Time Evolution of a Generic Quantum Universe

Motivation:
Analyze the phenomenological implication of an evolutionary quantum dynamics for the gravitational field associated to a generic cosmological model.

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Summary of talk

I. Evolutionary quantum gravity

II. The generic quantum universe

III. Quasi-classical limit of the model

IV. Phenomenology of the dust fluid

V. Discussion and conclusions

I. Evolutionary quantum gravity

Evolutionary theory: $\Psi = \Psi(t, \left\{h_{ij}\right\})$

Smeared Schrödinger equation

$$i\partial_t \Psi = \widehat{\mathcal{H}}\Psi \equiv \int_{\Sigma_t^3} d^3x \left(N\widehat{H}\right)\Psi \qquad (1)$$

Eigenvalue problem for $\chi = \chi_\epsilon(\left\{h_{ij}\right\})$

$$\widehat{H}\chi = \epsilon\chi, \qquad \widehat{H}_i\chi = 0 \qquad (2)$$

WKB approximation: $\chi \sim e^{iS/\hbar}$

$$\widehat{HJ}S = \epsilon \equiv -2\sqrt{h}T_{00}, \qquad \widehat{HJ}_iS = 0 \qquad (3)$$

A new matter contribution appears

$$\rho \equiv T_{00} = -\frac{\epsilon(x^i)}{2\sqrt{h}} \qquad (4)$$

It is a dust fluid co-moving with the slicing 3-hypersurfaces, i.e. $T_{\mu\nu} = \rho n_\mu n_\nu$

II. The generic quantum universe

Because of quantum fluctuations, a quantum Universe has to be described by a generic inhomogeneous model

$$H(x^i) = \kappa\left[-\frac{p_a^2}{a} + \frac{1}{a^3}\left(p_+^2 + p_-^2\right)\right] + \frac{3}{8\pi}\frac{p_\phi^2}{a^3} + \\ - \frac{a^3}{4\kappa l_{in}^2} V(\beta_\pm) + a^3\left(\rho_{ur} + \rho_{pg}\right) = 0 \quad (5)$$

i) In a synchronous reference: **dynamics, point by point, Bianchi IX**

ii) **Matter fields**:
scalar field ϕ (inflation field)
$\rho_{ur} = \mu^2/a^4$ (ultrarelativistic energy density)
$\rho_{pg} = \sigma^2/a^5$ (perfect gas energy density)

Evolutionary (canonical) quantization of this model: eigenvalue problem

$$\hat{H}\chi \equiv \left\{\kappa\left[\partial_a \frac{1}{a}\partial_a - \frac{1}{a^3}\left(\partial_+^2 + \partial_-^2\right)\right] - \frac{3}{8\pi}\frac{1}{a^3}\partial_\phi^2 + \\ - \frac{a^3}{4\kappa l_{in}^2} V(\beta_\pm) + \frac{\mu^2}{a} + \frac{\sigma^2}{a^2}\right\}\chi = \epsilon\chi \quad (6)$$

Integral representation for $\chi(a, \beta_\pm, \phi)$

$$\chi(a, \beta_\pm, \phi) = \int \theta_K(a) F_K(a, \beta_\pm, \phi) dK \qquad (7)$$

Adiabatic approximation $|\partial_a F| \ll |\partial_a \theta|$

$$\kappa \frac{d}{da}\left(\frac{1}{a}\frac{d\theta}{da}\right) + \left(\kappa \frac{K^2}{a^3} + \frac{\mu^2}{a} + \frac{\sigma^2}{a^2} - \epsilon\right)\theta = 0, \qquad (8)$$

$$-(\partial_+^2 + \partial_-^2 + \frac{3}{8\pi\kappa}\partial_\phi^2)F + \frac{a^6}{4\kappa^2 l_{in}^2} V(\beta_\pm)F = K^2(a)F \qquad (9)$$

Neglect the potential term

$$a^3 \ll \mathcal{O}\left(l_P^2 l_{in} \sqrt{\frac{< K^2 >}{|\bar{V}(\beta_\pm)|}}\right) \qquad (10)$$

The eigenvalue problem (6) reduces to a system of ∞^3 independent eigenvalue problem (in each space point isomorphic to a Bianchi I model)

Solutions of the above equations

$F_a(\beta_\pm, \phi)$ plane waves

$$\theta(a) = \omega(a)\,\mathrm{exp}\left[-\frac{1}{2l_P^2}\left(a + \frac{\epsilon l_P^2}{16\pi}\right)^2\right] \qquad (11)$$

Toward the singularity $a \to 0$

$$\omega(a) = \sum_{n=0}^{\infty} c_n a^{n+\gamma}, \qquad c_0 \neq 0 \qquad (12)$$

where $\gamma = 1 - \sqrt{1 - K^2}$, $K^2 = k_\beta^2 + k_\phi^2$

coefficients obey the recurrence relations

$$c_n = -f(n,\gamma)\left\{\left[-\frac{\epsilon}{8\pi}\left(n + \gamma - \frac{3}{2}\right) + \frac{\sigma^2}{8\pi l_P^2}\right] c_{n-1} + \right.$$
$$\left. + \left[-\frac{2}{l_P^2}(n + \gamma - 2) + \left(\frac{\epsilon}{16\pi}\right)^2 + \frac{\mu^2}{8\pi l_P^2}\right] c_{n-2}\right\} \qquad (13)$$

with $f(n,\gamma) = ((n+\gamma)(n+\gamma-2) + K^2)^{-1}$

We required the wavefunction to decay at large a (potential term important) the series (12) must therefore terminate

$$\epsilon_{n,\gamma} = \frac{\sigma^2}{l_P^2(n+\gamma-1/2)} \tag{14}$$

$$2(n+\gamma) = \left(\frac{l_P \epsilon_{n,\gamma}}{16\pi}\right)^2 + \frac{\mu^2}{8\pi}. \tag{15}$$

Ground state $n = 0$ eigenvalue

$$\epsilon_{0,\gamma} = -\frac{\sigma^2}{l_P^2(1/2-\gamma)} \tag{16}$$

for $\gamma < 1/2$ is negative

The real ground state, according to equation (15) for $\mu^2 = 0$ ($\gamma \sim 1.8 \cdot 10^{-3}$)

$$\epsilon_0 \simeq -\frac{2\sigma^2}{l_P^2} \tag{17}$$

associated positive dust energy density

III. Quasi-classical limit of the model

- The quasi-classical limit is reached before the potential term becomes important

$$l_{in} \gg \sqrt{\frac{|\bar{V}(\beta_\pm)|}{< K^2 >}\frac{1}{|\epsilon|}} \tag{18}$$

- Ground state probability density: Gaussian distribution

$$|\theta_0(a)|^2 = \frac{1}{2\pi l_P^2} \exp\left[-\frac{1}{l_P^2}\left(a - \frac{\sigma^2}{8\pi}\right)^2\right] \tag{19}$$

  Singularity-free behavior of the early Universe

- Classical limit of the spectrum

$$\epsilon_{n,\gamma} = \frac{\sigma^2}{l_P^2(n+\gamma-1/2)} \sim \frac{1}{n} \tag{20}$$

  For large occupation numbers n, our quantum dynamics overlaps the Wheeler-DeWitt approach

IV. Phenomenology of the dust fluid

We put by hands a cut-off length: minimal length l per particle in the perfect gas $(l \geq l_P) \Rightarrow \rho_{pg} \leq \mathcal{O}(1/l_P^4), (\sigma^2 \leq \mathcal{O}(l_P))$

The spectrum is limited by below

$$|\epsilon| \leq \mathcal{O}(1/l_P) \tag{21}$$

Contribution to the actual critical parameter

$$\Omega_{dust} \sim \frac{\rho_{dust}}{\rho_T} \sim \mathcal{O}\left(\epsilon \frac{d_H^2 l_P^2}{a_T^3}\right) \tag{22}$$

$\rho_T \sim \mathcal{O}(10^{-29} g/cm^3)$ present critical density
$a_T \sim \mathcal{O}(10^{28} cm)$ present radius of curvature
$d_H \sim \mathcal{O}(10^{27} cm)$ Hubble size of the Universe

Thus for $\epsilon_0 \sim \mathcal{O}(1/l_P)$

$$\Omega_{dust} \sim \frac{d_H^2 l_P}{a_T^3} \sim \mathcal{O}\left(10^{-60}\right) \tag{23}$$

Therefore no phenomenology can came out from our dust fluid

V. Discussion and conclusions

- A dust fluid (Planck mass particle) is a good choice to realize a clock in quantum gravity.

- From a phenomenological point of view, no evidence appears of the non-zero eigenvalue in the Universe critical parameter. So an evolutionary quantum cosmology overlaps, in the generic inhomogeneous case, the Wheeler-DeWitt approach.

- Good indications on the solution of the horizon paradox. In fact, if (like here) the mean size of the primordial Universe is comparable to the classical horizon at the Planck time, no real puzzle arises about its later strong uniformity.