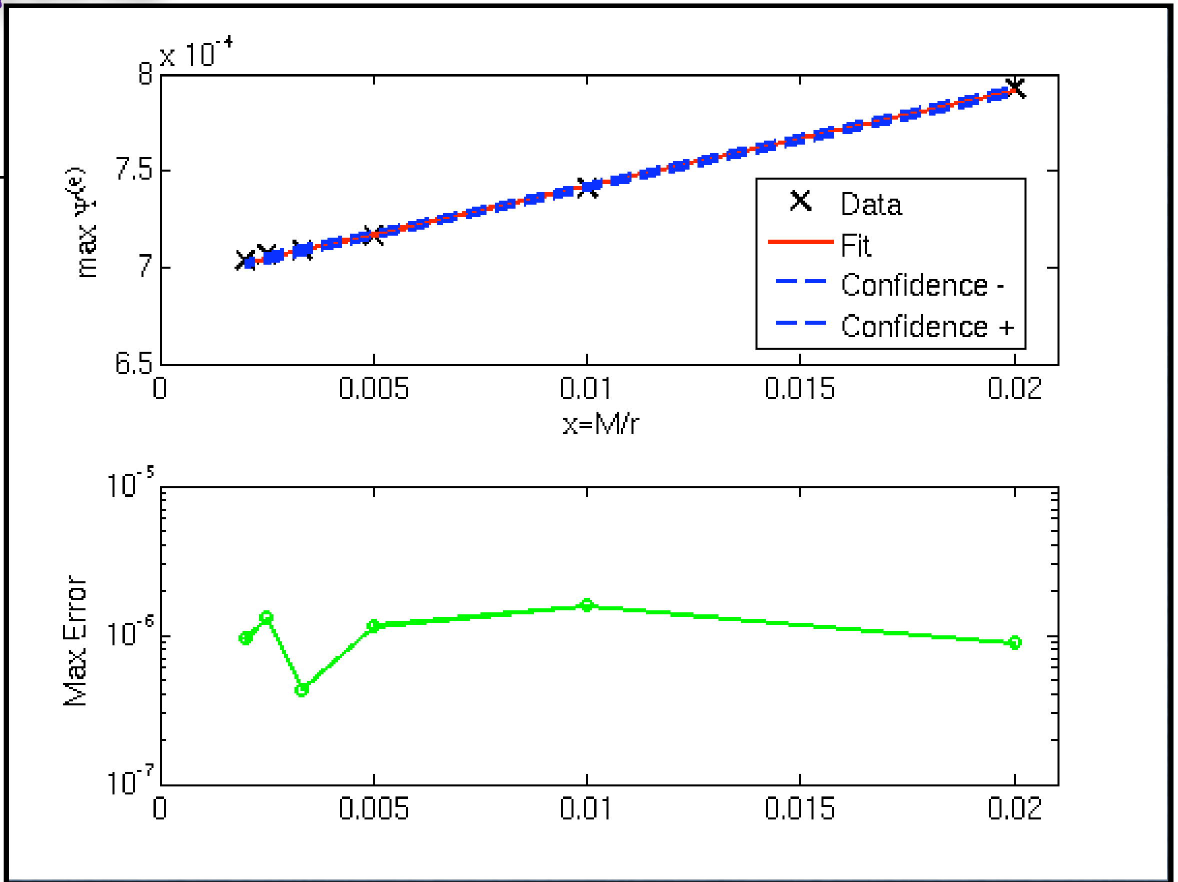
$$\tau_{p_1} = 2.28 \ \text{ sec } \ (2\%)$$

3. Finite extraction effects



3. Finite extraction effects





$$\max \Psi^{(e)}(r) \sim A^{\infty} + A^{1}\frac{M}{r} + \dots$$

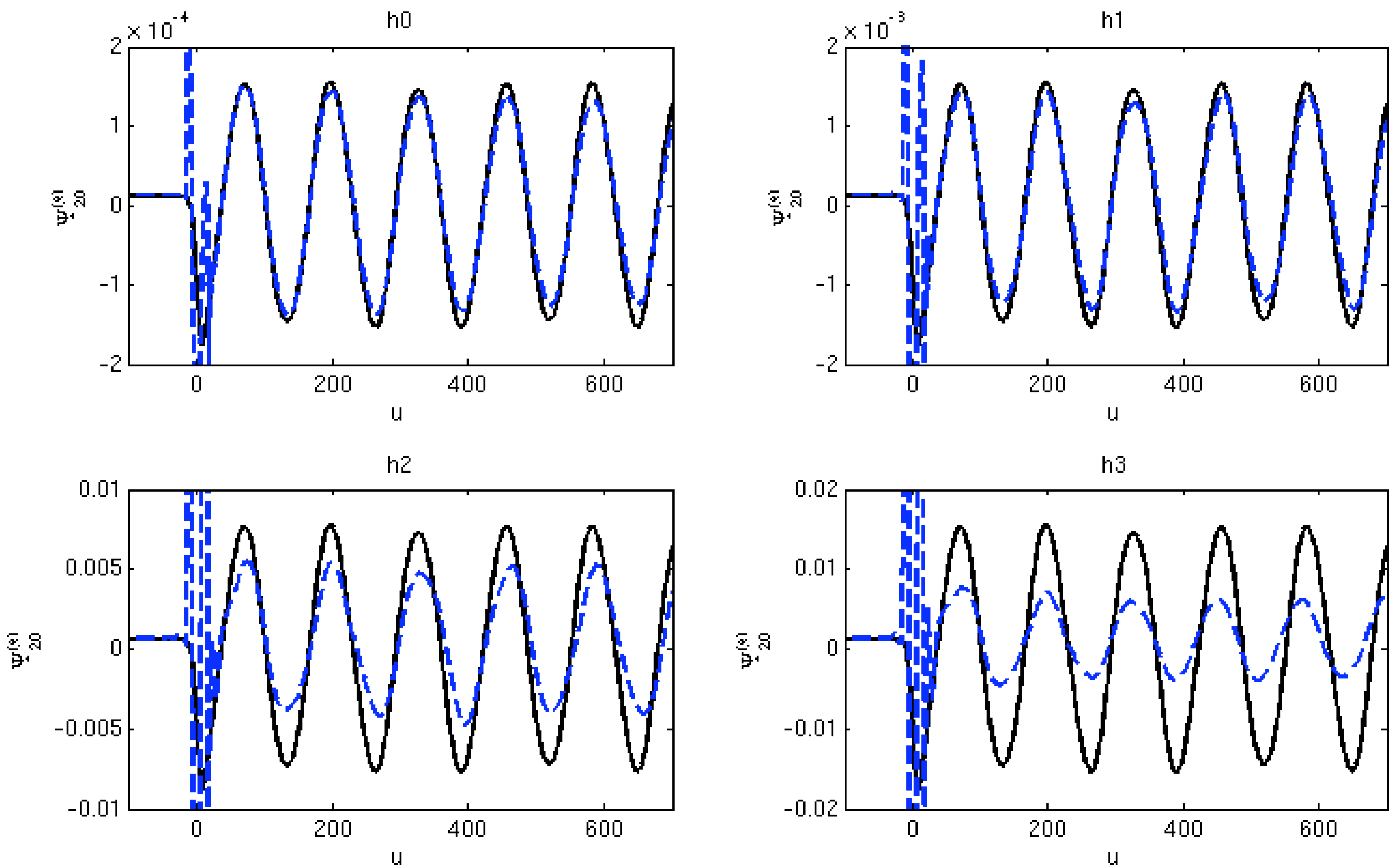
r [M]	$\delta A / A^{\infty}_{fit}$
25	49.30%
50	8.90%
100	1.74%
200	1.62%

Comparing 1D Vs 3D Waves

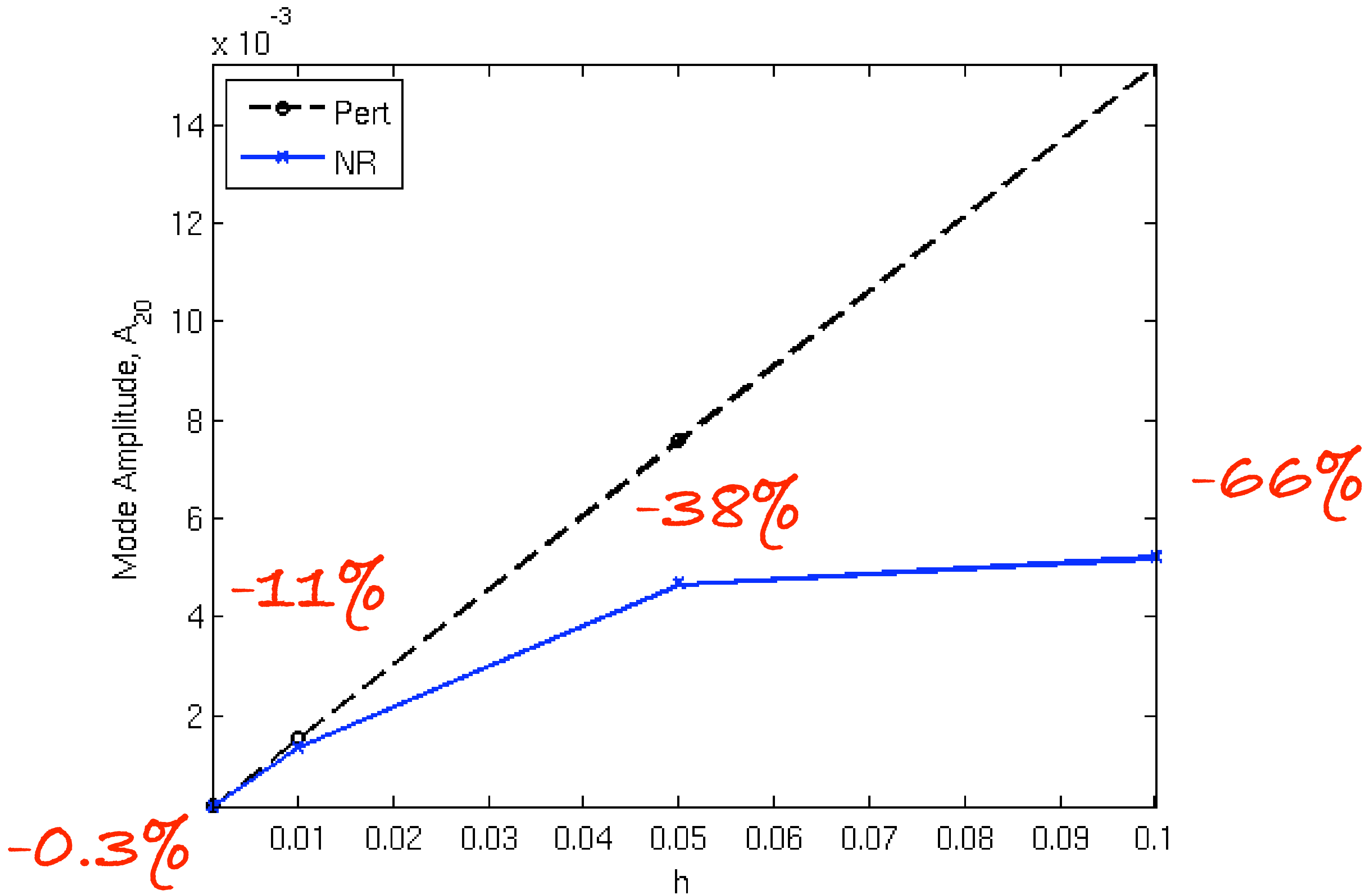
- Different values of the initial perturbation amplitude:

$$h = [0.001, 0.01, 0.05, 0.1] := [h0, h1, h2, h3]$$

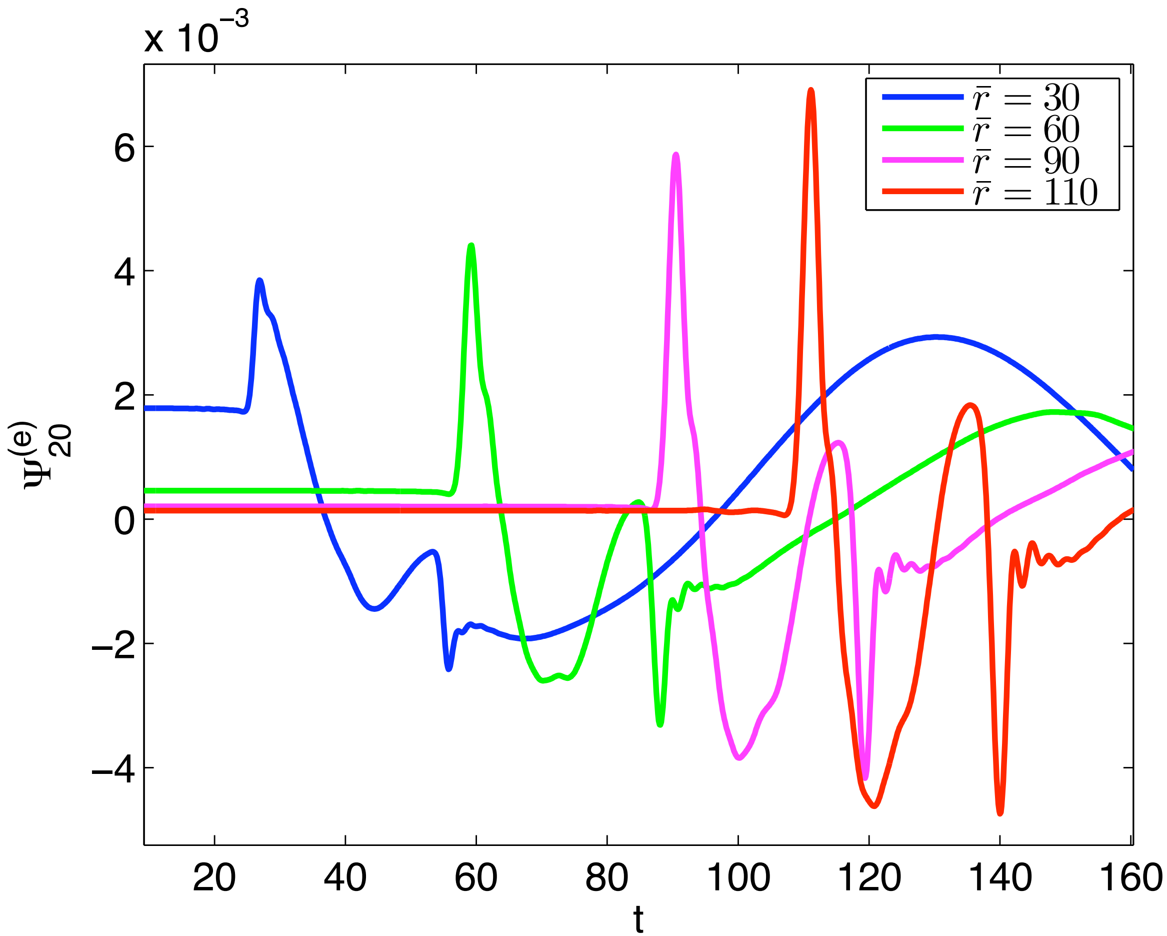
- Wave Extraction at $r = 80M$



h	ν_{3D}^{f} [Hz]	Diff.[%]	$\nu_{3D}^{p_1}$ [Hz]	Diff.[%]
h0	1578	0.2	3705	0.5
h1	1576	0.3	3705	0.5
h2	1573	0.5	3635	2.4
h3	1623	2.7	3565	4.3



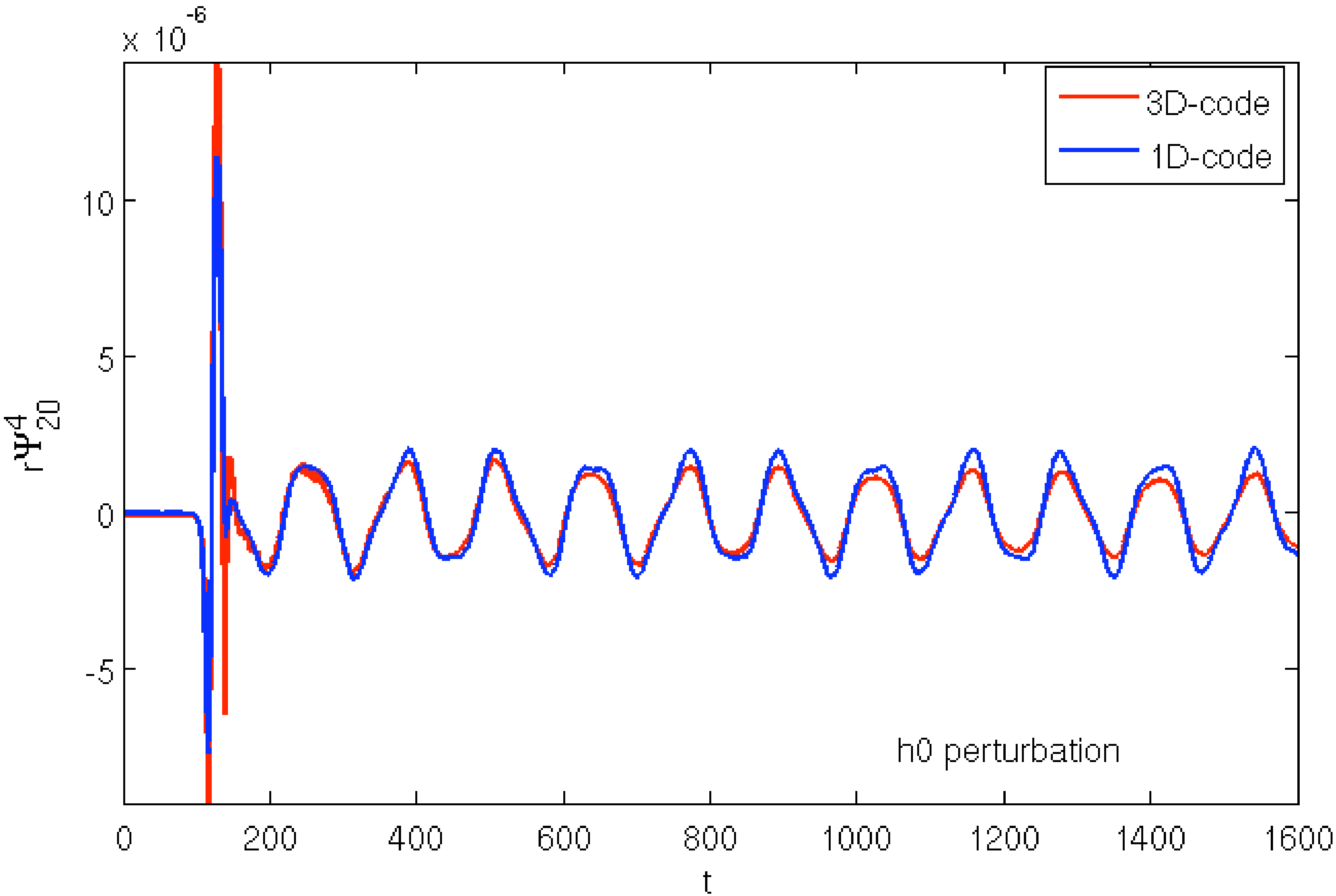
Initial "burst"



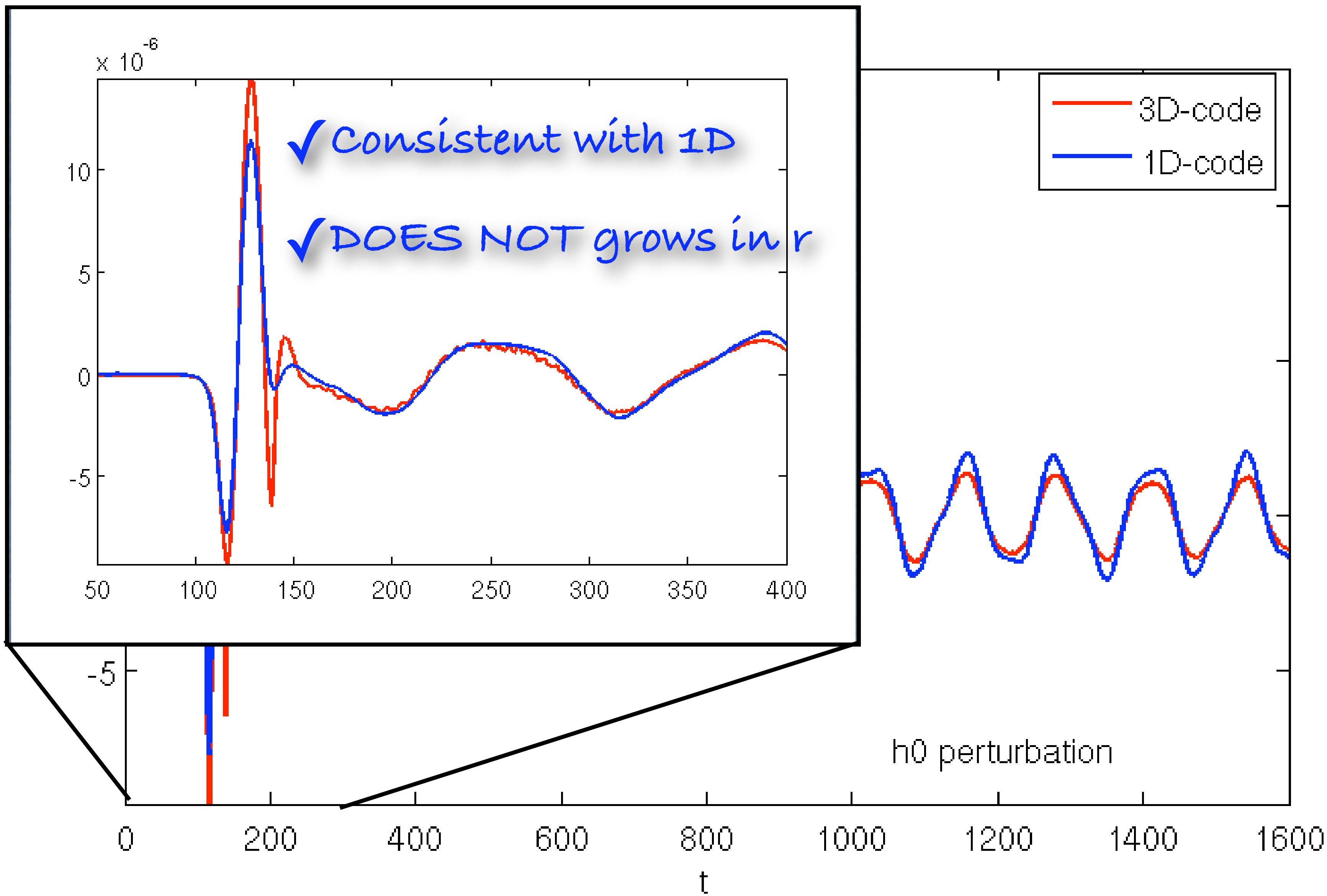
- ✓High frequency oscillations
- ✓linear grows in r
- ✓increases with resolution
- ✓greater for higher initial perturbation amplitude
- ✓smaller when full constraints are solved

unphysical and related to the constraint violation and to the Zerilli 3D extraction ...

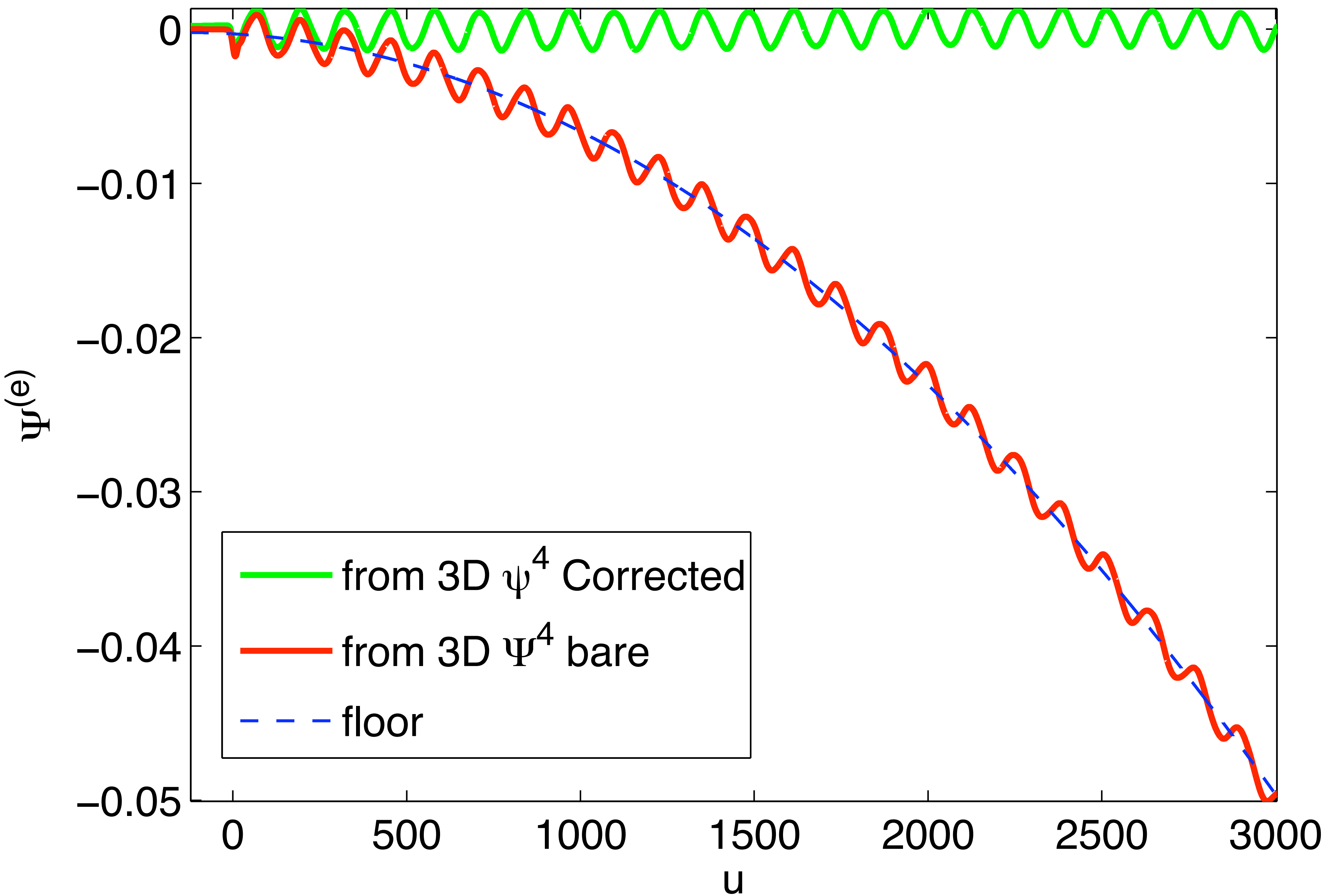
Ψ_4 extraction



Ψ_4 extraction



How to recover the Zerilli's ?



How to recover the Zerilli's ?

$$
\begin{aligned}
\Psi^{(e)}(t) &\propto \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \left\{ \lim_{r\to\infty} \left[r\Psi^4(t'',r) \right] \right\} \\
&= Q_0 + Q_1 t + \int_0^t dt' \int_0^{t'} dt'' \left\{ \lim_{r\to\infty} \left[r\Psi^4(t'',r) \right] \right\} \\
&= Q_0 + Q_1 t + \int_0^t dt' \int_0^{t'} dt'' \left(r\Psi^4(t'',r) + f(t'',r) \right) \\
&= Q_0 + Q_1 t + \left[\int_0^t dt' \int_0^{t'} dt'' r\Psi^4(t'',r) \right] + \sum_{k=2}^{n} F_k(r) t^k + \ldots
\end{aligned}
$$

How to recover the Zerilli's ?

1. "off-set" function

$$
\begin{aligned}
\Psi^{(\mathrm{e})}(t) &\propto \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \left\{ \lim_{r\to\infty} \left[r\Psi^4(t'',r) \right] \right\} \\
&= Q_0 + Q_1 t + \int_0^t dt' \int_0^{t'} dt'' \left\{ \lim_{r\to\infty} \left[r\Psi^4(t'',r) \right] \right\} \\
&= Q_0 + Q_1 t + \int_0^t dt' \int_0^{t'} dt'' \left(r\Psi^4(t'',r) + f(t'',r) \right) \\
&= Q_0 + Q_1 t + \left[\int_0^t dt' \int_0^{t'} dt'' r\Psi^4(t'',r) \right] + \sum_{k=2}^{n} F_k(r) t^k + \dots
\end{aligned}
$$

How to recover the Zerilli's ?

1. "off-set" function

$$\Psi^{(e)}(t) \propto \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \left\{ \lim_{r\to\infty} \left[r\Psi^4(t'',r) \right] \right\}$$

$$= Q_0 + Q_1 t + \int_0^t dt' \int_0^{t'} dt'' \left\{ \lim_{r\to\infty} \left[r\Psi^4(t'',r) \right] \right\}$$

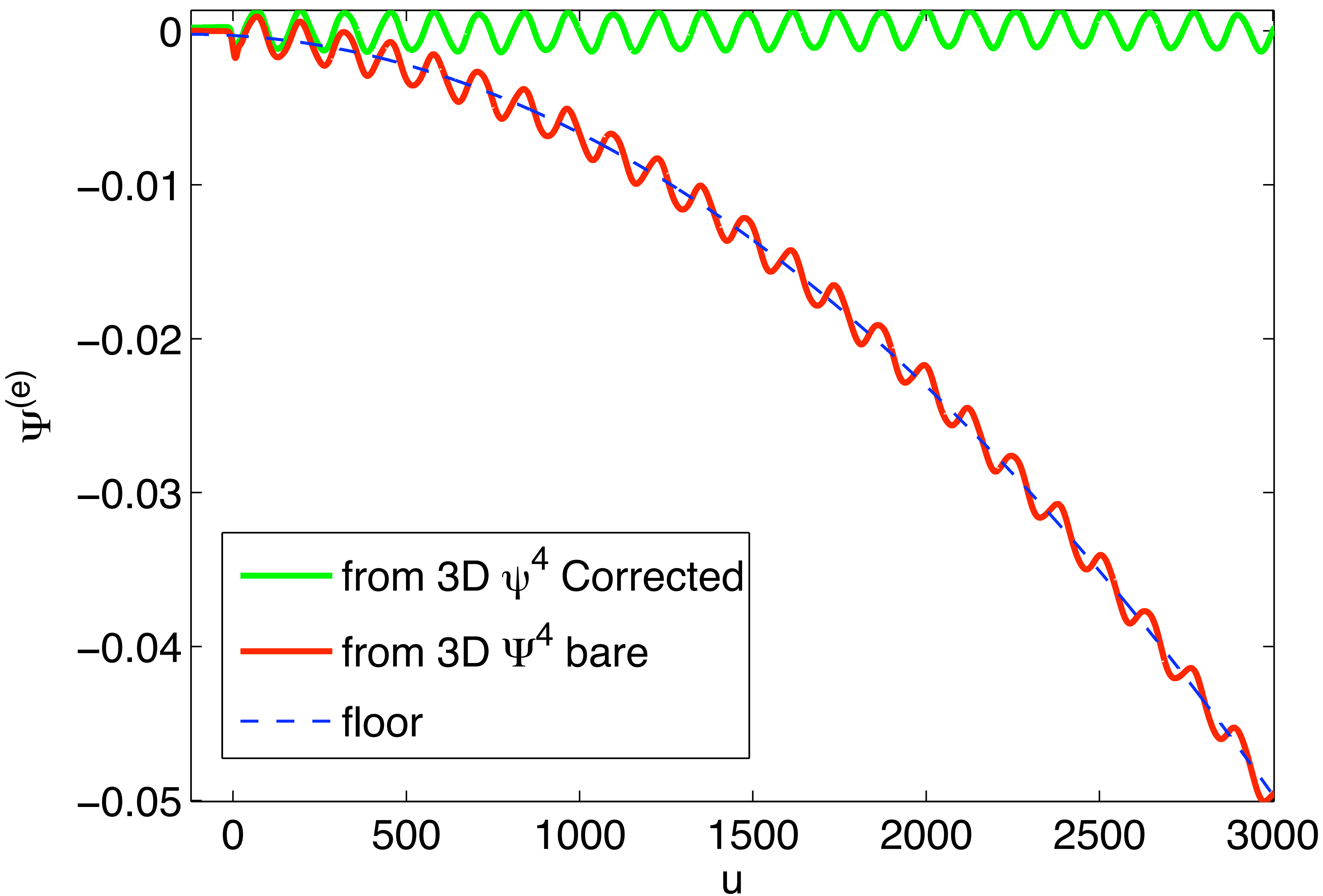
$$= Q_0 + Q_1 t + \int_0^t dt' \int_0^{t'} dt'' \left(r\Psi^4(t'',r) + f(t'',r) \right)$$

$$= Q_0 + Q_1 t + \left[\int_0^t dt' \int_0^{t'} dt'' r\Psi^4(t'',r) \right] + \sum_{k=2}^{n} F_k(r) t^k + \dots$$

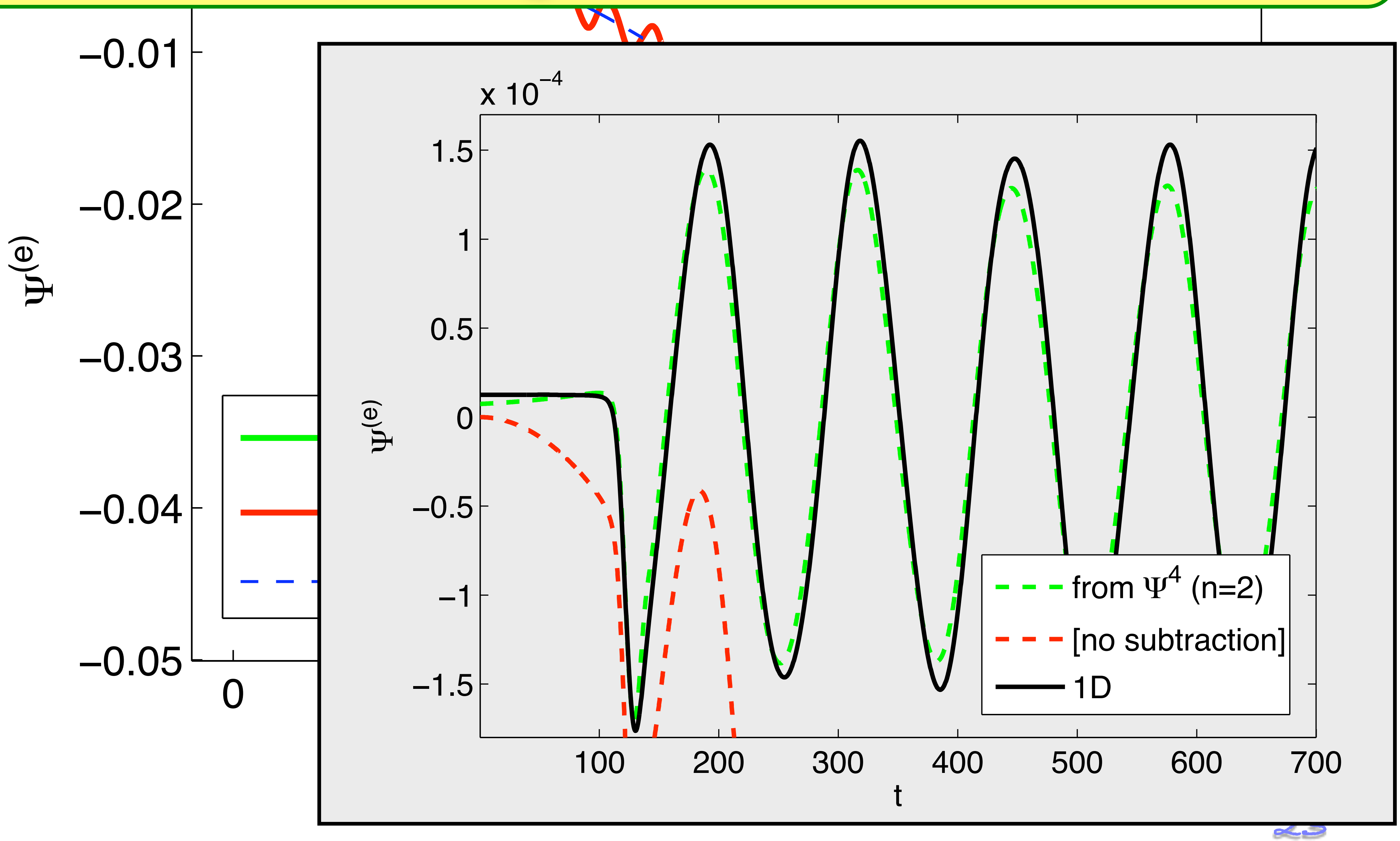
2. "slow" variation

Let's try:

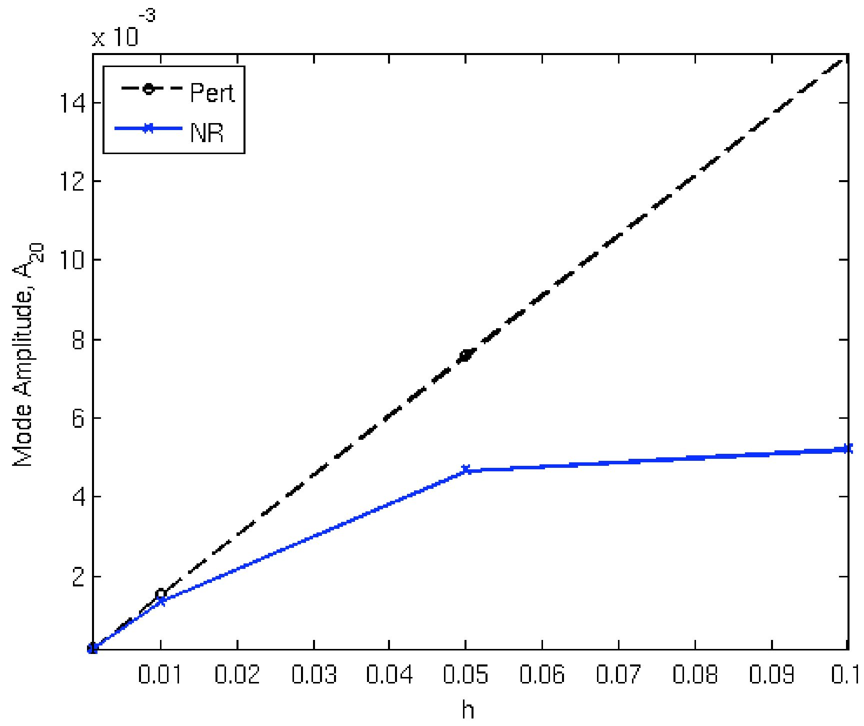
$$\Psi^{(e)}(t,r) \propto \int_0^t dt' \int_0^{t'} dt'' r\Psi^4(t'',r) + Q_0 + Q_1 t + F_2 t^2 + \dots$$


0
−0.01
−0.02
−0.03
−0.04
−0.05
$\Psi^{(e)}$
0
500
1000
1500
2000
2500
3000
u
from 3D ψ^4 Corrected
from 3D Ψ^4 bare
floor

(Once corrected for the floor) Zerilli from the Ψ4 extraction is perfectly consistent with the perturbative



Non-linear effects



Mode couplings:

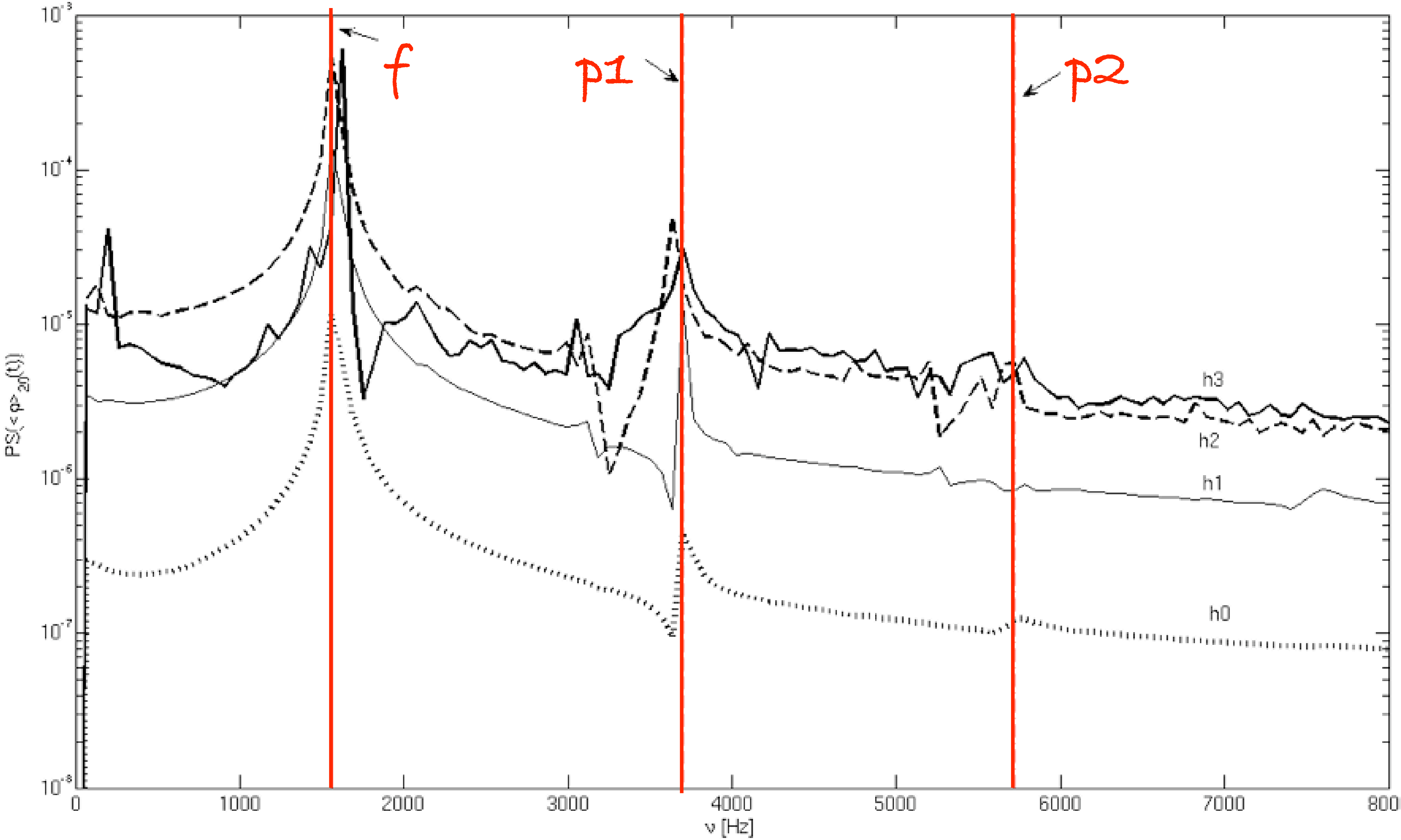
- ✓ non-axisymmetric + odd parity modes : suppressed
- ✓ radial modes
- ✓ ell=4,6 m=0,4 (grid)

STRATEGY:

Fourier analysis of $\langle\rho\rangle_{\ell,m}(t) = \int d^3x\rho(t,\mathbf{x})Y_{\ell,m}$

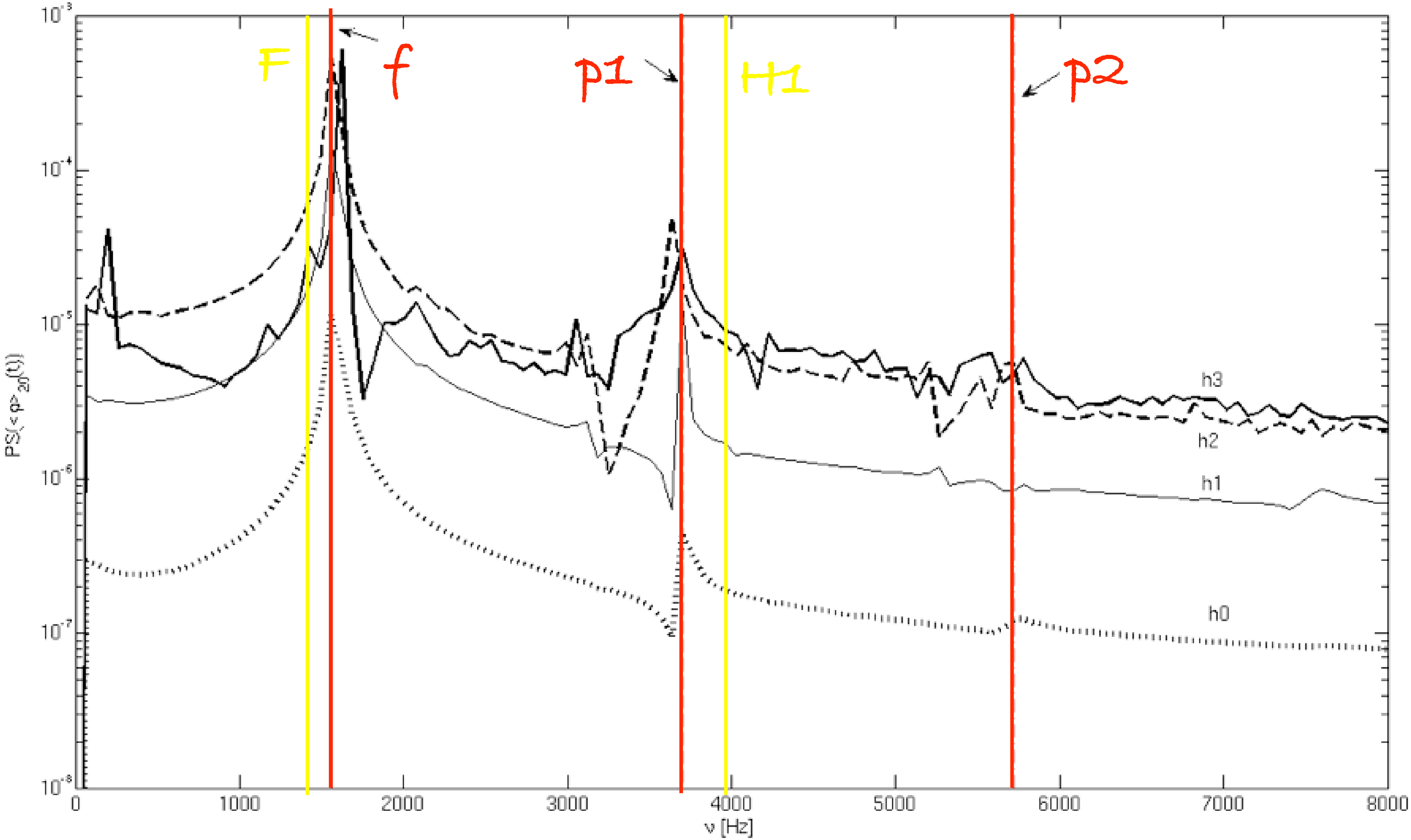
"Weak" couplings $\nu_{coupl} = \nu_1 \pm \nu_2$

$$\langle\rho\rangle_{2,0}$$



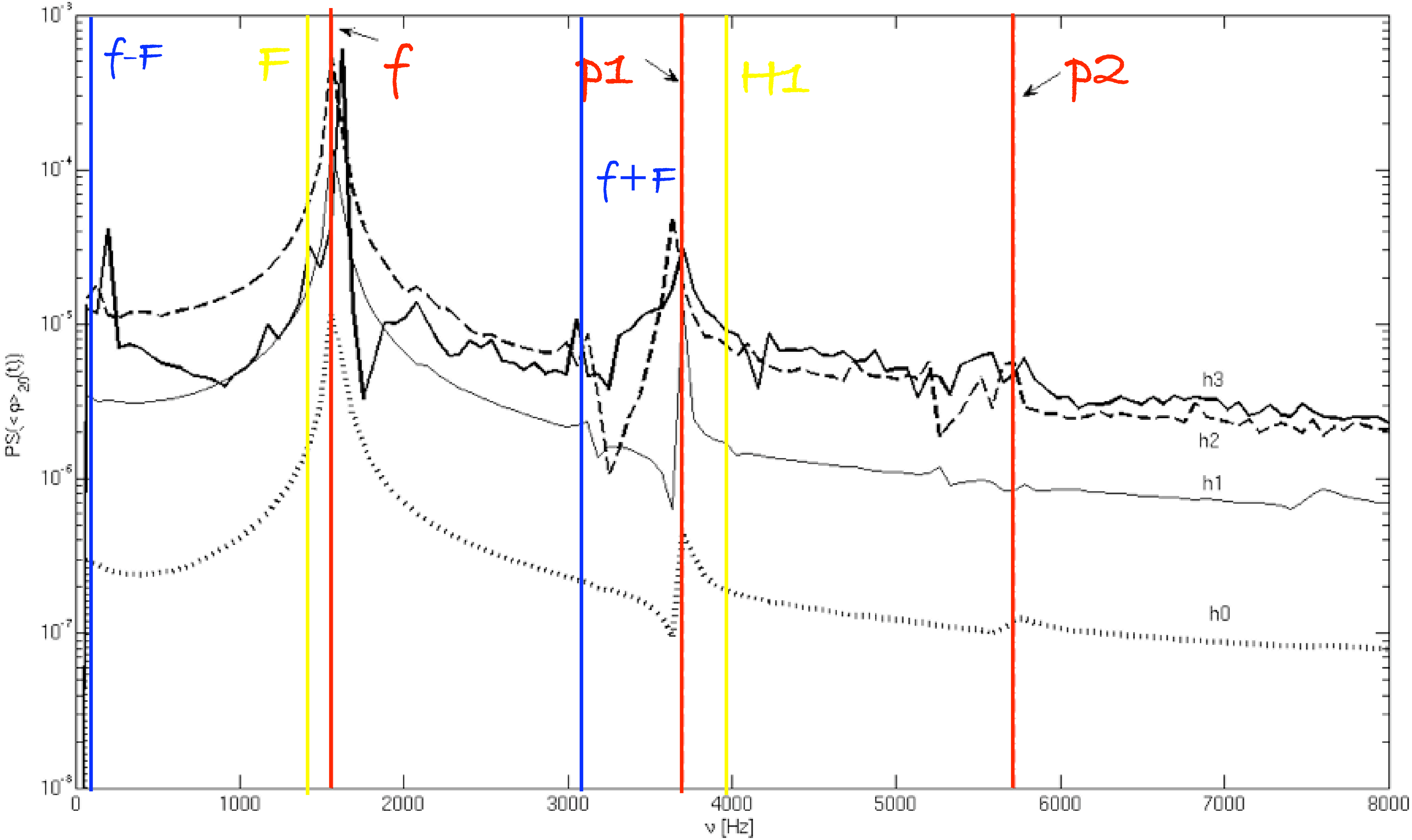
[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle \rho \rangle_{2,0}$



[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle \rho \rangle_{2,0}$



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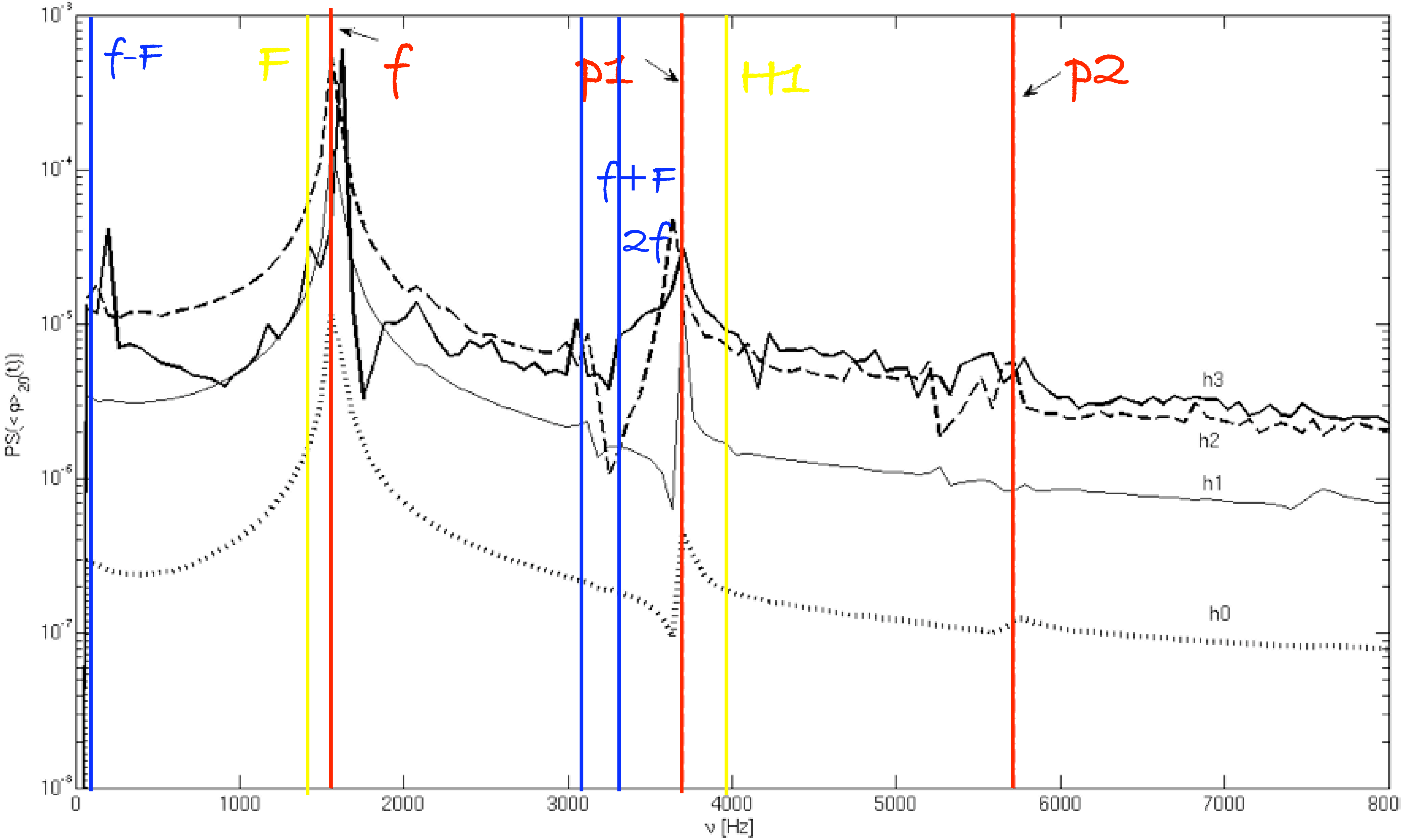
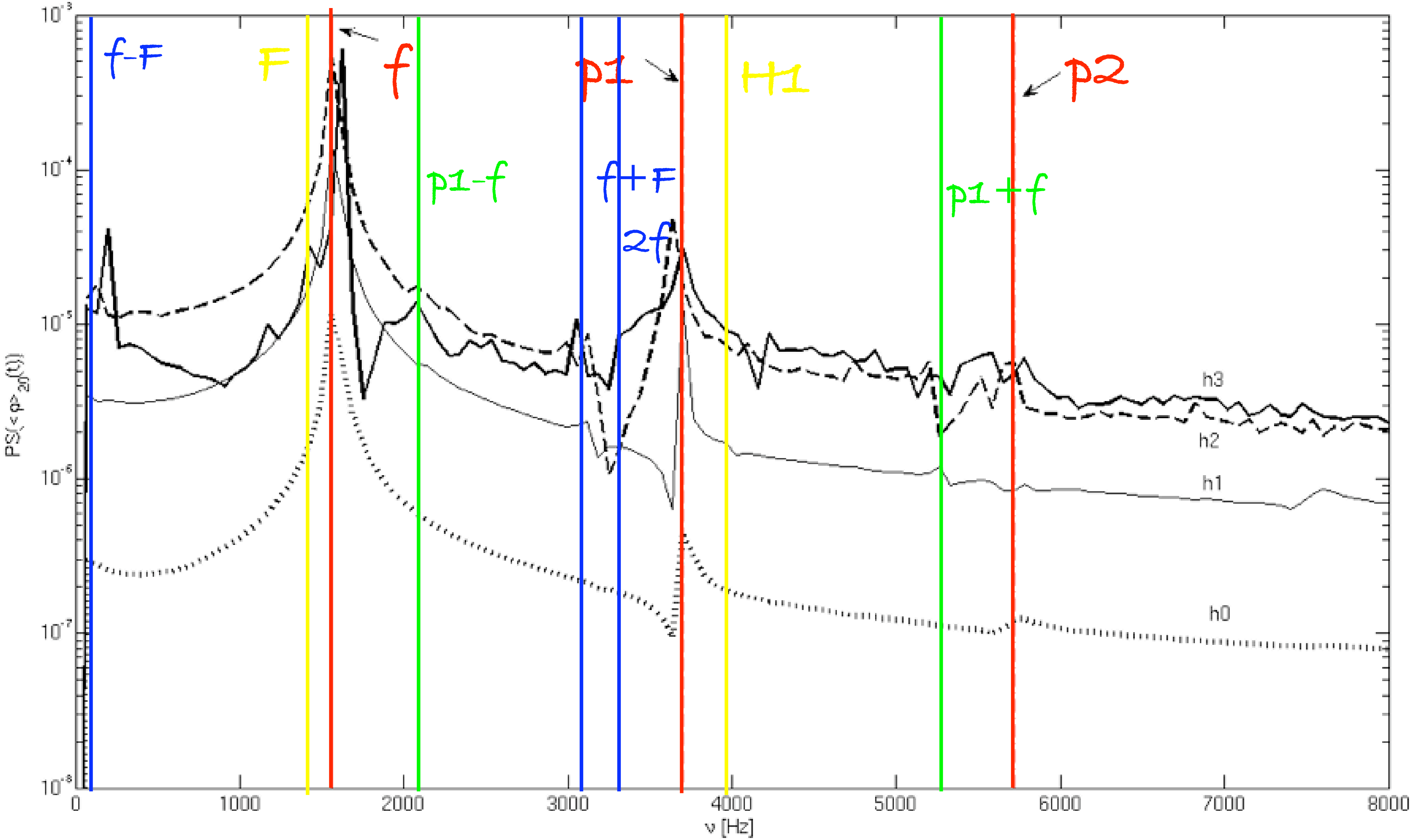
$\langle\rho\rangle_{2,0}$



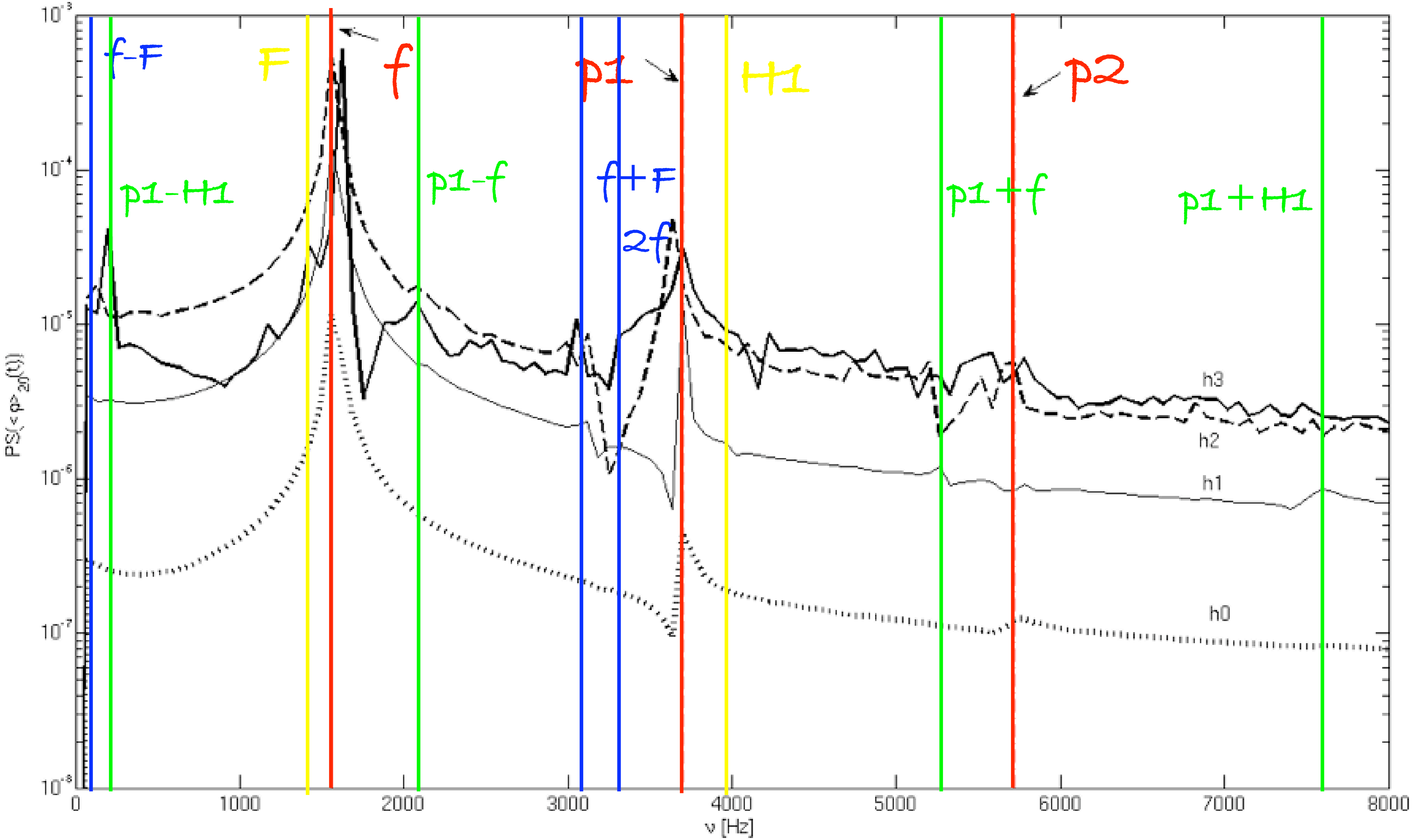
[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle \rho \rangle_{2,0}$



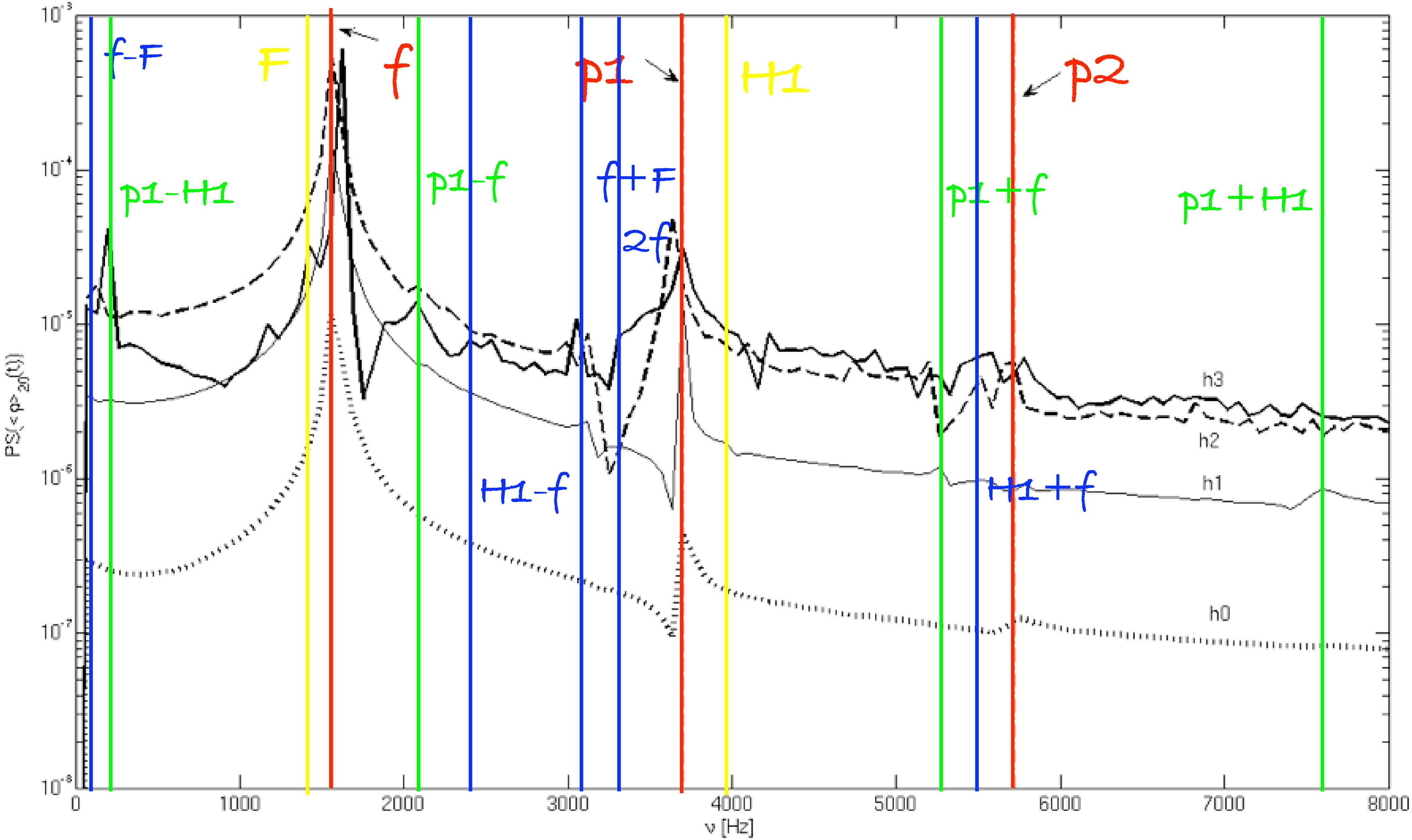
[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle\rho\rangle_{2,0}$



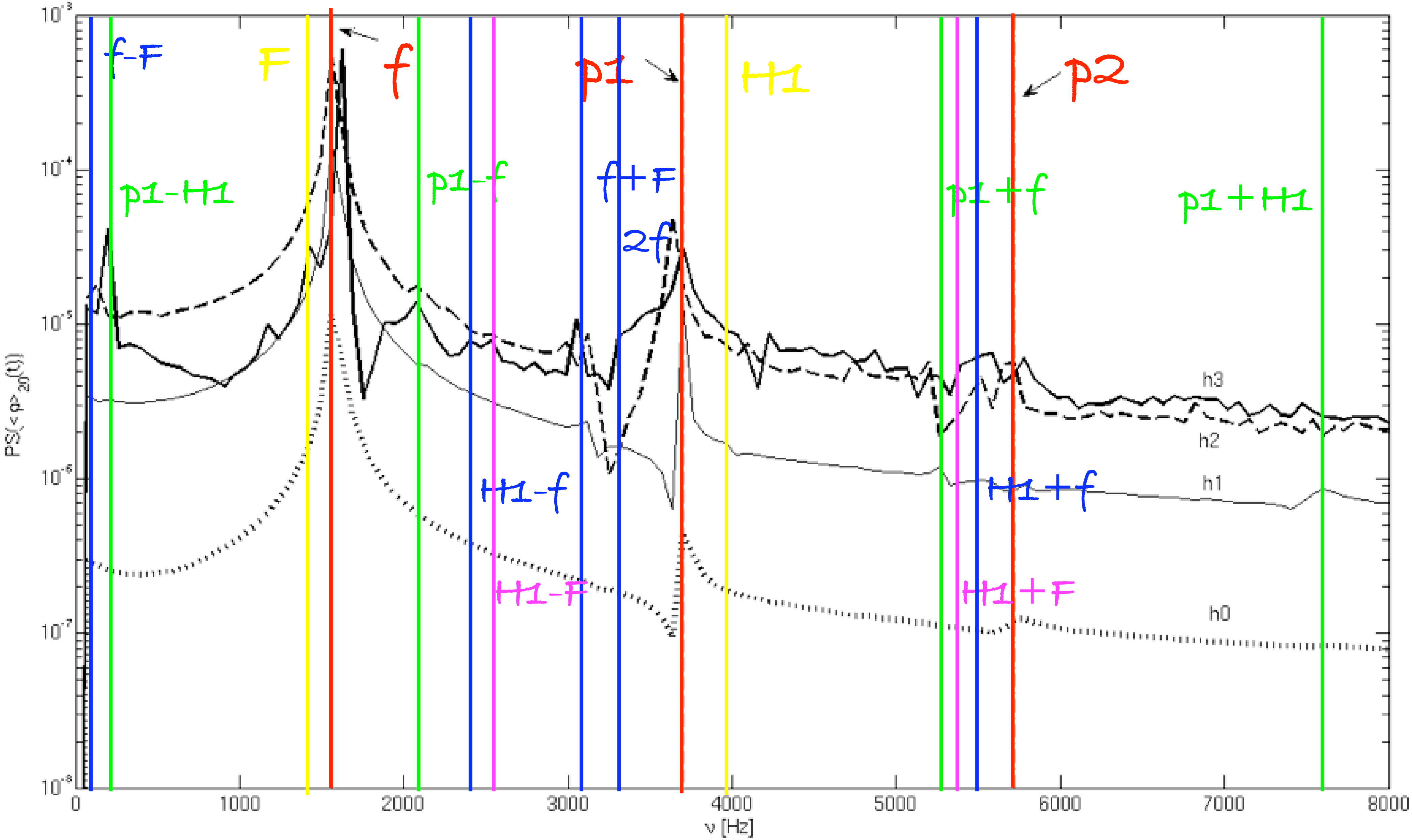
[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle\rho\rangle_{2,0}$



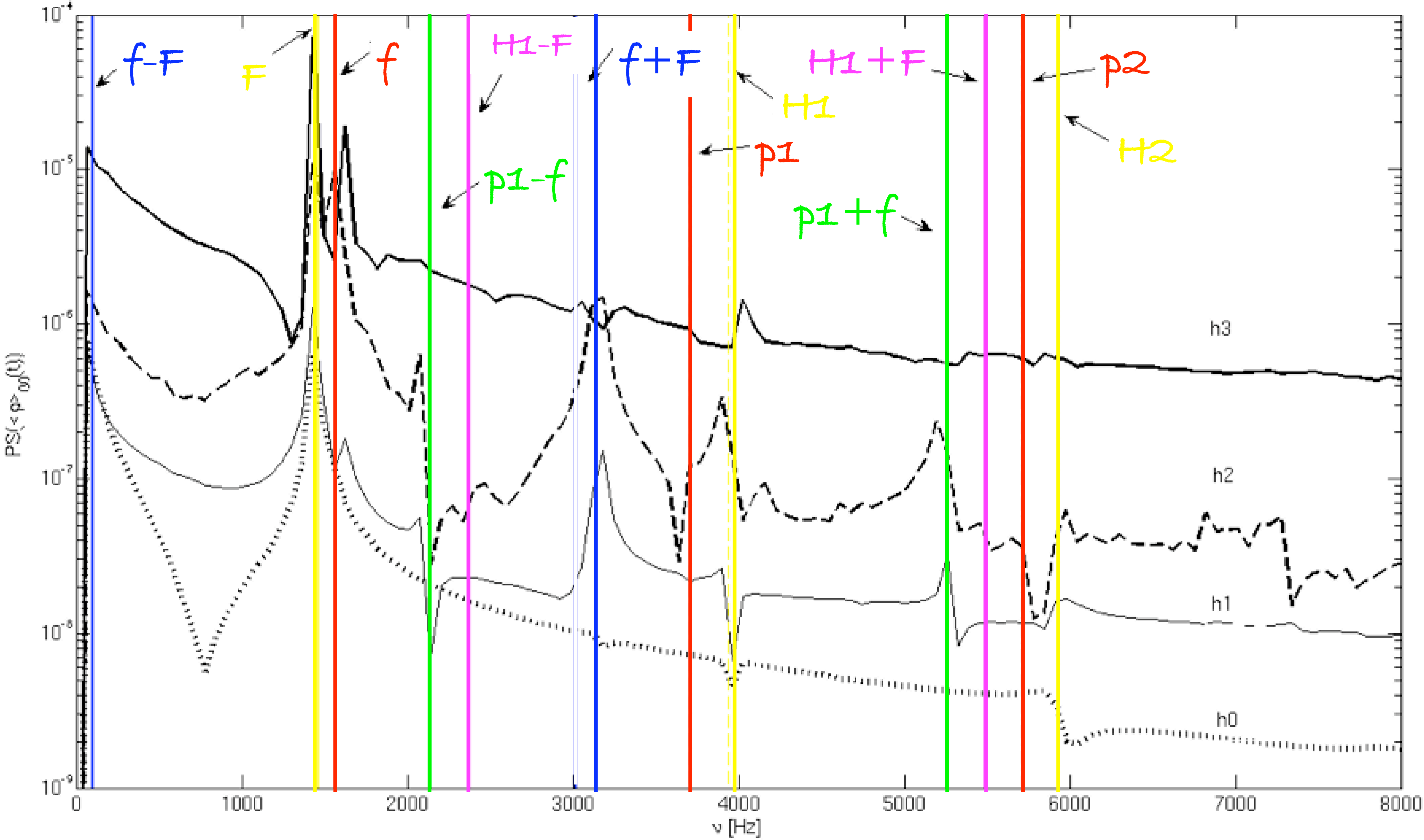
[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle \rho \rangle_{2,0}$



[Passamonti et al. 2006 / Dimmelmeier et al 2007]

$\langle \rho \rangle_{0,0}$



Quadrupole extraction

Functional form:

$$I_{ij}[\varrho] \equiv \int d^3x \, \varrho x_i x_j$$

No "Standard Quadrupole" in full GR.
Possible generalizations worth to try

$\longrightarrow$

SQF : $\varrho = \rho$

SQF1: $\varrho = \alpha^2 \sqrt{\gamma} T^{00}$

[Nagar et al. 2005]

SQF2: $\varrho = \sqrt{\gamma} W \rho$

[Blanchet et al 1990/ Shibata Sekiguchi 2003]

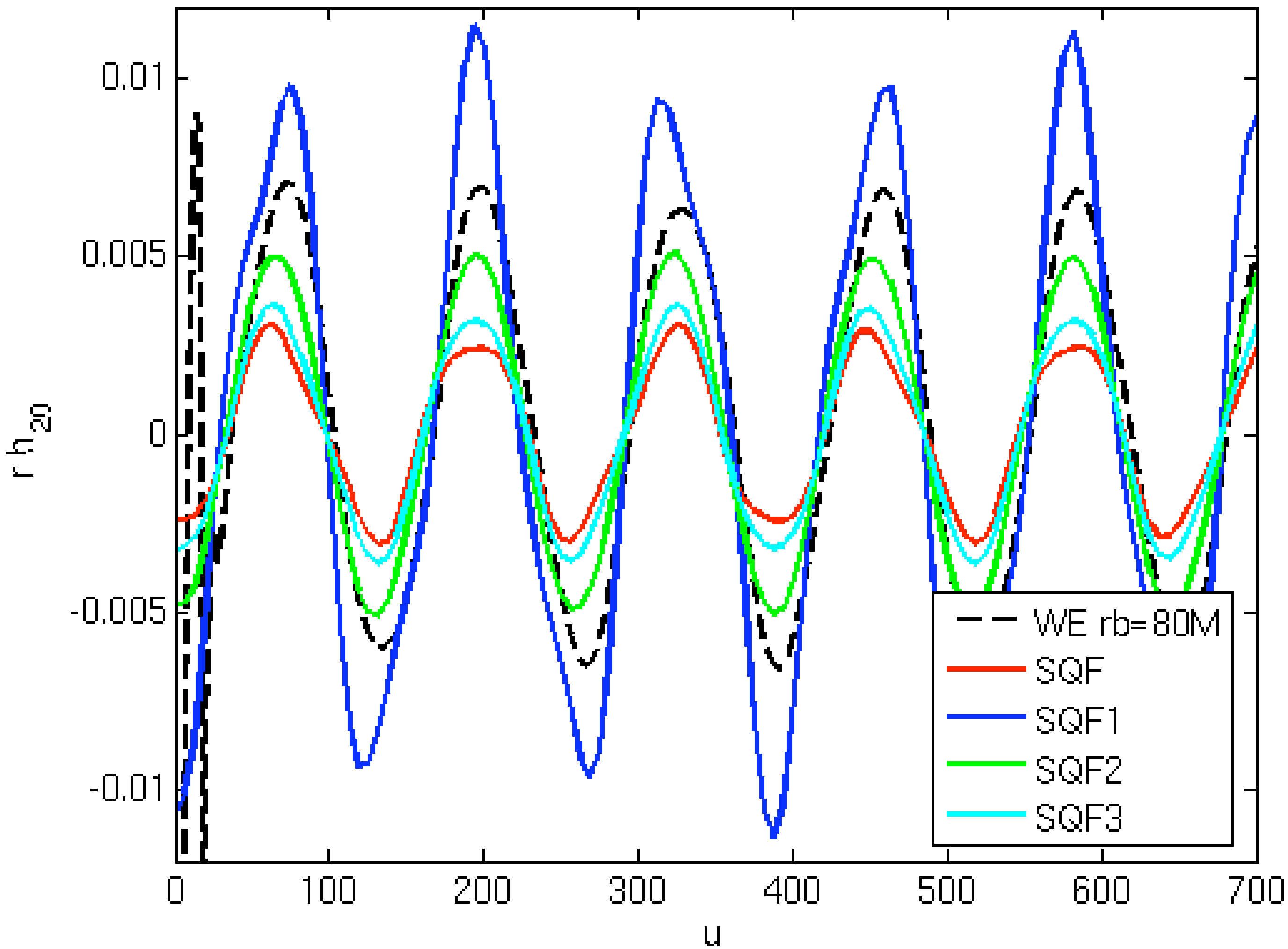
SQF3: $\varrho = u^0 \rho = \frac{W}{\alpha} \rho$

[Nagar et al. 2005]

Multipole:

$$r h_{2,0} = \sqrt{\frac{24\pi}{5}} (\ddot{I}_{zz} - \frac{1}{3} I)$$

(rem. $h_+ - i h_\times = \sum_{\ell,m} h_{\ell,m} \, {}_{-2}Y_{\ell,m}$)



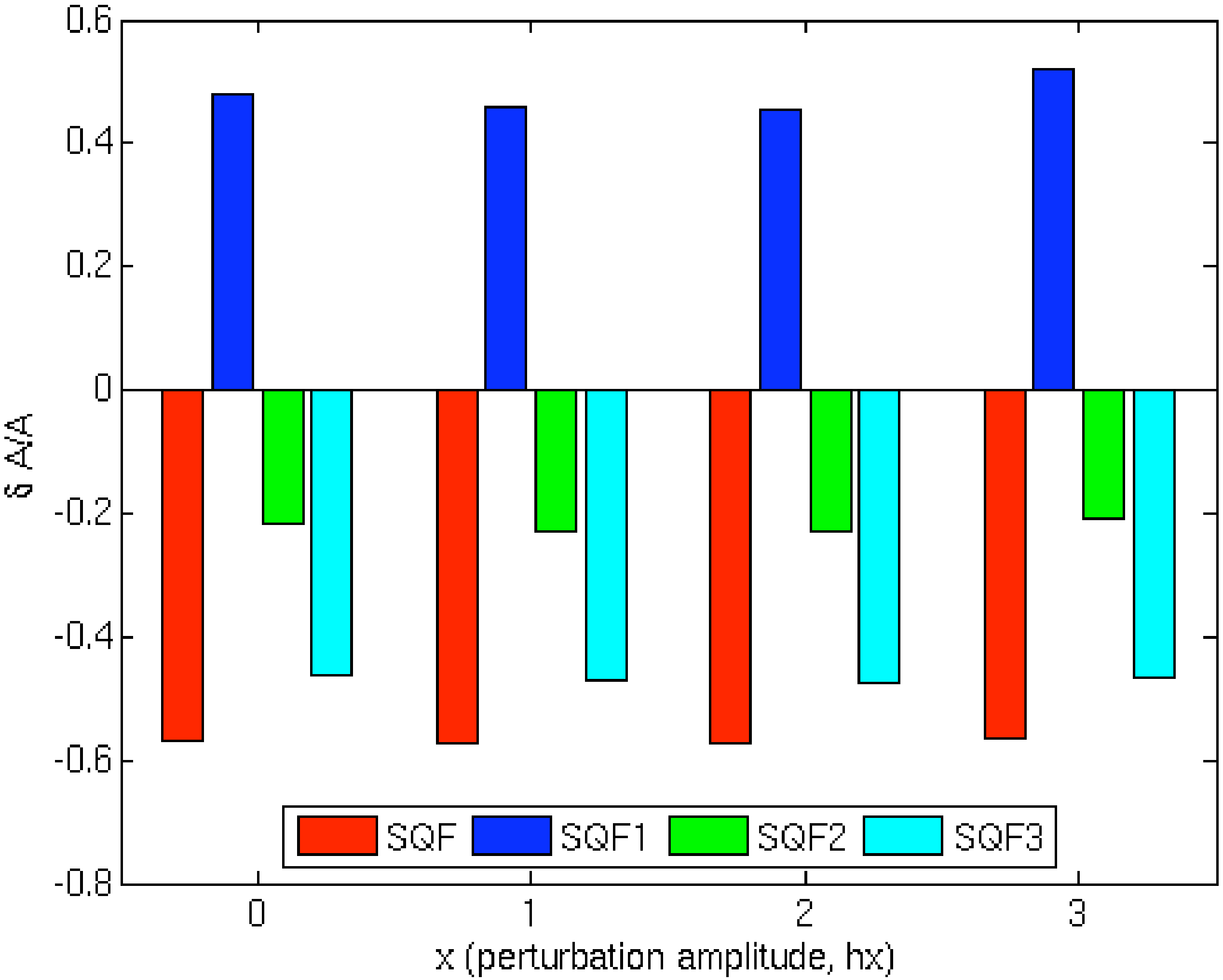
SQF : $\varrho = \rho$

SQF1: $\varrho = \alpha^2 \sqrt{\gamma} T^{00}$

SQF2: $\varrho = \sqrt{\gamma} W \rho$

SQF3: $\varrho = u^0 \rho = \frac{W}{\alpha} \rho$

- Frequencies : OK
- Differences in amplitude !



SQF : $\varrho = \rho$

SQF1: $\varrho = \alpha^2 \sqrt{\gamma} T^{00}$

SQF2: $\varrho = \sqrt{\gamma} W \rho$

SQF3: $\varrho = u^0 \rho = \frac{W}{\alpha} \rho$

- Frequencies : OK
- Differences in amplitude !

Summary

- Perturbative ID (Whisky_PerturbTOV)
- Evolve with both 3D FGR and 1D perturbative code
- Wave extraction: WaveExtract (Zerilli), Psikadelia (Psi4) and SQFs
- Compare results

Conclusions

- **Zerilli Extraction**
  - ✓ 3D Zerilli extraction consistent with Perturbative (linear regime)
  - ✓ Extraction r>80M
  - ✓ initial junk Zerilli Extraction

- **Ψ4 Extraction**
  - ✓ 3D Psi4 extraction consistent with Perturbative (linear regime)
  - ✓ Extraction r>80M
  - ✓ NO junk radiation
  - ✓ Off-set subtraction needed

Conclusions

- **Zerilli Extraction**
  - ✓ 3D Zerilli extraction consistent with Perturbative (linear regime)

- **$\Psi 4$ Extraction**
  - ✓ 3D Psi4 extraction consistent with Perturbative (linear regime)

Comparison with perturbative simulations indicates that

both method must be taken into account to extract accurate waveforms

Conclusions (cont.)

- **Quadrupole Extraction**
  - ✓ Frequencies arte properly captured
  - ✓ Amplitudes are underestimated [Shibata Sekiguchi 2003]
  - ✓ BEST: SQF2
- **Non-linear effects**
  - ✓ radial couplings
  - ✓ overtones couplings [Passamonti et al. 2006 / Dimmelmeier et al 2007]
  - ✓ self coulplings

Thank you very much!

REFerencies:

1. Nagar 2004
2.gr-.qc/0408041 2004
3.Nagar et al. 2004
4.Bernuzzi, Nagar & De Pietri 2008
5.Bernuzzi & Nagar 2008
6.Baiotti et al. 2008 [in preparation]