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Relativistic Corrections to Lunar Occultations

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Abstract

During a lunar occultation the light of a star is bent by the solar gravitational field, while the lunar profile is much less shifted by relativistic light bending. Relativistic corrections of order of milliseconds in the absolute timing of occultations have to be considered for this effect. The accuracy of absolute timing predictions is studied with the aid of WinOccult program in the cases of occultations of Saturn (May 22, 2007), Venus (June 18, 2007) and Pleiades (August 7, 2007), including the effect of irregularities of lunar limb's profile down to sub-arcsecond length scale.

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I. INTRODUCTION

Lunar occultations have been used for several purposes in astronomy. The geometry of an occultation is determined outside the atmosphere, therefore atmospheric refraction and seeing do not affect the timing of the phenomenon.

Ancient occultations recorded by Ptolemy in the Almagest were used recently for recovering the secular acceleration of the Moon [16]. This acceleration is $a_{Moon} \sim -23"/cy^2$ and after 23 centuries it implies a deviation from mean motion of 3.4°. First observations of lunar occultations were made by Timocharis in the third century before Christ (β Scorpii on Dec. 21, 294 b.C.) [12]. The accuracy of timing was $\sim$ 10 minutes, and at the average angular velocity of the Moon relative to fixed stars of $v_m = 0.5"/s$ it corresponds to $\sim 5' = 0.08°$.

The discovery of close multiple systems is possible from the accurate timing of occultations' light curves. Identifications of multiple stellar systems during lunar occultations become common in the last century. Spica's occultation of June 10, 1753 was observed by G. B. Audiffredi in Rome. He reported a non instantaneous disappearing of the star [14], which may be the first noted evidence of its multiplicity [18].

More recently occultations were used in astrometry to detect accurately the position of a star or a quasar, with a precise timing and knowledge of lunar orbit: this is the case of quasar 3C273, whose optical counterpart was identificated in 1962 when the Moon occulted its radio flux and re-examined two Saros cycles later in 2001 [13].

Nowadays accuracies of millarcsecs are common dealing with lunar occultations.

Relativistic corrections in lunar orbit are known also experimentally from Lunar Laser Ranging [1].

Relativistic effects due to gravitational light bending by the solar gravitational field are of order of milliarcseconds, and they will be here evaluated.

II. SOLAR ECLIPSES AND LUNAR EPHEMERIDES

The more diffused software for occultations' predictions is WinOccult issued by D. Herald [8]. It is based upon DE200 lunar ephemerides and stars' positions of Hypparcos catalogue. Occultations can be used to check the accuracy of lunar ephemerides. The lunar ephemerides used in Baily bead module of WinOccult can be adjusted in order to reproduce the observed

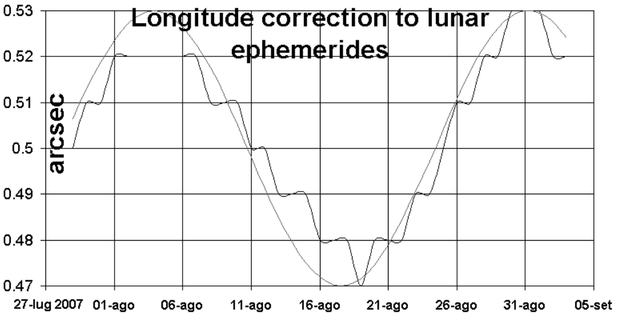

FIG. 1: Lunar longitude corrections depending on libration phases, for WinOccult program.

UTC timed sequence of beads near the centrality of the solar eclipse (technically speaking solar eclipses are lunar occultations of the Sun). The amount of corrections should not exceed 0.5" in right ascension and declination of the Moon, but the author expects no corrections at all, because DE200 Lunar ephemerides are issued up to relativistic effects [9].

A. Correction to Lunar Longitudes

There is a difference in lunar longitude among French VSOP87 and American (issued by Jet Propulsion Laboratories) DE200/LE200 lunar ephemerides of +0.50" in lunar longitude and -0.25" in latitude, it deals with the difference between the center of the figure and the center of mass of the Moon. It gives a systematic difference between eclipse predictions made by NASA and those made by other National institutions like the Bureau of Longitudes.

In the Occultation Prediction module of WinOccult, viewing lunar polynomials, that empyrical longitude correction appears to be 0.5"+0.03" $\cdot\ sin(2\pi/T)$ with T=27.2 days (draconitic month), and it follows the libration phases of the Moon (fig. 1).

B. Corrections to Mean Lunar Limb

The irregularities of lunar limb at subarcsecond scale are treated [10] [9] by defining three value of mean lunar limbs: average, upper and lower. Those mean limbs are used for general predictions: the average one for predicting the location of umbral path; the upper one for the beginning of totality and the lower one for its end (in this way the observers will be on time or less than 3 s in advance for each event). These predictions are regularly published by IMCCE and NASA in the occasion of solar eclipses.

True lunar limbs have to be used for local accurate predictions, or data reductions, in case of observations of Baily beads. The software which has an access to Watts' charts of lunar limbs [7] for various libration phases is Baily beads module of WinOccult, while Occultation module of WinOccult uses the average lunar limb, introducing an uncertainty on the absolute occultation timing of ±3 s. Dealing with planetary occultations of Saturn (May 22, 2007) and Venus (June 18, 2007) I have considered the local Watts' profile of the Moon to correct the predictions made with VSOP87 or DE200 ephemerides with mean limbs.

III. LUNAR LIMB PROFILE ACCURACY

Using Watts' profiles would reduce the uncertainty to the subarcsecond level of ±0.2" i.e. ±0.4 s, which is the intrinsic uncertainty on Watts limb's features. Morrison and Appleby [2] have extensively studied lunar occultations recorded in the second half of XX century and they found systematic corrections to be applyied to Watts' profiles. These corrections depend on Watts' angle (read counterclockwisely from lunar North pole) on the lunar limb. Their maximum amplitude is 0.2". After that systematic correction random errors up to 0.2" are still possible.

An example of this kind of uncertainty is evident in the case of grazing occultations [4]: the spatial resolution of Watts' profile is 0.01° in Watts angle corresponding to ~ 0.08" at mean lunar distance from Earth. Since 1" at the same distance corresponds to 2 Km, details on lunar limb are sampled each 160 m, with an accuracy of ±200 m.

It is therefore possible to discover new features during grazes [15] or to miss the occultation for few meters, as in the case of Electra's graze of August 7, 2007.

IV. OCCULTATIONS DATA AND PREDICTIONS

A. Electra's Graze on August 7, 2007

At the location of $41°54'55.5"$ N and $12°14'39.3"$ E we were at the Southern limit for grazing occultation of Electra, the 3.7 star of Pleiades. The mean lunar limb would occult

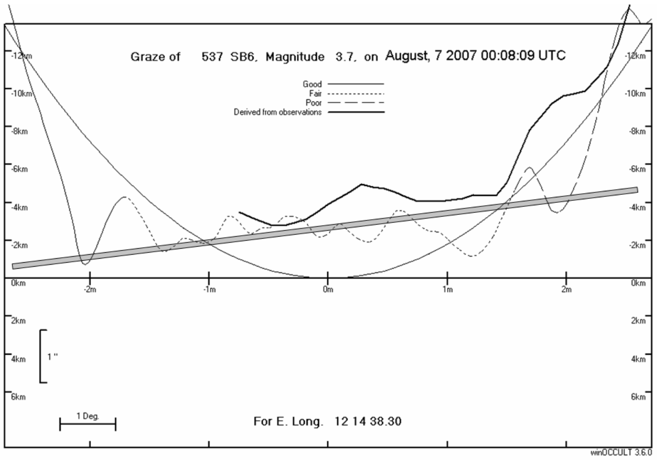

FIG. 2: Geometry of the graze of Electra at observer's location. The plot of the profile includes the corrections to the Watts charts determined by Morrison & Appleby MNRAS (1981) 196, 1013 as a correction to the profile height above the mean limb. The profile deduced from past grazes is independent of the datum used for the Watts charts (and thus is not subject to these corrections. For the longitude 12°14′39.3" E in the x-axis there is the time in minutes before and after graze. In the y-axis the Km in (-) or outside (+) the shadow of the Moon. The inclined line is the grazing star's path relative to the Moon.

the star for the same longitude at latitude 41°53′34" N, being us 1′21.5" N of that location we were 81.5" N, i.e. 2.51 Km inside the mean limb shadow.

The Moon at that time was moving toward northern latitudes with an angle of 13.92° with respect to celestial parallels. The corresponding Position Angle is PA=76.08°. The lower part of lunar limb shown in figure 2 has Watts Angle WA=171.69°; the lunar North pole (WA=0° was at PA=347.40° = −12.6° that night, therefore the PA of graze is $PA_g =$ 159.09°. An horizontal line corresponds to a motion perpendicular to PA_g i.e. at 69.09°. The star appear to move with an angle of 7° with respect to horizontal. The motion of the Moon is Eastward, so a star near the limb moves towards rising PA, from 160° to 180°

B. Pleiades August 7, 2007

In the table 1 the reapparitions of Pleiades from dark limb of the Moon have been recorded during the series of occultations of august 7, 2007.

The calculated timings are between parentheses and are rounded to the closest integer second (only seconds are indicated); these times are taken from Occultation Predictions

TABLE I: Pleiades Occultations of August 7, 2007. Reapparitions

Star name	obs.UTC(calc.)	W. A. [^{o}]	h ["]	v ["/s]	res. ["]
V0624 Tau	0:19:10.5(09)	276	-0.90	0.58	1.58
V1187 Tau	0:22:02.3(02)	315	0.90	0.41	-0.92
SAO 76119	0:36:05.6(04)	267	0.55	0.59	0.19
Celaeno	0:38:28.8(28)	239	-0.46	0.52	0.69
SAO 76136	0:41:40.9(31)	228	0.21	0.44	4.05*
Taygeta	0:56:08.7(07)	268	0.02	0.59	0.78
SAO 76152	0:57:14.1(13)	233	-0.41	0.48	0.77
Maia	1:03:36.1(35)	236	-0.90	-0.50	1.28
Asterope	1:15:08.7(07)	272	-0.63	-0.59	1.43
			Average		0.73*±0.80

of WinOccult. They are calculated relatively to mean lunar limb. The height h ["] of Watts profile with respect to mean lunar limb with Morrison and Appleby corrections are in the table, with the velocity v ["/s] of the lunar profile corresponding to each Watts angle ($PA = WA - 12.6°$ for that night). The final average has been obtained eliminating the data on SAO 76136. The effect due to the rounding processes can be estimated adding $-0.5 \pm 0.5 \cdot$random(x) s to the predicted values. The data are obtained from an audio record of the occultation, synchronized to UTC.

Personal equation [3] PE has been considered as 0.34 s. The final result is rather independent on PE which can range between 0.3 and 0.8 s yielding similar results.

The observed OT and the real RT time of the event is given by the equation OT=RT+PE. The calculated time CT=CT(mean limb)+h_{Moon}/v_{Moon}.

The final differences between OT and CT includes PE and uncertainties on Watts profiles.

Finally the ephemerides of the Moon appear to be in advance of 0.7" $\pm$0.8" with respect to real observations. The above correction to the VSOP87 longitude of the Moon, of 0.52" for that night fits that 0.7" shift of the ephemerides with respect to observed value. From those data the 0.52" empyrical correction seems not justified.

C. Venus June 18, 2007

The reapparition of Venus limb during lunar occultation of June 18, 2007 has been accurately videorecorded with UTC synchronization at it occurred at 15:52:29.0 UTC at location East Long. 12°27′7.1” Lat. 41°52′44.8” Altitude 64 m. From WinOccult Occultation Predictions the tabultated time of reapparition (of the center of the planet) is 15:53:04, with 81 s of partial stage of disk on the lunar limb. Therefore the reapparition of firs limb is calculated for 15:52:23.5, 40.5 s before the apparition of the center of Venus above the mean lunar limb.

At the Watts angle WA=256° the lunar limb exceedes of h=1.30” the mean limb, for libration phases 6.22° in longitude and −2.59° in latitude. The Moon's velocity (orbital motion + parallax) relative to Venus is 0.31”/s, so Venus is expected to appear 4.2 s later at 27.7 s, because of limb's mountain, still 1.3 s before than observed.

Again the longitude correction, posed to +0.50”, gives an extra 1.6 s second of advance of lunar ephemerides with respect to the observed Moon, whithout this correction the prediction for the emersion of Venus would be 29.3 s and O-C=-0.3 s. This final difference would correspond to 0.093” which is within the uncertainty of Watts' profile: there the mountain is 93 milliarcsec smaller than Watts' atlas, but it includes the rounding error to the near integer second in the prediction of reapparition's time.

D. Saturn May 22, 2007

Applying the same considerations of the case of Venus, there is a difference of O-C=+0.3 s in the disappearance of Saturn's disk at location East Long. 12°27′7.1” Lat. 41°52′44.8” Altitude 64 m, while at Specola Vaticana (location East Long. 12°39′0.9” Lat. 41°44′50.2” Altitude 350 m) O-C=+1.0 s. The difference between the two locations may be due only to prediction's rounding errors.

At both locations UTC timings were accurate at levels better than 0.1 s. Calculated times include the effect of a lunar limb's mountain of h”=1.32” at Watts angle 125°.

The observations show the lunar ephemerides in advance of 1 s with respect to real Moon. At the angular speed of Moon relatively to Saturn (0.41”/s) this correspond to a shift of 0.41” again compatible with the above longitude correction of 0.5”.

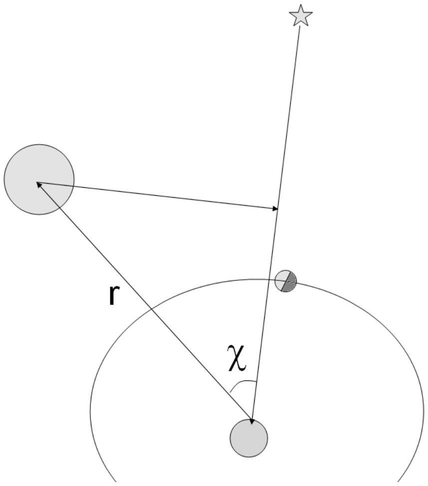

FIG. 3: Geometry of a lunar occultation, with χ and r shown.

Also the case of Saturn seems to reject such correction: there is a similar advance of the lunar ephemerides with respect to the observed Moon always of $\sim= 0.5$".

V. RELATIVISTIC CORRECTIONS TO OCCULTATION PREDICTIONS

Thanks to the optimal astrometry available for bright stars we can check light gravitational bending during lunar occultations with maximum exactness.

Using IMCCE or USNO ephemerides, the coordinates of stars from Hypparcos catalogue and the corrections to the mean lunar limb from Watts charts it is possibile to detect relativistic effects with an apparatus capable of millisecond accuracy.

The Gravitational light bending during a lunar occultation is properly a differential effect, but we can neglect in first approximation the contribution due to the lunar gravitational field and we consider only the solar field. This effect for a star at χ degrees of elongation from the Sun is given by the equation (see fig. 3):

$$\delta\chi = \frac{4GM_\odot}{c^2 r \tan(\chi/2)}$$

At $\chi = 75.2°$, the elongation of the Pleiades during the occultation of Aug. 7, 2007, and at the observer's position r=1 AU $\delta\chi$ =10.64 milliarcsec.

This bending shifts radially away from the Sun the stars' apparent position; in practice it shifts the star backwards of 10 milliarcsec in the lunar longitude direction in this case. That is a delay with respect to ephemerides of 17.2 milliseconds, well measurable with time accuracy of photoelectric photometry.

Those relativistic corrections vary with solar elongations and affect systematically, to

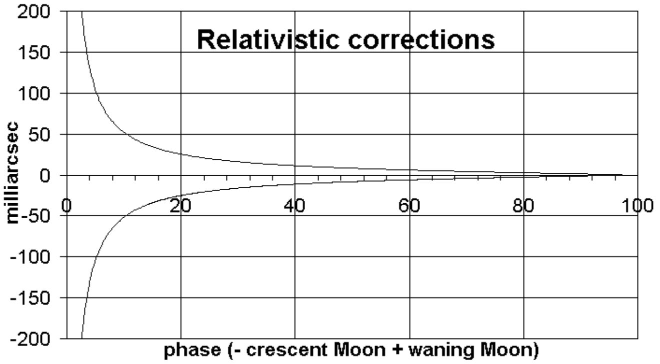

FIG. 4: Relativistic corrections versus lunar phases. Negative corrections correspond to phases of crescent Moon.

the level < 10 milliarcsecs, all Morrison and Appleby corrections to Watts' datum and all observational corrections to Watts' profiles. This correction is still smaller than the uncertainty of < 200 milliarcsecs for each feature of Watts map, but it has to be considered when more accurate lunar profiles will become available.

In our cases the temporal accuracy has been down to 1/25 s (videorecords with standard camcorder), and at the angular velocity of the Moon it corresponds to 77 milliarcsecs of accuracy in the case of Venus' occultation.

In case of planetary occultations, in particular Venus, the observer position can be posed in the planet, and the calculation follows with corresponding elongation angles. In the case of Saturn the effect is similar to a star with the same elongation (1.4 milliarcsec or 2 millisec of time), while for Venus is much smaller, being Venus at 0.72 AU from the Earth. The Moon, for the same reason of Venus, undergo a negligible bending effect, $\sim 1\mu$arcsec, while its own gravitational field gives 0.026 milliarcsec at grazing position for a star [5] [6].

A. Ephemerides Accuracy

Ephemerides VSOP87 and DE405/LE405 show agreement for Venus in that occultation within 10 milliarcsecs. At 15:52 UTC of June 18 the center of Venus, as seen from the location of observation (true equator; equinoxe of the date, from IMCCE website) was located at DE405 R.A. 9h 0m 9.70931s dec. +18° 55' 26.7597" VSOP87 R.A. 9h 0m 9.70813s +18° 55' 26.7670". The separation bewteen the two predictions (which include relativistic corrections)is 16.7 milliarcsec in right ascension and 7.3 in declination, 18.2 milliarcsec in total. For comparison the relativistic precession for Venus perihelion is 86.3 milliarcsec per year, which multiplied by the orbital eccentricity gives the observable effect of 0.59 milliarcsec per year [17]. For reference solar ephemerides VSOP87 and that one used by US Naval Observatory in the Nautical Almanac are close within 0.1 s [11], or 4 milliarcseconds.

VI. CONCLUSIONS

Relativistic light bending is detectable during lunar occultations modulated by the elongation from the Sun. The uncertainties on lunar limb features are presently larger than this effect. This problem can be partially overcome by observing the same star occulted after a sidereal month (there is a small change in libration phase). Reduction procedures have to consider explicitly the corrections to lunar longitude.

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