Graviton Propagator in LQG: a tool to test Spinfoam Models

Emanuele Alesci

Centre de Physique Théorique de Luminy, Marseille
ENS, Lyon

III Stueckelberg workshop,
Pescara, July 2008

Plan of the talk

- Brief summary of LQG
- The LQG graviton propagator (diagonal components)
- Tensorial structure of the propagator
- Problems of the Barret Crane model and a proposal for a solution
- New models: what do we know?
- Some lines of research with preliminary results (more questions than answers)
- Conclusions

Loop quantum gravity in brief

i) connection formulation of GR (Ashtekar 86, Barbero 95) **:**

$\mathbf{q} \rightarrow \mathbf{A}, \mathbf{E}$ $\mathbf{E}$ = triad field ("gravitational electric field") $\mathbf{A}$ = spin connection

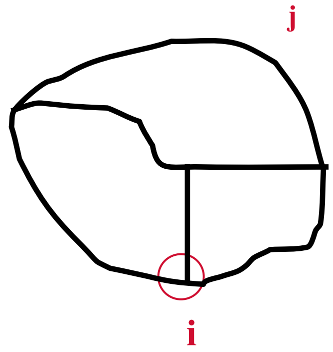
GR as a constrained hamiltonian system:
G=0 (gauss constraint)
V=0 (vector constraint)
H=0 (hamiltonian constraint)

ii) quantum theory:

States: $\Psi(A)$ Operators: A, $\mathbf{E}(x) = -i \hbar G\, \delta / \delta A(x)$

Loop states: $\Psi_\alpha(A) = \langle A \mid \alpha \rangle = \mathbf{Tr}\ \mathbf{e}^{\int_\alpha A}$ (Smolin Rovelli 88)

Spin network states: $\Psi_S(A) = \langle A \mid S \rangle$ (Smolin Rovelli 95)

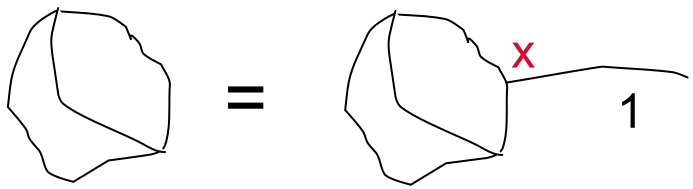


| S >=|Γ,j,i> Γ Graph

j SU(2) quantum number associated to links

i SU(2) quantum number associated to nodes (intertwiner)

E(x) acts on spin network states as a **Grasping Operator**
creating a new link in rep 1

E(x) 

Interpretation of the spin network states | S >

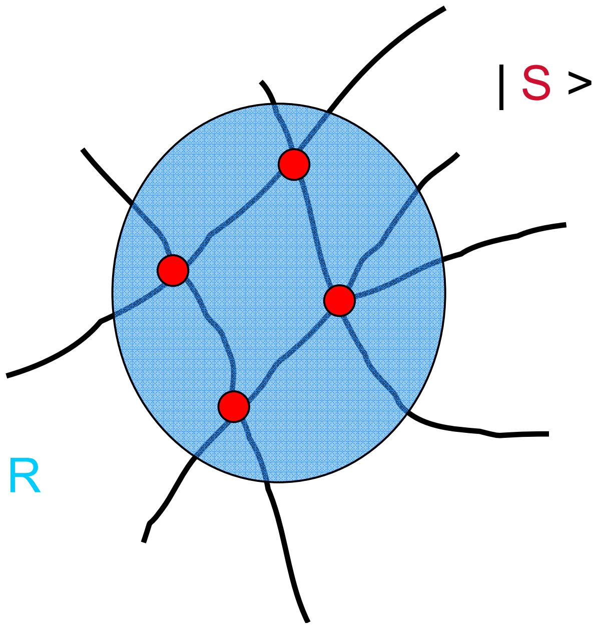
Look at the geometrical operators: Volume and area operators

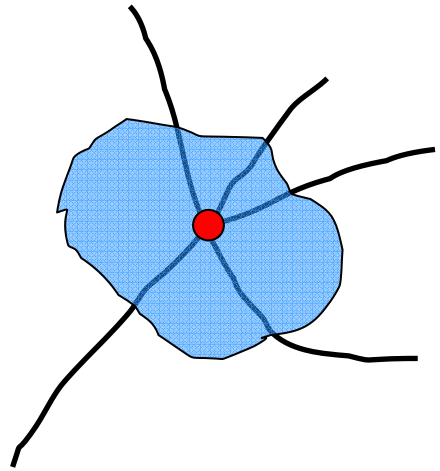
Volume

V(R) function of the gravitational field

↓ Quantization

- $\hat{V}$(R) is a well-defined self-adjoint operator
- The Volume operator receives a discrete contribution for each node of | S > inside R (Smolin, Rovelli 95)





NODE

"chunk of space"

with quantized volume

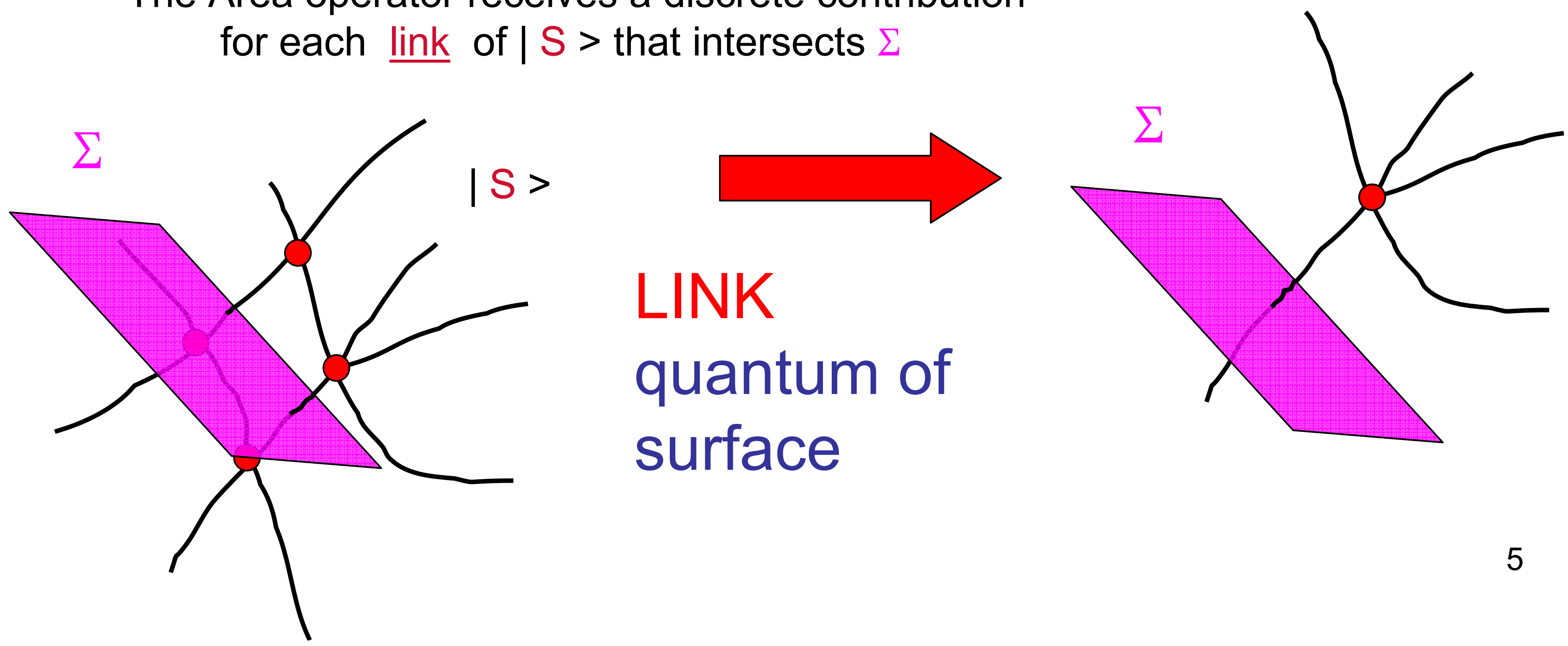
Intertwiners carry quantum numbers of Volume

Area

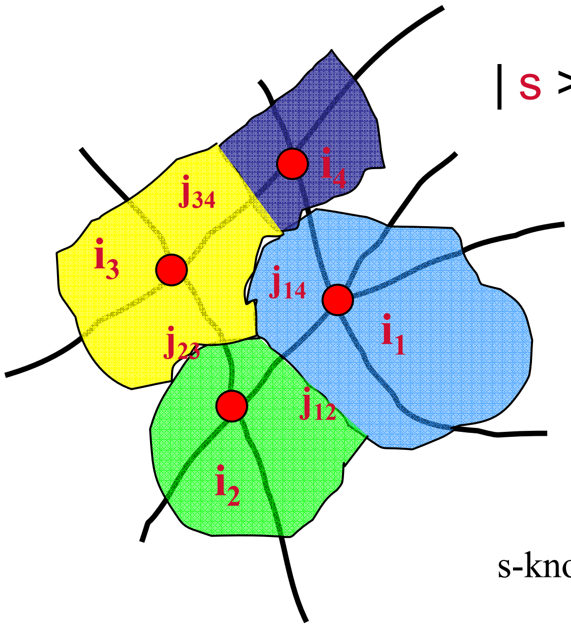
$A(\Sigma)$ function of the gravitational field

↓ Quantization

- $\hat{A}(\Sigma)$ is a well-defined self-adjoint operator (Smolin, Rovelli 95)
- The Area operator receives a discrete contribution for each link of $|S>$ that intersects Σ



s-knot state



$| s >$ Diffeomorphism invariant spinnetwork,
Equivalence class of spinnetworks
Under Diffeomorphisms
We loose the information about the localization:

$$| s > \; = \; | \Gamma , j_l , i_n >$$

s-knots

Intertwiners of the nodes: quantum numbers of volume

Spin of the links: quantum numbers of area

Γ is not a graph in the manifold but the information about the Connectivity between the elementary quantum chunks of space

- Spin networks are not excitations in space: they are excitations of space.

→ Background independent QFT

- **Discrete structure of space at the Planck scale**
- **Area and volume are quantized**

LQG Graviton Propagator

How to compute a propagator in a diff invariant theory?

$$G_0(x,y) = \int D\phi(x)\ \phi(x)\phi(y)\ e^{iS[\phi]}$$

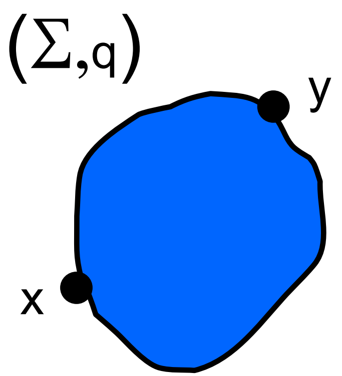
Rovelli's strategy based on:

- Boundary formulation
- Define a new function with the information on the background around which the propagator is defined in the boundary state via q (the classical value of the field on Σ)

Ψ a state picked on a geometry **q**=(q,p) (metric and extrinsic curvature) of Σ

$$\mathbf{G}_{\mathbf{q}}^{abcd}(\mathbf{x},\mathbf{y}) = \int [D\gamma]\ h^{ab}(\mathbf{x})\ h^{cd}(\mathbf{y})\ W[\gamma]\ \Psi_{\mathbf{q}}(\gamma)$$

$$\mathbf{G}^{abcd}_{\mathbf{q}}(\mathbf{x},\mathbf{y}) = \int [D\gamma]\ h^{ab}(\mathbf{x})\ h^{cd}(\mathbf{y})\ W[\gamma]\ \Psi_{\mathbf{q}}(\gamma)$$



- $\int D\gamma$ ⟹ $\Sigma_{\text{s-knots}}$ from LQG : s-knots represent discrete geometries

- $W[\gamma]$ ⟹ W[s] Spinfoam amplitude from GFT: dynamic for the s-knots states

- Ψ_q ⟹ Coherent boundary state picked on the mean geometry **q**

(No clear procedure to select it, not like in usual QFT, where it can be constructed from the propagation kernel L. Doplicher, Mattei, Speziale, Testa, Rovelli)

- $h^{ab}(x)$ ⟹ Graviton operator from LQG : built using grasping opertors E

LQG Graviton Propagator

$$\mathbf{G}^{abcd}_{\mathbf{q}}(x,y) = \sum_{s,s'} \langle W|s'\rangle\langle s'|h^{ab}(x)\ h^{cd}(y)|s\rangle\langle s|\Psi_{\mathbf{q}}\rangle$$

(Modesto Rovelli 05)

Bianchi, Modesto, Rovelli,Speziale have found for the DIAGONAL components of the propagator

$$G(L) \sim i\frac{8\pi\hbar G}{4\pi^2}\frac{1}{|x-y|_q^2} \sim i\frac{\hbar G}{L^2}$$

Which is the correct graviton propagator component

This is only valid for $L^2 >> \hbar G$
For small L the propagator is affected
by pure quantum gravity effects

Equivalent to Newton law

Derivation based on the use of the Barret Crane (98) model as Spinfoam vertex

This result has reinforced the idea that the Barret Crane model is able to reproduce General Relativity in the low energy limit

Tensorial Structure

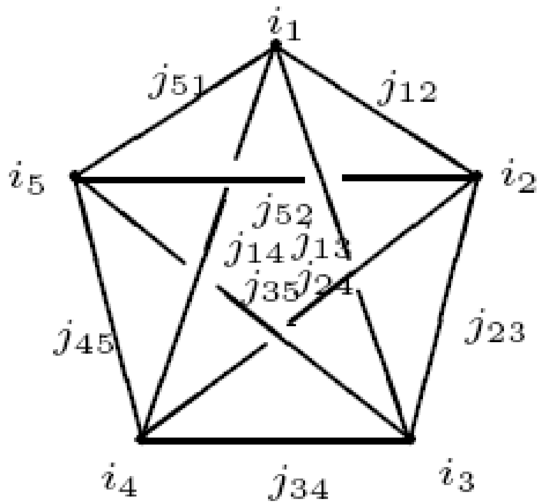
Try to extend the results of Bianchi, Modesto, Rovelli, Speziale

To compute the complete tensorial structure

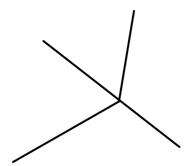
$$\mathbf{G}_{\mathbf{q}}^{abcd}(x,y) = \sum_{s,s'} \langle W|s'\rangle\langle s'|h^{ab}(x)\ h^{cd}(y)|s\rangle\langle s|\Psi_{\mathbf{q}}\rangle$$

To first order in λ $\langle W|s\rangle = W[s] = W[\Gamma,\mathbf{j},\mathbf{i}]$ the dominant term, is for Γ :

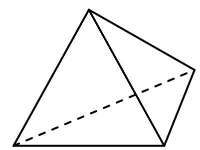
$s =$



Each 4 valent node



Dual of a thetrahedron



We have 5 intertwiners and 10 spins as variables

Inserting resolution of the identity, using the base $|s\rangle = |\Gamma, \mathbf{j}, \mathbf{i}\rangle$

$$\mathbf{G}_{\mathbf{q}}^{abcd}(x,y) = \sum_{\mathbf{j},\mathbf{j}',\mathbf{i},\mathbf{i}'} W(\mathbf{j}',\mathbf{i}') \; < \mathbf{j}',\mathbf{i}'|h^{ab}(x)\; h^{cd}(y)|\mathbf{j},\mathbf{i} > \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

The recipe: Three ingredients

$W(\mathbf{j},\mathbf{i}) = W[\Gamma_5,\mathbf{j},\mathbf{i}]$ PROPAGATION KERNEL

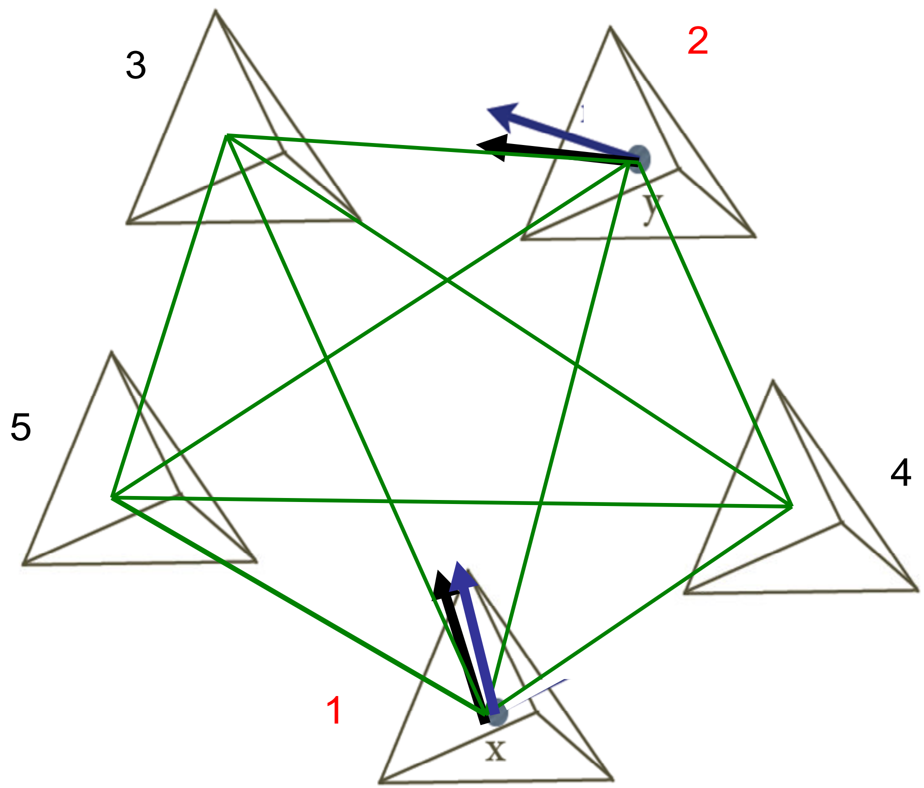
$\Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i}) = \Psi_{\mathbf{q}}[\Gamma_5,\mathbf{j},\mathbf{i}] = \langle \Gamma_5,\mathbf{j},\mathbf{i}|\Psi_{\mathbf{q}}\rangle$ BOUNDARY STATE

$< \mathbf{j}',\mathbf{i}'|h^{ab}(x)\; h^{cd}(y)|\mathbf{j},\mathbf{i} >$ QUANTUM OPERATORS

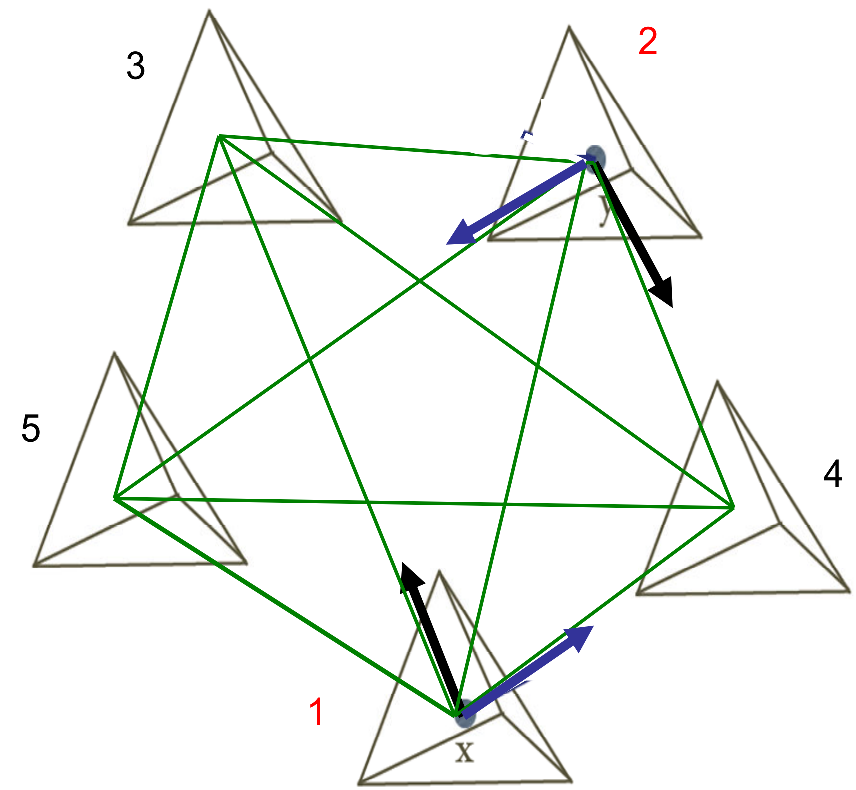
Now explicit dependence on the interwiners i

Consider the propagator projection on the normals $n_a^{(ni)}$ to the triangle t_{ni} that bounds the tetrahedra n and i and so on

$$\mathbf{G}_{\mathbf{q}\,n,m}^{ij,kl} := \mathbf{G}_{\mathbf{q}}^{abcd}(x_n, x_m)\; n_a^{(ni)} n_b^{(nj)}\; n_c^{(mk)} n_d^{(ml)}$$



Diagonal components:
Area correlators



Not-Diagonal components:
Angle correlators

Since $h^{ab} = g^{ab} - \delta^{ab} = E^{ai}E^b_i - \delta^{ab}$ defining $E_n^{(ml)} = E^a(\vec{x}) n_a^{(ml)}$

We have to compute

$$\mathrm{G}_{\mathrm{q}\,n,m}^{ij,kl} = \langle W | \left(E_n^{(ni)} \cdot E_n^{(nj)} - n^{(ni)} \cdot n^{(nj)}\right)\left(E_m^{(mk)} \cdot E_m^{(ml)} - n^{(mk)} \cdot n^{(ml)}\right) | \Psi_\mathrm{q} \rangle$$
$$= \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i})\left(E_n^{(ni)} \cdot E_n^{(nj)} - n^{(ni)} . n^{(nj)}\right)\left(E_m^{(mk)} \cdot E_m^{(ml)} - n^{(mk)} . n^{(ml)}\right)\Psi_\mathrm{q}(\mathbf{j},\mathbf{i})$$

Understand the action of the non diagonal operator E.E
on the spin networks states. They are double grasping operators

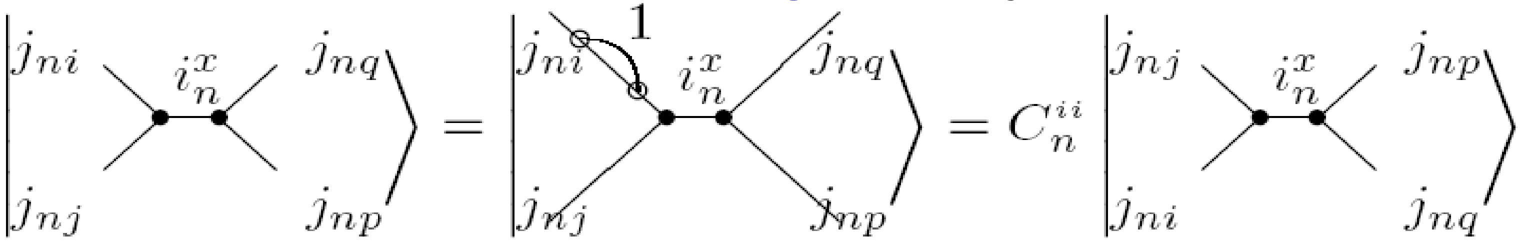
$$E_n^{(ni)} \cdot E_n^{(nj)} \quad |\Gamma, \mathbf{j}, \mathbf{i}\rangle$$

Use of Recoupling Theory

Action of the quantum operators

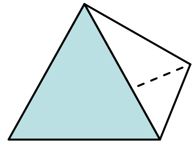
ii) The action of the operators EE is diagonal if i=j

$E_n^{(ni)} \cdot E_n^{(ni)}$



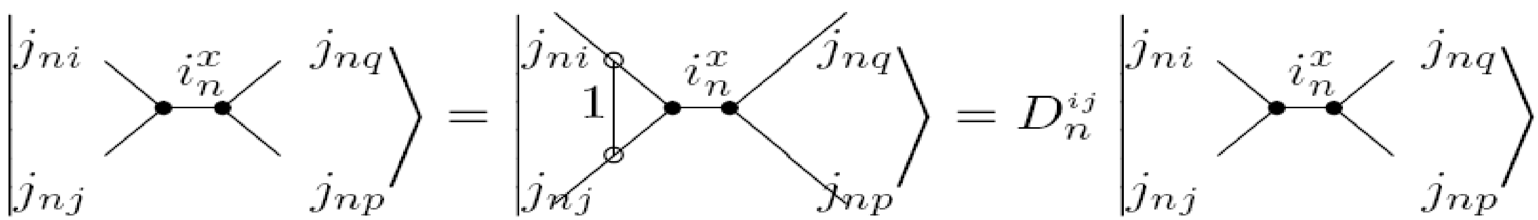
It is the Area operator,it reads the Casimir $C_n^{ii} = C^2(j_{ni})$ of the link j_{ni}

In our simplicial picture the area of the triangle



ij)

$E_n^{(ni)} \cdot E_n^{(nj)}$

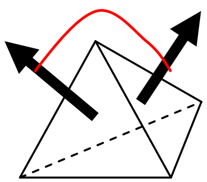


The actions of the operators EE with i≠j is diagonal or not depending on the node pairing. It involves directly the intertwiner dependance of the node

This is the operator associated

with the dihedral angle

between the triangles dual to the grasped links



New boundary state

To compute the DIAGONAL terms was sufficient a state of the kind

$$\Psi_{\mathrm{q}}[\mathrm{j},\mathrm{i}] = C \exp\left\{ -\frac{1}{2j^0} \sum_{(ij)(mr)} \alpha_{(ij)(mr)} \left(j^{(ij)} - j^0\right)\left(j^{(mr)} - j^0\right) + i\Phi \sum_{(ij)} j^{(ij)} \right\}$$

q is the geometry of the 3d boundary (Σ,*q*) of a spherical 4d ball, with linear size L >> √ħG

Ψ*q*(*s*) is a Gaussian state with correlation matrix α

peacked on the “background” spins j^0

The Φ are the background dihedral angles between tetrahedra
(Variables coniugate to spins). They code the *extrinsic* 3-geometry *q*

The operators call into play the intertwiner i, we have to consider the kinematics
of intertwiners and introduce an intertwiner dependance in the boundary state

New state

Old one

$$\Psi_q[j,i] = C \exp\left\{ -\frac{1}{2j^0} \sum_{(ij)(mr)} \alpha_{(ij)(mr)} \left(j^{(ij)} - j^0\right)\left(j^{(mr)} - j^0\right) + i\Phi \sum_{(i,j)} j^{(ij)} \right\} \cdot$$

$$\cdot \exp\left\{ -\sum_n \left(\frac{(i_n - i^0)^2}{4\sigma_{i_n}} + \sum_{a \neq n} \phi_{j_{na}\, i_n} \left(j^{(na)} - j^0\right)\left(i_n - i^0\right) + i\chi_{i_n}\left(i_n - i^0\right) \right) \right\}$$

Also gaussian in the intertwiners around the background value i^0 (background dihedral angles) with variance σ , phase factor χ, correlation spin-intertwiner ϕ.

Fixing these parameters we can create
a semiclassical 4-symplex,
picked on classical values of areas and angles (4-d and 3-d)

Calculation with BC vertex

In the calculation of the diagonal terms, was used a BC vertex with a projection map

$$W(\mathbf{j},\mathbf{i}) = W(\mathbf{j}) \prod_n \langle i_{BC} | i_n \rangle = W(\mathbf{j}) \prod_n (2 i_n + 1)$$

Where W(j) is the 10j symbol

Map Simple SO(4)–>SU(2) supported by the physical interpretation

We have to compute terms of the kind,

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(D^{ij}_n - n^{(ni)} \cdot n^{(nj)} \right) \left(D^{kl}_m - n^{(mk)} \cdot n^{(ml)} \right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

Keeping the dominat terms (we are interested in the large j^0 limit)

Intertwiners and spins as variables

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = j_0^2 \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(\frac{2}{\sqrt{3}} \delta i_n - \delta j_{ni} - \delta j_{nk} \right) \left(\frac{2}{\sqrt{3}} \delta i_m - \delta j_{mk} - \delta j_{ml} \right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = j_0^2 \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(\frac{2}{\sqrt{3}} \delta i_n - \delta j_{ni} - \delta j_{nk} \right) \left(\frac{2}{\sqrt{3}} \delta i_m - \delta j_{mk} - \delta j_{ml} \right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

$$W(\mathbf{j}) \approx e^{iS_{Regge}} + e^{-iS_{Regge}} + D$$

Barrett, Williams, Baez, Christensen, Egan, Freidel, Louapre

$$\Psi_{\mathbf{q}}[\mathbf{j},\mathbf{i}] \approx e^{-\frac{1}{2j^0}\sum \alpha_{(ij)(mr)}\delta j^{ij}\delta j^{mr} + i\sum \Phi \delta j^{ij}} e^{-\sum_n \frac{3(\delta i_n)^2}{4j^0} - i\sum_a \frac{3}{4j^0}\delta j^{an}\delta i_n + i\frac{\pi}{2}\delta i_n}$$

$$S_{\text{Regge}}(j_{nm}) = \Phi \sum_{nm} j_{nm} + \frac{1}{2} G_{(mn)(pq)} \delta j_{mn} \delta j_{pq}$$

The rapidly oscillating phase in the state (green) cancel or double the phase in the dynamic (green).
Only the term without phase survives (This was the key feature of the Diagonal terms)
BUT now there is also a phase term (pink) in the state
UNCOMPENSED by the dynamics

PROBLEM OF THE MODEL:
THE DYNAMICS DOESN'T SPEAK WITH THE INTERTWINERS

$i\frac{\pi}{2}\sum_p i_p$ The Phase Factor is not compensated by the dynamics

SUPPRESS THE SUM

If we proceed with the calculation, we can recast the problem introducing the 15 components vectors $\delta I^{\alpha} = (\delta j^{ab}, \delta i_n)$ $\delta\Theta^{\alpha} = (0, \chi i_n)$ and the 15 x 15

Correlation Matrix M that contains the 3 free parameters of the gaussian plus dynamics

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = \mathcal{N}' j_0^2 \int d\delta I^{\alpha} \left(\frac{2}{\sqrt{3}} \delta i_n - \delta j_{ni} - \delta j_{nj} \right) \left(\frac{2}{\sqrt{3}} \delta i_m - \delta j_{mk} - \delta j_{ml} \right) e^{-\frac{M_{\alpha\beta}}{j^0} \delta I^{\alpha} \delta I^{\beta}} e^{i\Theta_{\alpha} \delta I^{\alpha}}$$

We get a sum of terms of the kind

$$\left(\frac{M^{-1}_{\alpha\beta}}{j_0} - M^{-1}_{\alpha\gamma} \Theta^{\gamma} M^{-1}_{\beta\delta} \Theta^{\delta} \right) j^0 \to \infty$$

Dominant term CONSTANT

→ Wrong large distance propagator

The Barret Crane model don’t reproduce GR in the low energy limit !!!

Proposal

Unfreeze the intertwiners degrees of freedom

We make an hypotesis

(done before the new vertices were created):

We consider a vertex that in the large distance expansion has the same asymptotic behavior as the Barrett-Crane vertex on the spins j, and it has also a dependence on the intertwiners i. Guided by the compensation present in the diagonal case we assume a vertex which asymptotic expansion up to second order is

$$W_{Asymp}(\mathbf{j},\mathbf{i}) = e^{i\frac{G}{2}\delta j \delta j} e^{i\Phi \delta j} e^{i\chi_{i_n}\delta i_n} e^{i\phi_{j i_n}\delta j \delta i_n} + e^{-i(\text{same expression})}$$

Same as BC but with the crucial phase (pink) in the intertwiner variable able to compensate the one in the boundary state.

Correlation spin-intertwiner usefull but not crucial.

The same kind of terms as before becomes

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = \mathcal{N}' j_0^2 \int d\delta I^\alpha \left(\frac{2}{\sqrt{3}}\,\delta i_n - \delta j_{ni} - \delta j_{nj}\right)\left(\frac{2}{\sqrt{3}}\,\delta i_m - \delta j_{mk} - \delta j_{ml}\right)\, e^{-\frac{M_{\alpha\beta}}{j^0}\delta I^\alpha \delta I^\beta}\; e^{i\Theta_\alpha \delta I^\alpha}$$

The propagator is then a sum of terms of the kind

$$\frac{M'^{-1}_{\alpha\beta}}{j_0}$$

Right large distance behavior:

Remember $A = 8\pi\hbar G\,\sqrt{j^0(j^0+1)}$

$$\frac{k\hbar G M'^{-1}_{\alpha\beta}}{L^2}$$

$M'^{-1}_{\alpha\beta}$ Contains a linear combination of the derivatives of Regge Action and of the correlation matrix in the gaussian

The complete tensorial structure

In the Euclidean theory the Graviton Propagator in the harmonic gauge is

$$G^{\mu\nu\rho\sigma}_{linearized} = \frac{1}{2L^2}\left(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\rho\sigma}\right)$$

Using symmetrized gaussian state containing at least 5 free parameters as Boundary state we can reproduce exactly the same tensorial structure and the same behavior of the linearized theory

WE CAN FIND THE GRAVITON PROPAGATOR FROM LQG

THIS RESULT HAS MOTIVATED THE SEARCH FOR AN ALTERNATIVE MODEL ABLE TO REPRODUCE GR

Engle-Pereira-Rovelli, Alexandrov, Livine-Speziale, Freidel-Krasnov

- ARE THE NEW MODELS ABLE TO REPRODUCE THE GRAVITON PROPAGATOR?
- DO THEY SHOW THE PROPOSED INTERTWINER'S DEPENDANCE?

The new Models

EPR MODEL

$$A^{\gamma}_{EPR}(j_{ij}, i_n) = \sum_{i_n^-, i_n^+} 15j\left(j_{ij}\tfrac{(1+\gamma)}{2}, i_n^-\right)\, 15j\left(j_{ij}\tfrac{|1-\gamma|}{2}, i_n^+\right) \bigotimes_n f^{i_n}_{i_n^- i_n^+}$$

FK MODEL

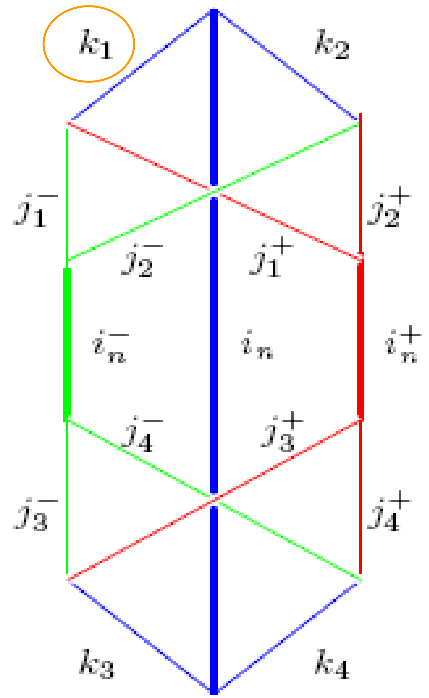
$$A^{\gamma}_{FK}(j_{ij}, i_n, k_n) = \sum_{i_n^-, i_n^+} 15j\left(j_{ij}\tfrac{(1+\gamma)}{2}, i_n^-\right)\, 15j\left(j_{ij}\tfrac{|1-\gamma|}{2}, i_n^+\right) \bigotimes_n f^{i_n, k_n}_{i_n^- i_n^+}$$

All these new models show the proposed SU(2) intertwiner dependance contained in the fusion coefficients f

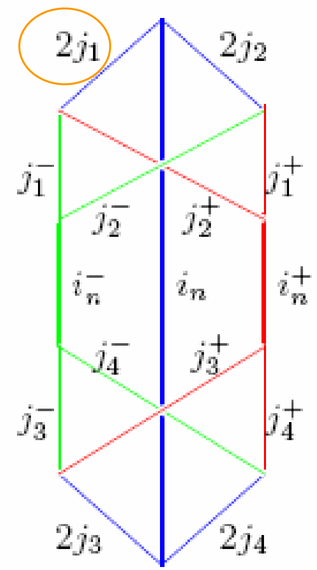
In this language the BC model is

$$A_{BC}(j_{ij}) = \sum 15j\left(j_{ij}\tfrac{1}{2}, i_n^-\right)\, 15j\left(j_{ij}\tfrac{1}{2}, i_n^+\right) \bigotimes \dim i_n^+ \,\delta_{i_n^+, i_n^-}\, \delta_{i_n, 0}$$

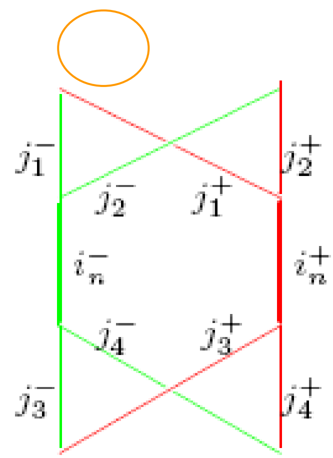
The new fundamental object is f that defines the new models:
It is a map from the space of the SU(2) (SO(3) in the EPR case) intertwiners to the space of the SO(4) intertwiners.



f_{FK}



f_{EPR}



f_{BC}

Note that they differ only in the way in which the two channels $j^+ + j^-$ compose in the resulting SU(2) representation (k for FK, 2j for EPR, 0 for BC)

What do we know about the new Models ?

1. The semiclassical limit? Do the new vertices show the proposed phase?
2. Is there any link of the models with the canonical approach at dynamical level (at kinematical level: yes EPR model) and in particular with Thiemann hamiltonian constraint?

Here we present some research directions to answer the first question

To answer the first question we need :

- The asymptotic of the 15j symbol :
  missing: very complicated in terms of recoupling theory,

- An exact or at least asymptothic formula expression of the f :
  we have found a simplification for f_{FK} and the exact analytic expression for f_{EPR}

To answer the second question we need : E.Alesci, K.Noui, F.Sardelli to appear

- An extension of the 3d constuction of Perez and Noui to these models, to obtain a physical scalar product involving the hamiltonian operator that defines the new models:
  we have found a Physical scalar product able to reproduce these spinfoam amplitudes

No clear relation with an Hamiltonian constraint like Thiemann's one

1. The semiclassical limit

We consider two possible approach to study the semiclassical limit:
NUMERICAL, ANALYTICAL

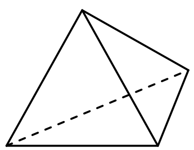
NUMERICAL

The only information appeared in the numerical approach is contained in the evolution of wave packets with the flipped vertex

E.Magliaro, C.Perini, C.Rovelli CQG(2008) arXiv:0710.5034

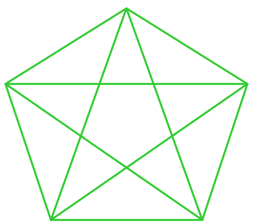
The process described by one vertex can be seen as the dynamics of a single cell in a Regge triangulation of general relativity.

The tested process is the evolution of an initial state formed by four coherent tetrahedra



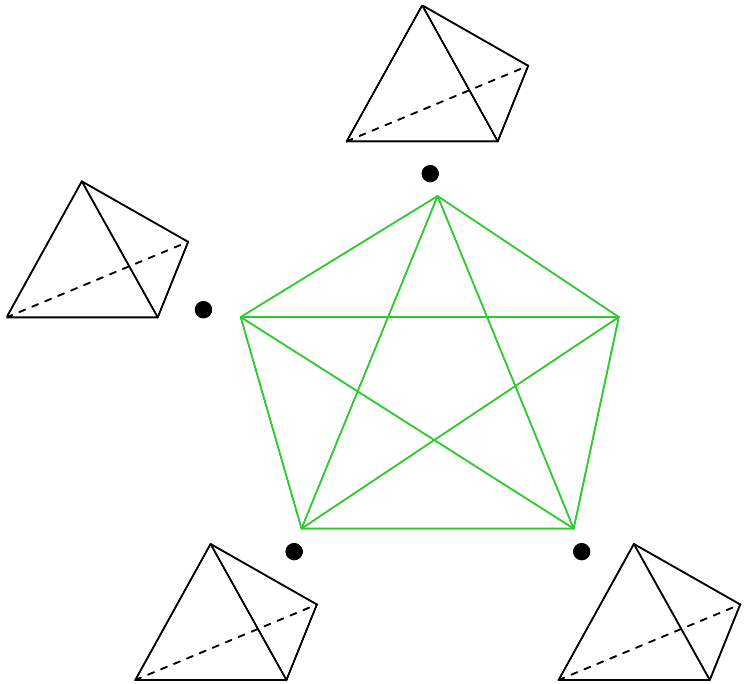
$$\psi(i) = N\sqrt{d_i}\, e^{-\frac{3}{4j_0}(i-i_0)^2 + i\frac{\pi}{2}i}$$

Coherent tetrahedron
S.Speziale, C.Rovelli

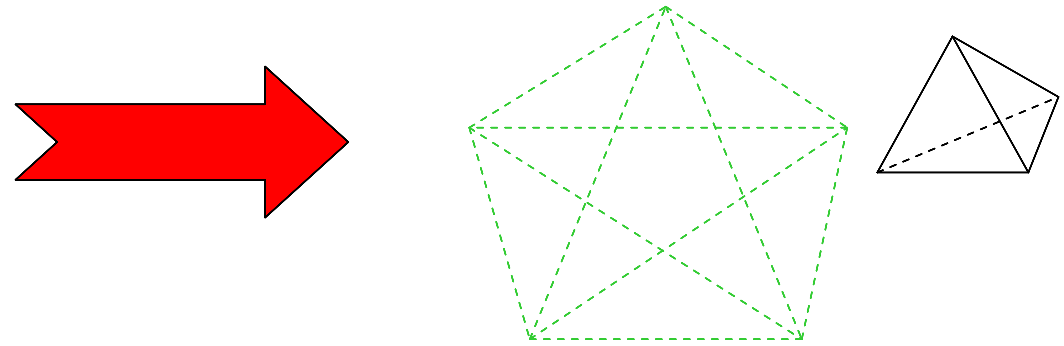


$$W(i_n) = \sum_{i_n^+, i_n^-} 15j\left(\frac{j_0}{2}, i_n^+\right) 15j\left(\frac{j_0}{2}, i_n^-\right) \prod_n f^{i_n}_{i_n^+ i_n^-}$$

Propagation kernel



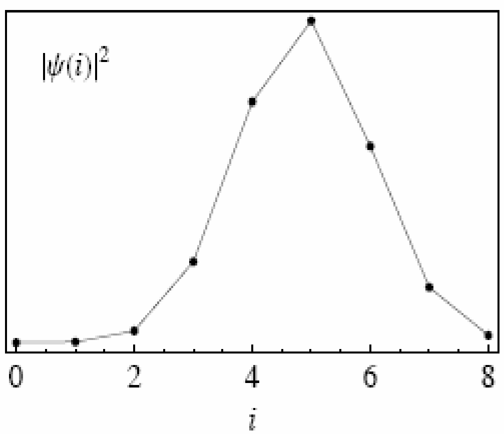
- Indicate contraction



$$\phi(i) = \sum_{i_1 \dots i_4} W(i_1, \dots, i_4, i) \prod_{n=1}^{4} \psi(i_n)$$

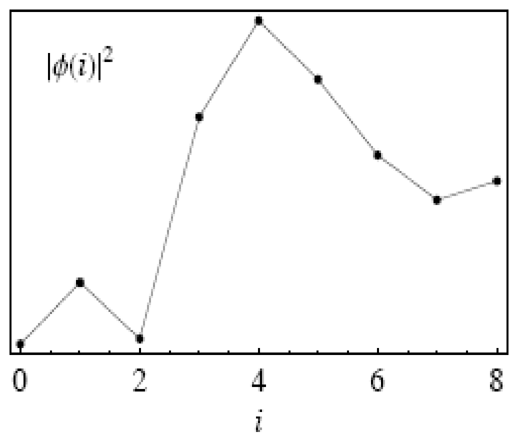
Is the evolved state a semiclassical tetrahedron with the right mean value?

Input



J=4

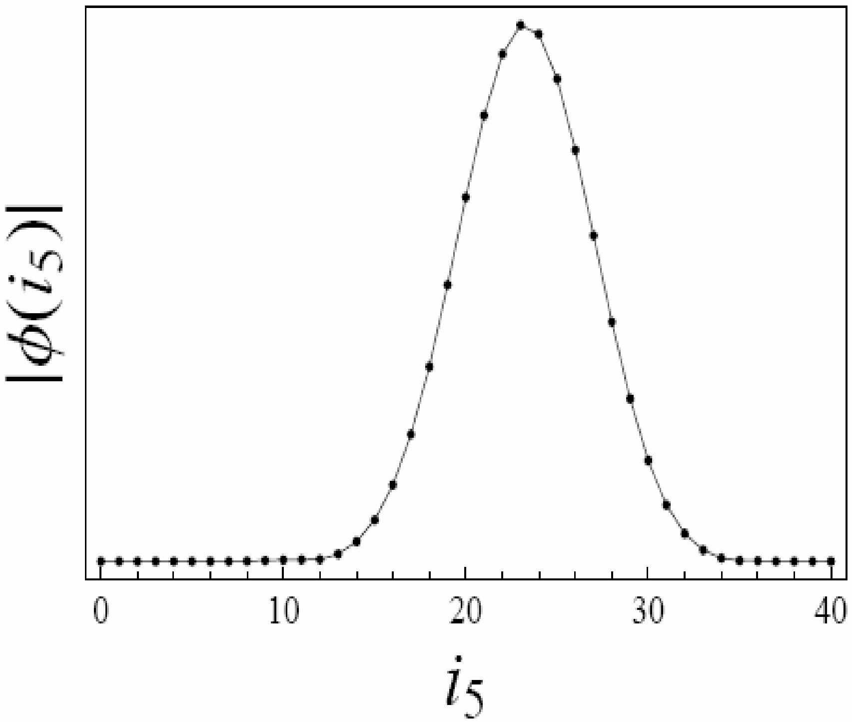
YES



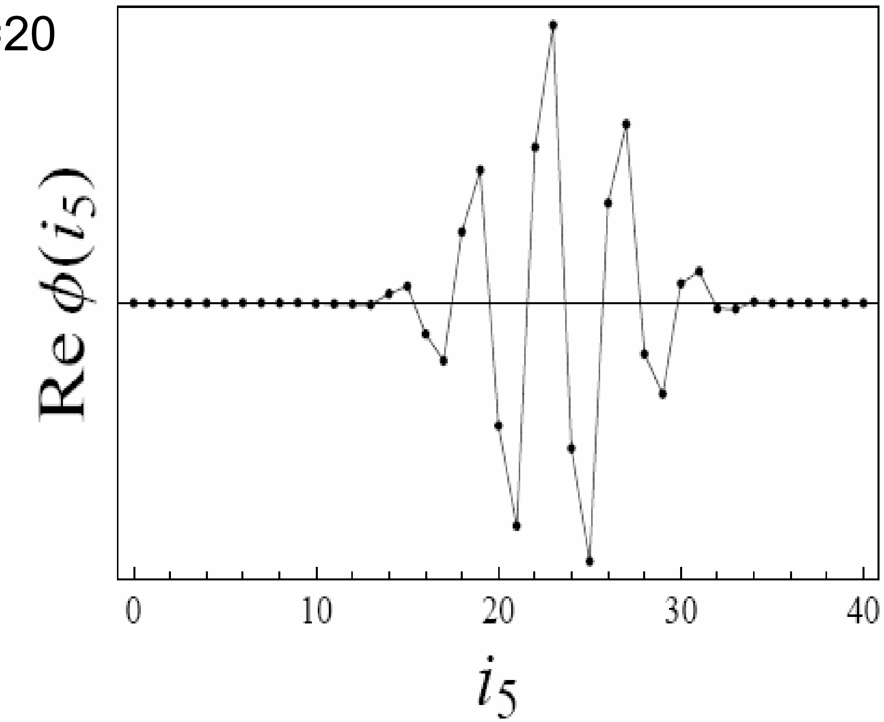
Output

The flipped vertex amplitude appears to evolve four coherent tetrahedra into one coherent tetrahedron, consistently with the flat solution of the classical Einstein equations

We have improved the previous result numerically,
E.Alesci,E.Bianchi E.Magliaro, C.Perini, "Intertwiners dynamics in the flipped vertex" work in progress (also Igor Khavkine is working on the same subject)



J=20



Not only the mean values but the entire gaussian shape is exactly reproduced

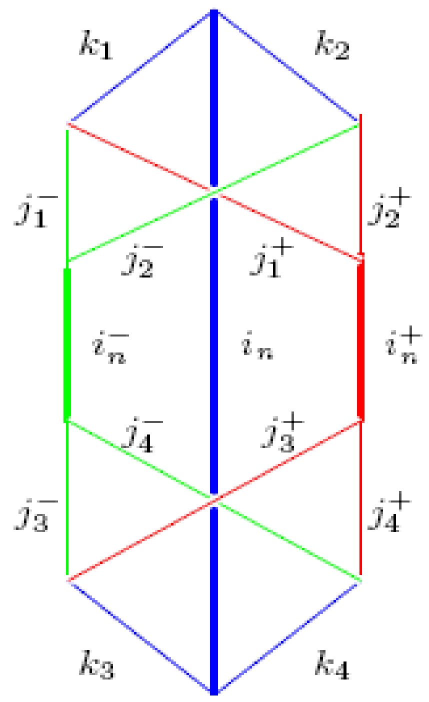
The phase in the outgoing state $(\pi/2)$ should come from the vertex
This could indicate that the vertex has the appropriate phase $(\pi/2)$

Analytic: results on f

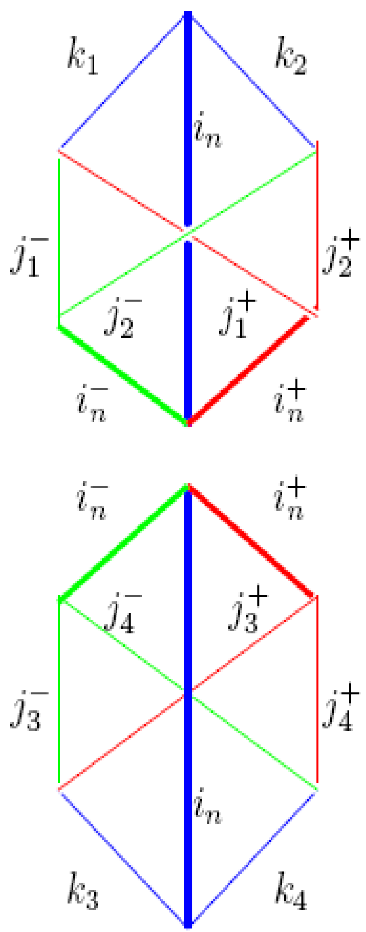
E.Alesci,E.Bianchi E.Magliaro, C.Perini,

"Asymptotic properties of the EPR fusion coefficients" to appear

Playing with recoupling theory



=



$$= \begin{Bmatrix} j_1^+ & j_1^- & i_n^+ \\ j_2^+ & j_2^- & i_n^- \\ k_1^+ & k_2^- & i_n \end{Bmatrix} \begin{Bmatrix} j_3^- & j_4^- & i_n^- \\ j_3^+ & j_4^+ & i_n^+ \\ k_3^+ & k_4^- & i_n \end{Bmatrix}$$

PRODUCT OF 2 9j SYMBOLS

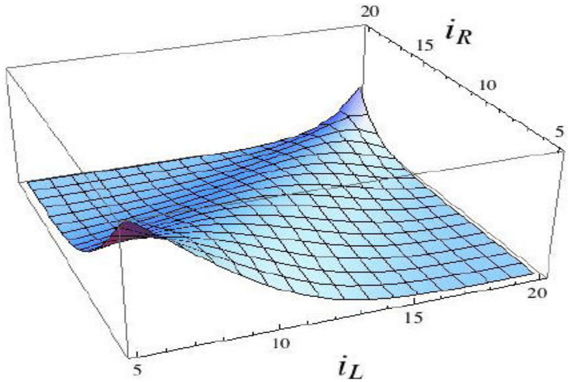
In the case $k=j^++j^-$ (EPR model) simple analytical formula, involving only factorials and a single Clebsh Gordan coefficient (no sums!)

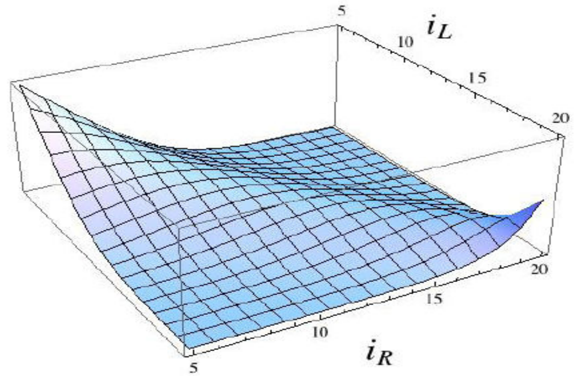
Information about the boundary state

In the special case of all j's equal (used to compute the wave packets propagation), The f_{EPR} has a simple asymptotic expression.

The dominant term is

$$f^{i=i_R+i_L}_{i_R,i_L} \approx N_i \, e^{-\frac{(i_L-i_R)^2}{\sigma_i}}$$





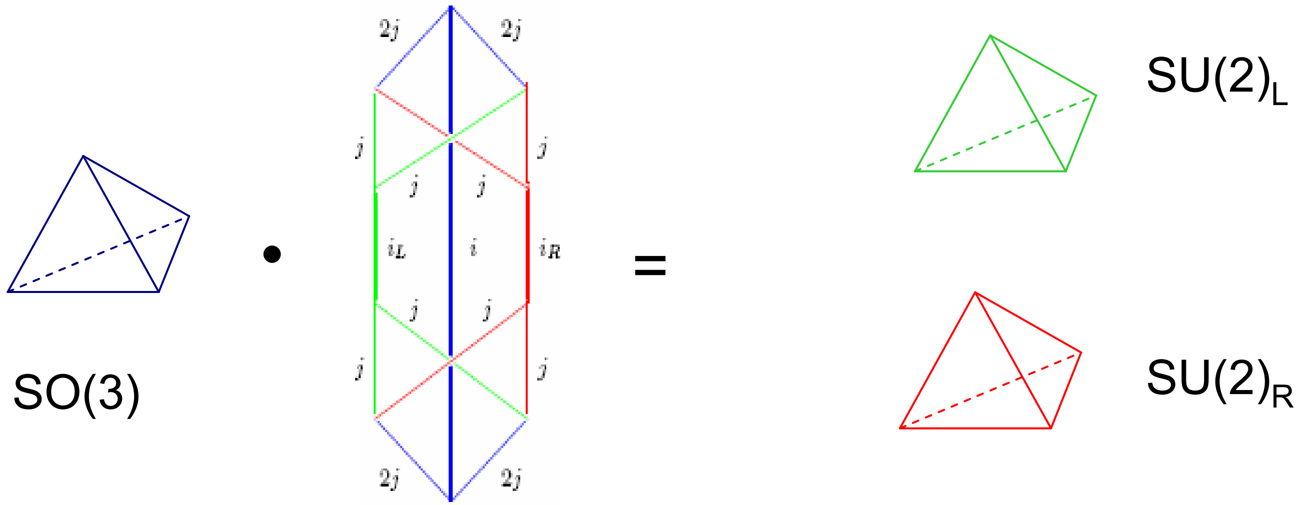
The σ has a remarkable feature: for $i=i_0$ (the value of the angle in the classical region) it is exactly the σ of the coherent tetrahedron.

Does the vertex itself codes the information about the boundary state? (as in ordinary free quantum field theory).

No answer for the moment but it is an interesting possibility…

Factorization of the Dynamics

The contraction of f_{EPR} with an SO(3) coherent tetrahedron
produces two SU(2) coherent tetrahedra; Centered around $i_0/2$ with the SAME phase



If we think to the wave packets propagation we have Two consequences:

- Dynamic at leading order factorizes
- This indicate that the EPR model could have the correct phase dependance to reproduce the propagator

To confirm this prediction we need exact informations on 15js: still missing….

Conclusions

CHANGE THE DYNAMICS: THE BARRET CRANE MODEL HAS NO INTERTWINER DEPENDANCE; USING A BC VERTEX WE ARE NOT ABLE TO REPRODUCE THE RIGHT LONG DISTANCE BEHAVIOR OF THE GRAVITON PROPAGATOR. In this sense the BC VERTEX DOESN'T WORK

Alesci, Rovelli Phys.Rev.D 76,104012 (2007), arXiv:0708.0883

THIS RESULT HAS MOTIVATED THE SEARCH FOR ALTERNATIVE MODELS

Engle-Pereira-Rovelli, Alexandrov, Livine-Speziale, Freidel-Krasnov

Full tensorial structure and right long distance behavior:

ASSUMING A VERTEX
WITH NON TRIVIAL INTERTWINER DEPENDANCE,
IT IS POSSIBLE TO RECOVER
THE FULL GRAVITON PROPAGATOR OF THE LINEARIZED THEORY
FROM LQG USING ROVELLI'S TECHNIQUES TO COMPUTE SCATTERING AMPLITUDES IN BACKGROUND INPEPENDENT FORMALISM

Alesci, Rovelli Phys.Rev.D77, 044024 (2008), arXiv:0711.1284

THIS RESULT GIVES INDICATIONS ON THE BEHAVIOR THAT AN ALTERNATIVE VERTEX CAN HAVE TO REPRODUCE GR

On the new models

Indications E.Alesci,E.Bianchi E.Magliaro, C.Perini

- Good numerical behavior for the packets propagation
- Analythical formulas for f:
  Great simplifications in the numerical calculations
  Factorization of SO(3) dynamics in left and right ones
  The vertex seem to show the required phase numericaly
  (without it the wave propagation could not be possible)

Open Questions

- The vertex know the boundary state? (like in free QFT)
- Integral formulation can give asymptotics?
  It gives the projector that realize the vertex. E.Alesci, K.Noui, F.Sardelli to appear
- Can this projector be related to the canonical theory?