

# Guidelines for measuring solar radius with Baily beads analysis

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By inspection of central eclipses videorecords, data of Baily beads timings are retrievable. Knowing the lunar limb profile at the moment of the eclipse we evaluate the excess or defect of solar limb when the Sun is assumed at its standard radius. Two procedures of data analysis are here presented: one based on limb heights and the other on times. While these methods are based upon Occult 4 software, they can be used with other ephemerides and new lunar profiles. The example of 2006 total eclipse data, with its remarkably negative value of  $\Delta R = -0.41'' \pm 0.04''$  is presented.

solar eclipses, observations, ephemerides, solar radius variations

## 1 Introduction

Baily beads were first noticed by the english astronomer Francis Baily during the annular eclipse of May 15, 1836. They are produced by the light of photosphere through the valleys of the lunar limb at the onset of totality or annularity.

The idea of measuring the solar diameter using total eclipse data can be placed back to Edmund Halley who coordinated the observation by several fellow of the Royal Society of the eclipse of 1715, total over London. Halley reported two observations near the umbral limits, used for the solar diameter's estimation by Dunham et al. (1980). Again in 1869 the observations of a total eclipse over the United States were coordinated by Simon Newcomb (1870), for getting observations near the path's limits. Ernest Brown at Yale University did the same work for the eclipse of 1925, total over New York (Sofia et al., 1983), and from 1970s total eclipses were observed from umbral path's limits with the explicit purpose of determining solar radius variations with few hundredths of arcsecond of accuracy (Fiala et al., 1994; Dunham et al., 2005). Independently Kubo (1993) found values of solar diameter with accuracy of 0.01 arcsec for total eclipses observed in 1970, 1973, 1980 and 1991 using DE200/LE200 ephemerides and corrections to Watts profile given by lunar occultations.

If the eclipse's observer is on the centerline, the duration of totality is proportional to the solar diameter, through the angular topocentric velocity of the Moon relative to

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the Sun. Baily beads timings (Herald, 1976) are also influenced by solar diameter, and totality's start and end are particular cases of such beads.

To increase the statistic Dunham proposed to use Baily beads in connection with lunar profile data, obtaining dozens of data for each eclipse and statistical accuracies down to 0.02 arcsec. Video records of total eclipses by projection on a screen or direct vision through a filter, with enough angular resolution, allow to see Baily beads disappearances (D) and reappearances (R). The ultimate limit of such method is the knowledge of lunar limb features, now available only from Watts atlas and updates from lunar occultations or grazes at all libration phases. To see unambiguously solar radius variation between two eclipses Dunham proposed to use only polar beads, because the corresponding lunar valleys have always nearly the same depths from one eclipse to another, since the lunar libration in latitude is nearly zero during a solar eclipse. Polar valleys' depths are being upgraded by lunar grazing occultations of stars, with accuracies better than a few hundredths of arcsecond.

In this paper I show how to obtain  $\Delta R$  from Baily beads data of both North and South limits or only one of them. Allowance for  $\Delta T$ ,  $\Delta_{long}$  and  $\Delta_{lat}$  small variations is left to correct ephemerides or unaccurate timings. This method uses residuals obtained with Occult 4.0.5 software and it is suitable for a spreadsheet.

## 2 Typical data format

A video made in occasion of a total eclipse for measuring the solar diameter is accompanied by information on observer position, telescope, filter, detector and timing method. Usually the position is given by a portable GPS receiver. The measurement of the longitude and latitude after a couple of minutes with GPS receiver is rather accurate, if no obstacles are present and most of the horizon is visible from the observing location. The accuracy is on the last digit, around  $\pm 0.1''$  both in longitude and latitude ( $\pm 3\text{m}$ ), and it is trustable, since no more selective availability is affecting commercial GPS. Generally the GPS is set on WGS84 datum, with quote expressed above mean sea level. Different is the case of the height, which can be wrong of  $\pm 10\text{m}$ , without averaging GPS data over several hours. After that average, the greatest uncertainty in the height measurement arises from the coarseness of the look-up table of conversion between WGS84 ellipsoid and mean sea level, used in the GPS unit. Position data are inputs for Occult 4 Software, which can be downloaded at the address <http://www.lunar-occultations.com/iota/occult4.htm> .

The video can be analyzed with Limovie, or by eye using a frame per frame tool as Quick Time 7.0, and a set of Baily beads events is prepared. Each event is characterized by D-disappearing event or A-appearing event; UTC of the event, Watts (axis) angle of the bead, height of the standard solar limb above (or below) the lunar valley.

The Watts angle is identified from the video with uncertainty due to the scale factor of the image and to the absence of an absolute reference point. A simulation of the eclipse circumstances, as the "Limb plot" or "Eclipse Image" windows of "Baily Beads" module of Occult 4, help the identification of the valley which are producing D or R

events at given times. This module uses the Watts (1962) profiles of lunar limb referred to the lunar center of mass, and includes the possibility to see also lunar occultation data recorded in the same libration phases.

Often Watts profiles and real lunar features differ slightly. As an example in the 2006 total eclipse at Northern limit, the totality was expected (by Watts profile) to start with D event at  $348^\circ$ , while the last photospheric light came from a (presumed) shallower valley at  $357^\circ$  which was deeper in reality. Same thing at Southern limit, where the observed sequence of D events at  $142^\circ$  and  $147^\circ$  is reversed with respect to simulated ones, due to a deeper valley at Watts angle  $147^\circ$ .

The uncertainty on Watts profile is like a random error, estimated by the author (Watts, 1962) to be around  $\pm 0.10''$ , but being actually more like  $\pm 0.20''$ .

### 3 Limb's heights residuals

The first method of estimation of solar radius correction to standard value of  $959.63''$  is based on limb's heights residuals. A residual is the height of the solar limb at the time of D/R event above (+) or below (-) the Watts limb. Since in Occult 4.0.5, released in August 28, 2008, there is the possibility to plot lunar occultations over the Watts profile, one can choose to use or not the Morrison and Appleby (1981) mean corrections to the lunar profiles (used for all the following data).

Occultations data and event sequence help to discard data from which the simulated profile departs too much from the real one. The alternative is to include all data available, leaving to the increased statistics the reduction of the uncertainty.

Considering, for example, the data from 2006 Northern limit (table 1 D and R with lowercase G) with Appleby and Morrison correction we obtain a standard deviation of such residuals (res.1) is  $\sigma=0.21''$ . It is to interpret as the aforesaid Watts limb's random error. The average value of res.1 is  $-0.40''$ , it means that the Sun is smaller than its standard value. The standard deviation of the average is  $\sigma/\sqrt{N}$  and the solar standard radius is in excess of  $0.40''\pm 0.09''$  with respect to the lunar valleys through which it should dis/appear at the same instant of each observed event of disappearance / reappearance. The correction to the standard radius is therefore  $\Delta R=-0.41''\pm 0.09''$ , for Northern limit data. This result has been obtained with Occult 4.0.5 using DE421/LE421 ephemerides. Small changes in the ephemerides, like a further shift in lunar longitude or latitude can slightly change such value as we can see in table 1 for res.3 and res. 4 columns. In Occult 4.0.5 all Watts profiles are referred to the center of mass of the Moon, through a shift of  $0.50''$  in longitude. No shift in latitude is made, while it is mentioned to be  $-0.25''$  in the website of IMCCE, Institut de Mécanique Céleste et Calcul des Ephémérides. In other words the observed lunar occultation data are referred directly to the center of mass of the moon - and hence the DE421 ephemeris position of the Moon - whereas the Watts charts are referenced to the center of figure, which must be corrected to the center of mass (and it is done in Occult 4). Therefore, the observed occultation data avoids the need for any ephemeris corrections and moreover the occultation data is also more reliable than the Watts

charts.

The exact value of the lunar latitude shift remains uncertain, Morrison and Appleby (1981) show a series of results based upon occultations, laser ranging and other methods, ranging around 0.50" of shift with various uncertainties. To allow for such a shift in lunar longitude it is to include a  $\Delta_{long}$  parameter, of the form  $\Delta_{long} \times \cos(WA + const)$ . The constant is determined by the direction of the lunar motion indicated by its Watts Angle and residuals are plotted as res.3.

The lowercase S and C stands for Sigismondi (31° 32' 17.6"N 26° 11' 32.3"E, 27 m) and Paolo Colona's (31° 32' 27.4"N 26° 11' 24"E, 27 m) locations at Southern limit. For both C and S, UTC timings are systematically shifted by  $\Delta T_S=0.04$  s: this value makes identical the average of residuals for both observers. Both C and S used a Meade 70ETX refracting telescope 70 mm diameter and F=350 mm, with a Tamron orange filter YA2. The telescope was projecting a 15 cm image of the Sun over a white screen and it was videorecorded with a SONY (S) and JVC (C) camcorder. The timing was obtained by filming the screen of the GPS Garmin II+. The lowercase G stands for Konrad Guhl's Northern limit (37° 23' 40.4"N 30° 46' 51.5"E, 600 m) location. For Northern limit  $\Delta T_N=0.00$  s. Guhl used a 50 mm telescope with focal length 540 mm and a density 4 Baader neutral filter. The images were videorecorded (VHS system) at the focal plane with a Mintronic camera and a Cuno type time inserter.

To each residual res.1 is added the product  $\Delta T \times v_{rel}(\alpha)$  to get res.2 columns. For Egyptian beads at 10:41 UT from lunar and solar ephemerides we retrieve a topocentric relative motion in right ascension  $v_{r,RA} = 0.313''/s$  and in declination  $v_{r,\delta} = 0.289''/s$ ; therefore the relative velocity of Moon to Sun was directed toward position angle  $PA_{rv}=47.3^\circ$  (PA is read from celestial North counterclockwise). The position angle of lunar North pole (zero of axis and  $0.21^\circ$  of Watts' angles) from lunar physical ephemerids during the eclipse was  $C=338.3^\circ$ , consequently  $v_{rel}(\alpha)$  at a given Watts angle  $\alpha$  is given by the formula  $v_{rel}(\alpha)=0.426'' \times \cos(WA-69.2^\circ)$ .

The corrections in res.3 and res.4 columns are displacements in lunar longitude and latitude. From the lunar ephemerides we have the following topocentric velocities  $v_{M,RA} = 0.356''/s$  and  $v_{M,\delta} = 0.304''/s$ , so the Moon is directed toward  $PA_M=49.5^\circ$  or  $WA_M=71.4^\circ$  and longitude correction is of the form  $\Delta_{long} \times \cos(WA - 71.4^\circ)$ , while the latitude correction is  $\Delta_{lat} \times \sin(WA - 71.4^\circ)$ . For Turkey at 10:56 UT we have  $v_{rel}(\alpha)=0.426'' \times \cos(WA-69.2^\circ)$  and  $PA_M=50.7^\circ$  or  $WA_M=72.6^\circ$ .  $\Delta_{long}=-0.006''$  correction is applied after  $\Delta T$  correction, and  $\Delta_{lat}=0.008''$  after  $\Delta_{long}$ . The final average of 35 beads in res. 4 column yields  $\Delta R=-0.41'' \pm 0.04''$ : the error here is statistical, obtained dividing the standard deviation for  $\sqrt{35}$ .

Table 1 Baily beads events for 2006, March 29 total solar eclipse. Data of Colona (C), Sigismondi (S) and Guhl (G). All residuals are calculated with Morrison and Appleby (1981) corrections of Watts profiles (1962).

event D/R	UTC	W.A.	res.1 $\Delta h$ ["]	res.2 w. $\Delta T$	res.3 w. $\Delta_{long}$	res.4 w. $\Delta_{lat}$
$D_C$	10:40:56.86	114.6	-0.71	-0.722	-0.726	-0.721
$D_C$	10:41:14.34	133.4	-0.35	-0.357	-0.360	-0.353
$D_C$	10:41:27.22	141.2	-0.13	-0.135	-0.137	-0.130
$D_C$	10:41:27.46	141.7	-0.05	-0.055	-0.057	-0.050
$D_C$	10:41:27.46	145.4	-0.18	-0.184	-0.186	-0.178
$R_C$	10:41:39.02	162.9	-0.01	-0.009	-0.009	-0.001
$R_C$	10:41:41.26	170.7	-0.53	-0.527	-0.526	-0.518
$R_C$	10:41:54.78	177.0	-0.54	-0.535	-0.533	-0.525
$R_C$	10:41:57.34	175.2	-0.44	-0.435	-0.434	-0.426
$R_C$	10:42:06.94	181.4	-0.77	-0.764	-0.762	-0.754
$R_C$	10:42:10.46	184.7	-0.76	-0.753	-0.750	-0.743
$R_C$	10:42:19.34	188.0	-0.46	-0.452	-0.449	-0.442
$R_C$	10:42:24.06	195.8	-0.54	-0.530	-0.526	-0.520
$D + R$	13 beads	Southern limit	no corr.	$\Delta T_S=0.04$ s	$\Delta_{long}=-0.006''$	$\Delta_{lat}=0.008''$
	averages ["]		$-0.420 \pm 0.072$	$-0.420 \pm 0.072$	$-0.420 \pm 0.071$	$-0.412 \pm 0.071$
$D_S$	10:40:27.10	94.0	-0.19	-0.205	-0.211	-0.208
$D_S$	10:40:33.46	100.1	-0.13	-0.145	-0.150	-0.146
$D_S$	10:40:49.90	109.2	-0.30	-0.313	-0.318	-0.313
$D_S$	10:40:53.58	111.4	-0.47	-0.483	-0.487	-0.482
$D_S$	10:40:57.18	114.6	-0.66	-0.672	-0.676	-0.671
$D_S$	10:41:04.66	122.4	-0.55	-0.560	-0.564	-0.558
$D_S$	10:41:14.22	133.4	-0.16	-0.167	-0.170	-0.163
$D_S$	10:41:29.58	141.6	-0.04	-0.045	-0.047	-0.040
$D_S$	10:41:33.18	147.0	-0.44	-0.444	-0.445	-0.438
$R_S$	10:41:29.06	162.8	-0.18	-0.179	-0.179	-0.171
$R_S$	10:41:39.18	170.8	-0.50	-0.497	-0.496	-0.488
$R_S$	10:41:53.94	175.2	-0.69	-0.685	-0.684	-0.676
$R_S$	10:41:55.26	177.0	-0.27	-0.265	-0.263	-0.256
$R_S$	10:42:06.62	181.4	-0.63	-0.624	-0.622	-0.614
$R_S$	10:42:09.54	184.7	-0.77	-0.763	-0.760	-0.753
$R_S$	10:42:22.94	195.8	-0.69	-0.680	-0.677	-0.670
$D + R$	16 beads	Southern limit	no corr.	$\Delta T_S=0.04$ s	$\Delta_{long}=-0.006''$	$\Delta_{lat}=0.008''$
	averages ["]		$-0.417 \pm 0.059$	$-0.420 \pm 0.058$	$-0.422 \pm 0.058$	$-0.415 \pm 0.058$
$D_G$	10:55:56.5	18.5	-0.12	-0.12	-0.124	-0.130
$D_G$	10:56:03.5	6.7	-0.36	-0.36	-0.362	-0.370
$D_G$	10:56:09.5	347.8	-0.69	-0.69	-0.691	-0.699
$D_G$	10:56:15.6	357.3	-0.24	-0.24	-0.242	-0.249
$R_G$	10:56:50.0	330.7	-0.44	-0.44	-0.439	-0.447
$R_G$	10:57:06.8	325.6	-0.58	-0.58	-0.578	-0.586
$D + R$	6 beads	Northern limit	no corr.	$\Delta T_N=0.00$ s	$\Delta_{long}=-0.006''$	$\Delta_{lat}=0.008''$
	averages ["]		$-0.405 \pm 0.086$	$-0.405 \pm 0.086$	$-0.406 \pm 0.086$	$-0.413 \pm 0.086$
$D + R$	35 beads	N and S limits	no corr.	$\Delta T_{N,S}$	$\Delta_{long}=-0.006''$	$\Delta_{lat}=0.008''$
	averages ["]		$-0.416 \pm 0.040$	$-0.418 \pm 0.039$	$-0.418 \pm 0.039$	$-0.414 \pm 0.039$

## 4 $\Delta R$ determination from relative timings

To avoid the uncertainty on ephemerides, or on the location of optical center with respect to the lunar mass center, and to avoid the procedure of  $\Delta T$  computation, one can use relative timings. The comparison is done between calculated and observed times at different  $\Delta R$ , as input for Occult 4.0.5.

## 5 Conclusions

The two methods presented, when  $\Delta T$  is found, are equivalent. The second one, of relative timings, is recommended with ancient and historical eclipses. The first one works either when both shadow limits are not available, or in the case of having only Northern or Southern limit observations, or centerline observations, and systematic errors in timing on ephemerides. The large negative correction to standard solar radius of  $\Delta R = -0.41'' \pm 0.04''$  occurred when mean sunspot number passed from 10.6 (March 2006) to 30.2 (April 2006, NOAA data), the largest variation for that year.

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