Blind estimation of common signal for the GW150914 event

Hao Liu

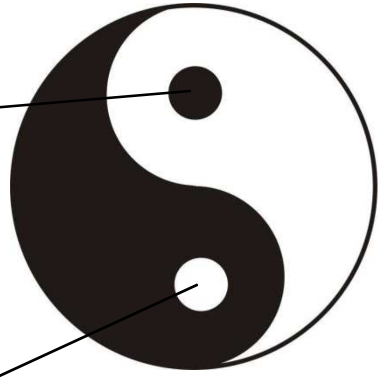
with James Creswell, Sebastian von Hausegger, Andrew D. Jackson & Pavel Naselsky

Niels Bohr Institute and the Discovery Center

For MG15@Rome, 07-2018

For GW150914

TaiJi



No matter how white, some black always remains.

Our knowledge is never perfect

The Taiji program in space for GWD @ CAS, China

See Arxiv:1805.10119 for introduction

Also TianQin, see Jun Luo's talk

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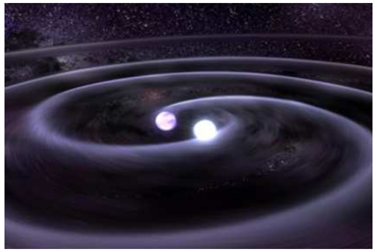
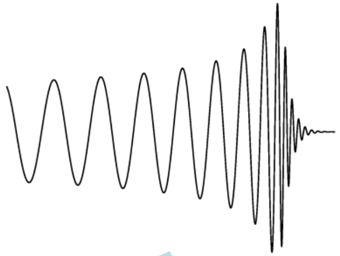
Assume GW150914 is a BH-BH merger event



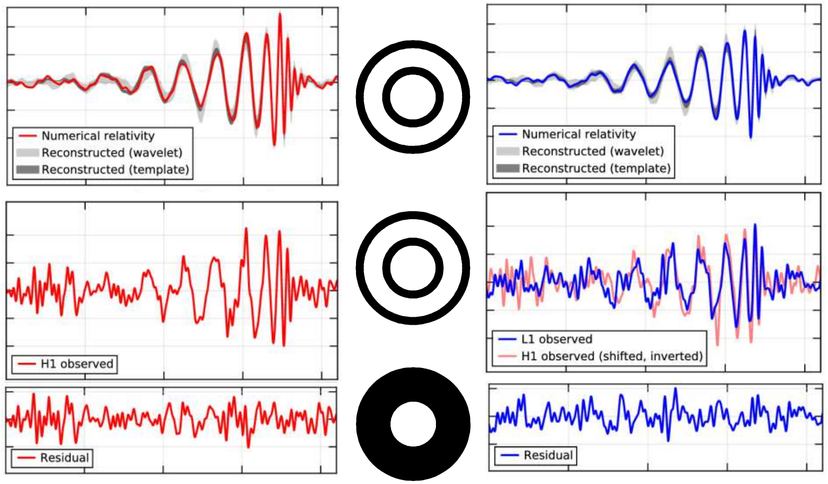
A unique physical signal



*Projection effects

*Instrument response

…



Residual (R) = Strain (S) - “projected common signal” (T)

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Two basic principles of detection

- Correlated signal.
  - Projection effect.
  - For convenience...
- Uncorrelated residuals.
  - Hanford and Livingston

These two principles are directly related to the likelihood

Likelihood functions are widely used, they can have various forms, but a standard core part is:

$$X^T C^{-1} X$$

Data or noise

Covariance matrix

A ***covariance matrix*** should have:

Diagonal terms (always non-zero)

Off-diagonal terms

- To assume zeros can greatly simplify the analysis, but this means very strong assumptions:
  - Perfect noise property.
  - Full knowledge of systematics.
  - Full knowledge of "Extra sources".

Have the off-diagonal terms been considered by LIGO?

- The probability of the data d_k given the parameters θ is

$$\mathcal{L}(d|\theta) \propto \exp\left(-\frac{1}{2}\sum_k \langle h(\theta) - d_k | h(\theta) - d_k \rangle\right)$$

where

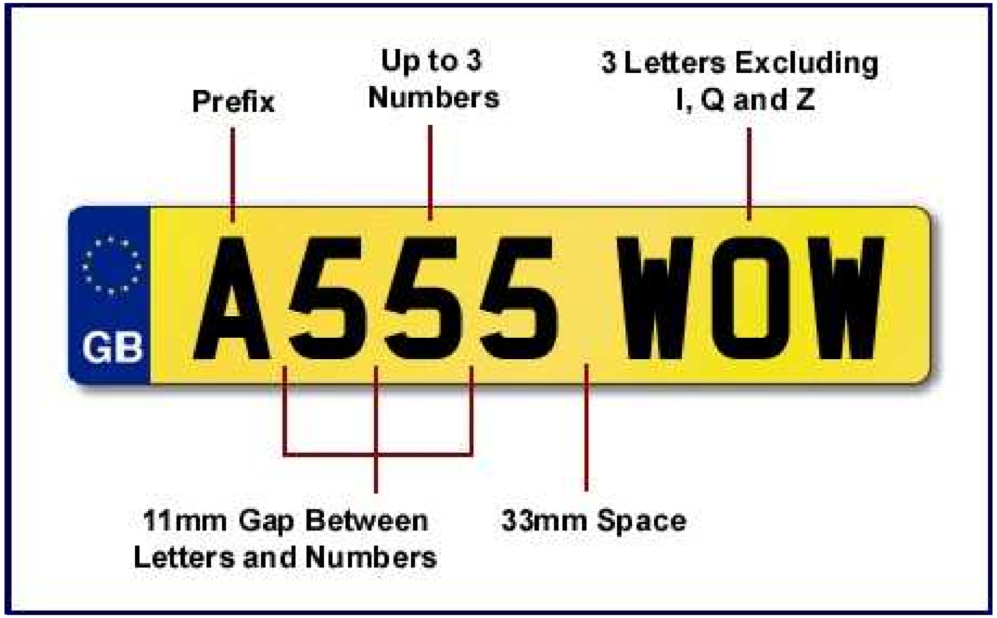
$$\langle h_1 | h_2 \rangle = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{h}_1^*(f) h_2(f)}{S_n(f)}\, df.$$

A practical issue to apply the basic principles: they both require an estimate of a “common signal”.

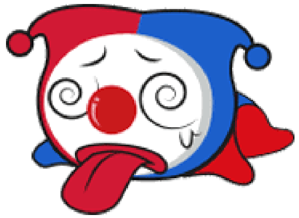
– Possibility 1: By using a GW *template bank*:
  - Advantage: much more sensitive.
  - Disadvantage
    - Affected by degeneracy (see James Creswell's talk in the HE8 session for more details).
    - This method is subject to many assumptions and constraints
      – **Assumption** that measured signal = simple template + noise.
      – **Assumption** of stationary Gauss noise (e.g., Needed to calculate power spectral density [PSD]).
      – **Complete** knowledge of the systematics (e.g., the glitch in the NS-NS merge).
      – **Complete** knowledge of “extra things”.
      – **One can easily see that all these are related to the off-diagonal terms of C.**

There are some interesting examples for a template analysis...

UK traffic police routinely use **template analysis** because they know *exactly* what to look for.

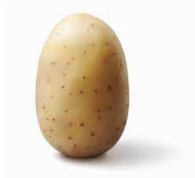


. . . But an Italian license plate would cause problems.

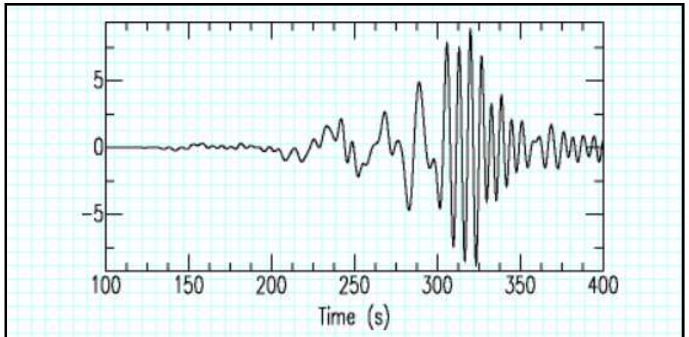


Even an artificial intelligence algorithm needs a "complete" set of templates.

I have a nice app for recognizing plants
One day it says...


=


LIGO's template set should include *all* possibilities (including terrestrial events). Otherwise, any match will be regarded as "proof" of a GW event.



A dispersed Rayleigh wave generated by an earthquake in Alabama near the Gulf coast, and recorded in Missouri.

Almost any catastrophic event, e.g. a seismic event, can have a "chirp"
(Just an example, not necessarily same to GW-chirp in all aspects)

"Incomplete" templates can lead to false positives and are potentially dangerous!

A practical issue: both principles require an estimate of a "common signal".

- Possibility 1: By using a GW *template bank.*
- Possibility 2: By a **blind estimation** based on **residual analysis**
  - Blind = no a priori assumption about templates and noise properties.
  - Robust but less sensitive.

Why residual analysis?

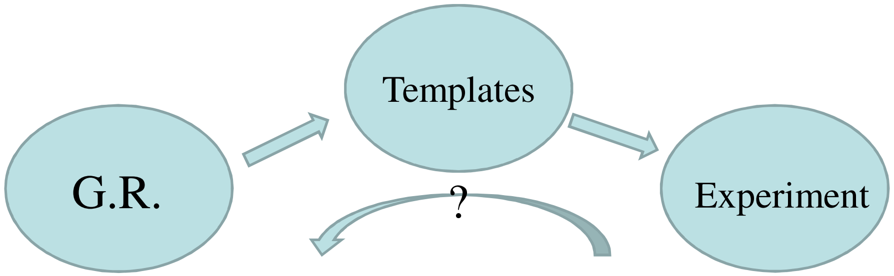
- For an improved likelihood approach (already mentioned).
- For other interesting proposals, like the *wormhole* (Pablo et al., Phys. Rev. D 97, 024040)
- To unleash the full power of the network

Residual analysis and full power of the network



…

- What can multiple GW detectors in a GW network do for us?
  - Better positioning
  - Better SNR
- However, the full power of the GW network can only be unleashed with the full analysis of the residuals.
  - Verification of the residual is “internal” to the GW network.
  - Feedback cycle.



What needs to be done

- Blind estimation of the common signal with consideration of the residuals.

A blind search for a common signal in gravitational wave detectors

Hao Liu[a,b], James Creswell[a], Sebastian von Hausegger[a], Andrew D. Jackson[c] and Pavel Naselsky[a]

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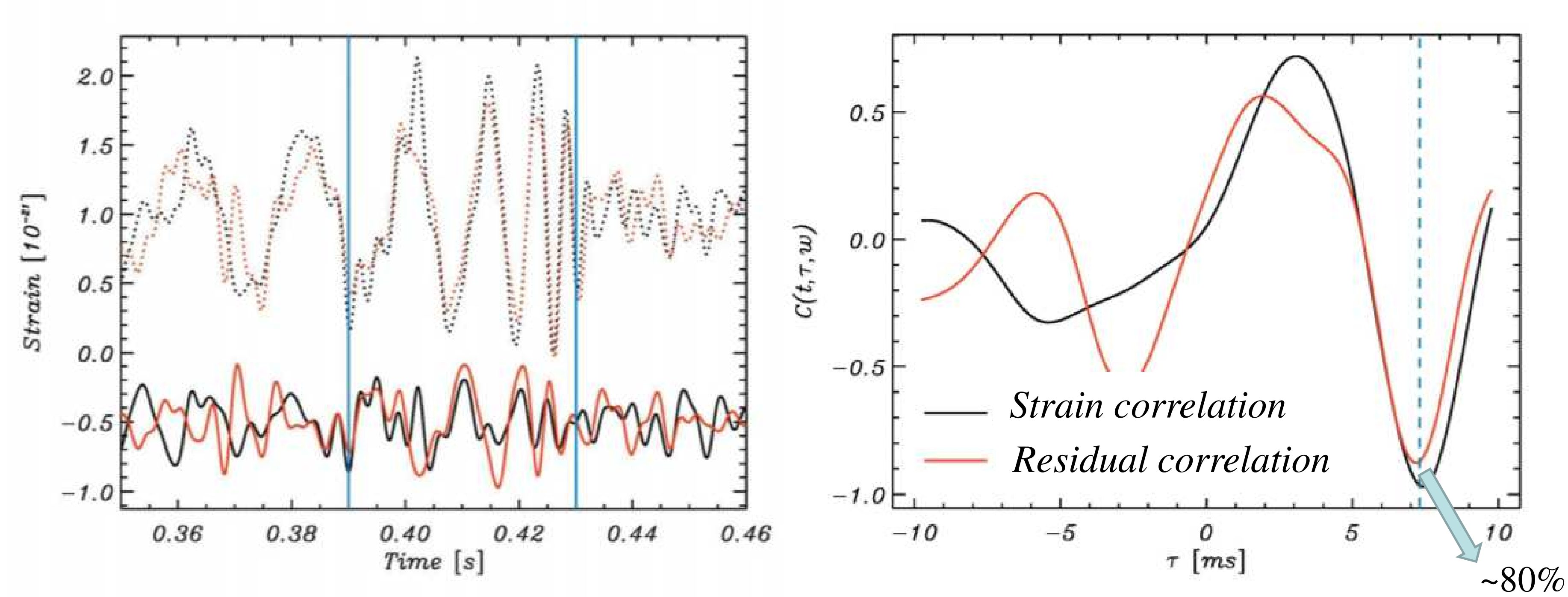
- This is based on discovery of the residual correlation

On the time lags of the LIGO signals

James Creswell[a], Sebastian von Hausegger[a], Andrew D. Jackson[b], Hao Liu[a,c] and Pavel Naselsky[a]

Published 9 August 2017 • © 2017 IOP Publishing Ltd and Sissa Medialab
Journal of Cosmology and Astroparticle Physics, Volume 2017, August 2017

We started with templates but soon found abnormal correlations in the residuals: “On the time lags of the LIGO signals”, Creswell et al., 2017, JCAP, 08, 013



Strong correlations between the Hanford and Livingston residuals were detected.

(This work has been thoroughly tested by various groups.)

See also: http://www.nbi.ku.dk/gravitational-waves/ for data and source codes

What does a ~80% residual correlation mean?

Projection of the GW signal to Hanford and Livingston

$$S_H(t) = P_H[G(t)] + n_H(t) \rightarrow G_1(t) = P_H^{-1}[S_H(t) - n_H(t)]$$

$$S_L(t) = P_L[G(t)] + n_L(t) \rightarrow G_2(t) = P_L^{-1}[S_L(t) - n_L(t)]$$

Two independent estimations

Naive estimation

$$G(t) = \frac{G_1(t) + G_2(t)}{2}$$

Problem: 100% anti-correlation

$$R_1(t) = G_1(t) - G(t)$$

$$R_2(t) = G_2(t) - G(t)$$

$$R_1(t) = -R_2(t)$$

Abnormal noise correlations indicate a non-physical solution.

Key issue to be discussed after finding the residual correlation

- **Can this problem be resolved by using the entire GW template bank?**
  - “Yes” implies a problem internal to the template bank.
  - “No” opens the door to new opportunities (not crisis!).

This also provides a positive response to the issue mentioned just now by David shoemaker

How to apply the two basic principles in a blind estimation

Naive estimation

$$G(t) = \frac{G_1(t) + G_2(t)}{2}$$

Assume that a reasonable estimation is A(t) instead of G(t)

- The initial guess of A(t) is simply white noise.
- Then it is improved by a random walk approach.

Correlated signal

Uncorrelated residual

$$C(x, y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

$C(A, G_1) \Uparrow$ $C(A, G_2) \Uparrow$ Diagonal.

$C(G_1 - A_1, G_2 - A_2) \Downarrow$ Off diagonal.

This looks like a linear model, but is actually non-linear.

1) Amplitude

2) No prior assumption for morphology (each point can change separately)

For technical details, see:

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A blind search for a common signal in gravitational wave detectors

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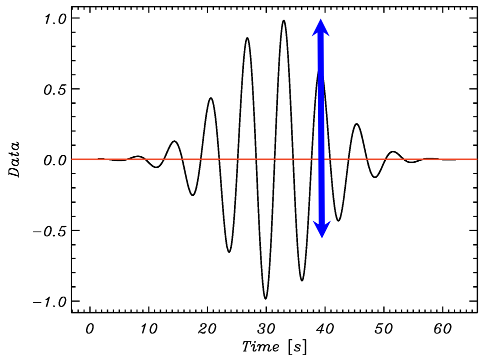
Blind estimation of the common signal

Correlation coefficients are highly non-Gaussian: $$C(x,y)=\frac{\sum_i (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_i (x_i-\bar{x})^2 \sum_i (y_i-\bar{y})^2}}$$

$$C(X,Y) \xrightarrow{\text{Fisher transform}} Z_{XY}=\frac{1}{2}\log\left(\frac{1+C_{XY}}{1-C_{XY}}\right),$$ To enhance Gaussianity

$$\log(L) \propto Z_{AX}^2 + Z_{AY}^2 - Z_{R_x R_y}^2$$

- Change the value of A at each point by a small amount to increase the likelihood, L, and random walk to "convergence". (The scheme is non-linear but stable.)

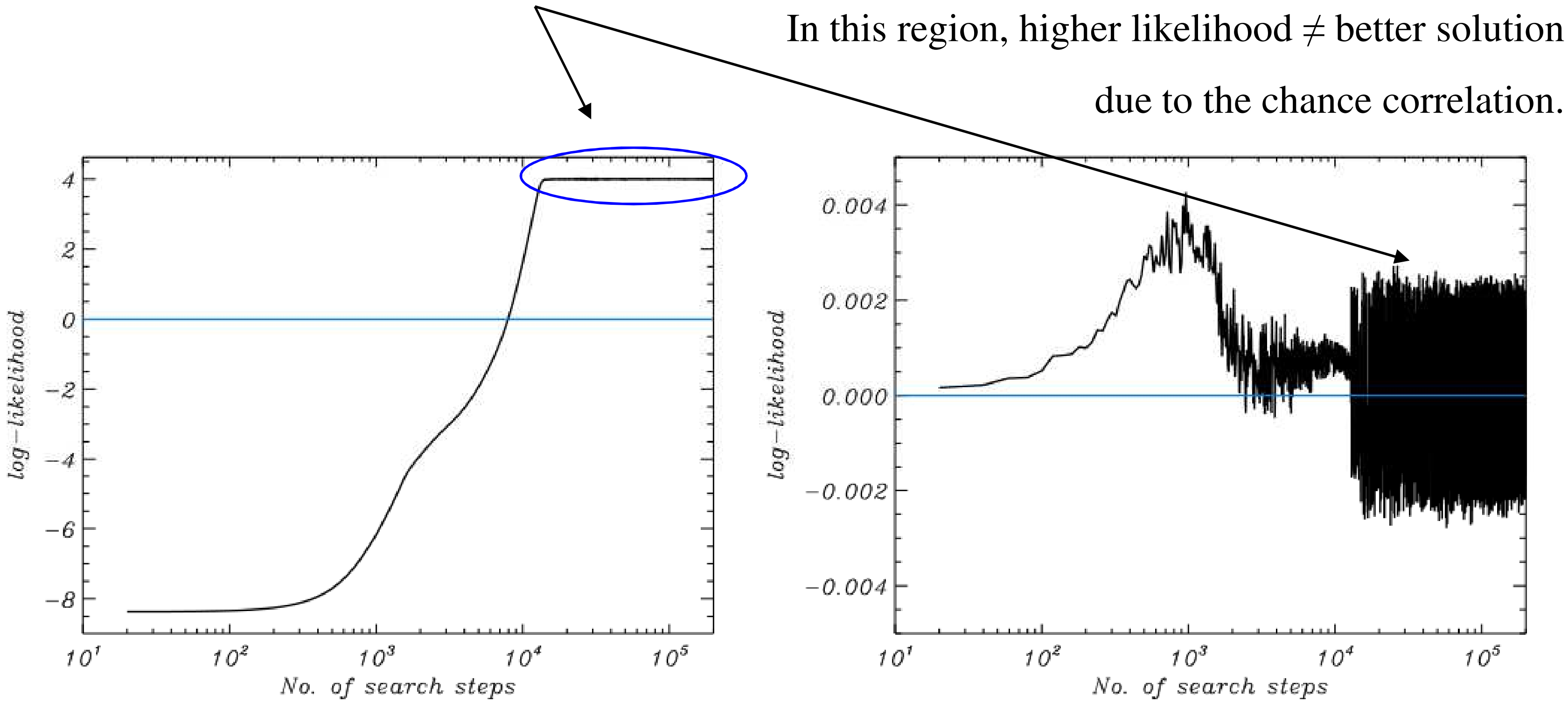


1) Determine a direction that gives higher likelihood.
2) Take a small step of random size in this direction.

Oscillation of the likelihood

In this region, higher likelihood ≠ better solution

due to the chance correlation.

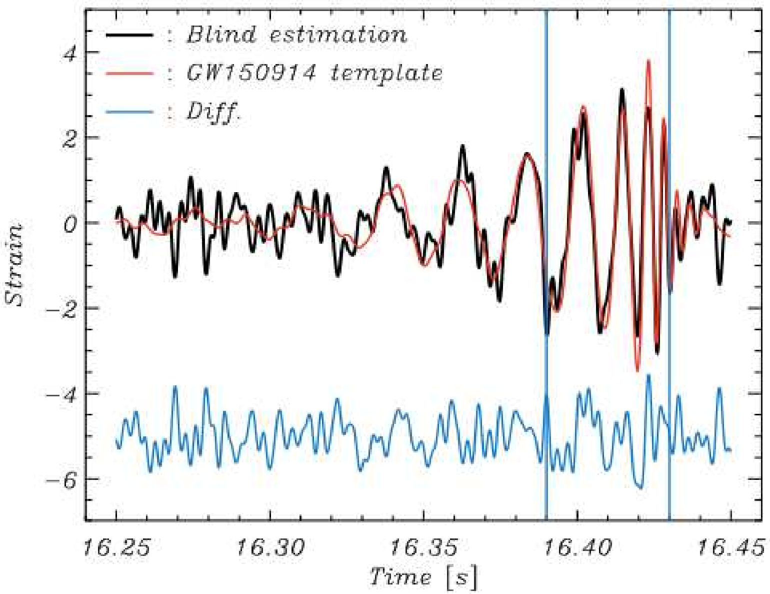


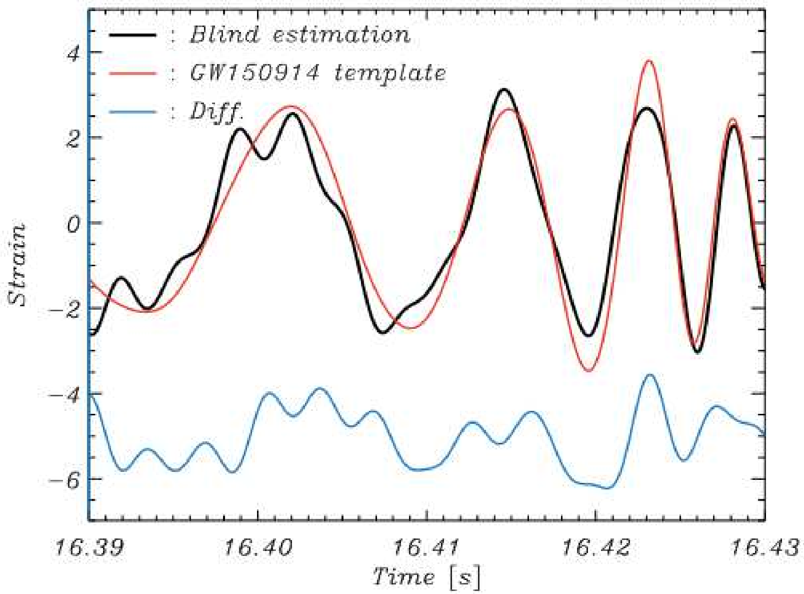
1) The *real* common signal lies in the oscillatory region, but it can not be determined precisely.

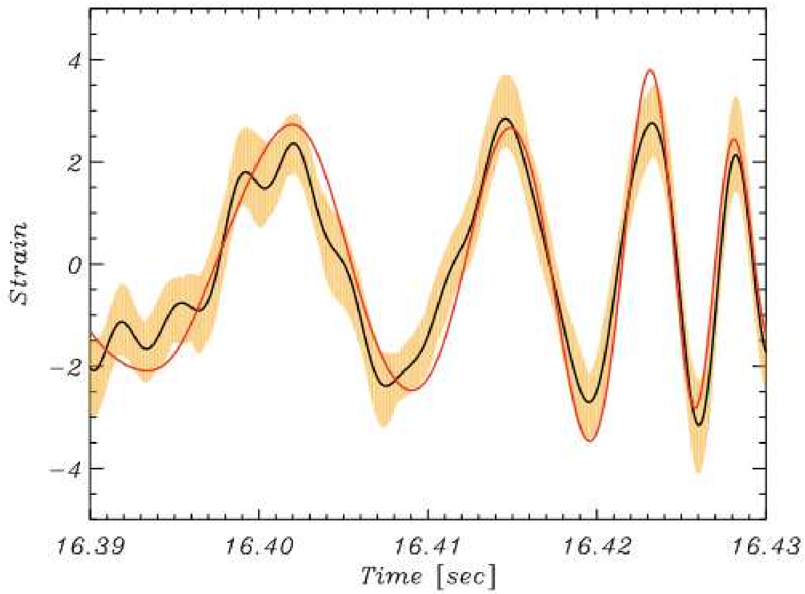
3) Oscillations reflect the family of solutions that are reasonably good, and each solution has its own realization of residual.

4) Range of oscillations can be given for each point.

Solution from the real data





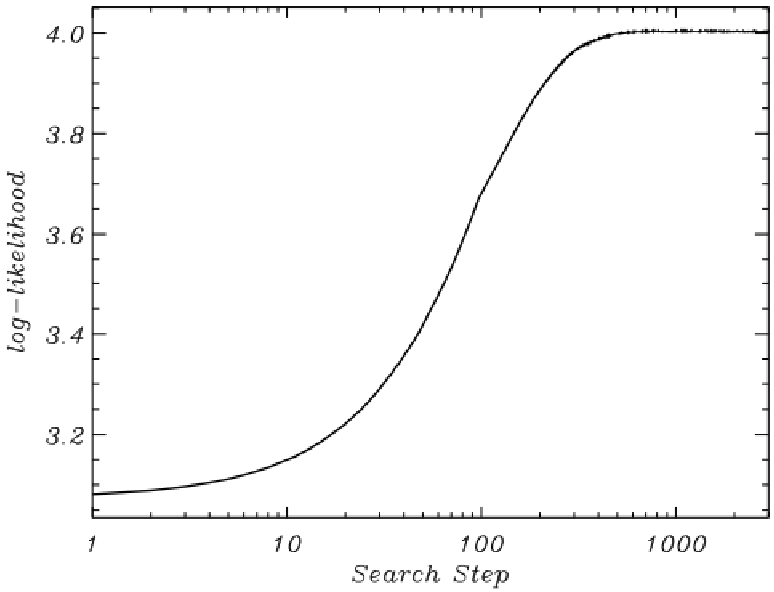


100 runs, each time with a new random initial guess
100,000 solutions obtained in each run.
Thus, the yellow band contains 10^7 solutions.

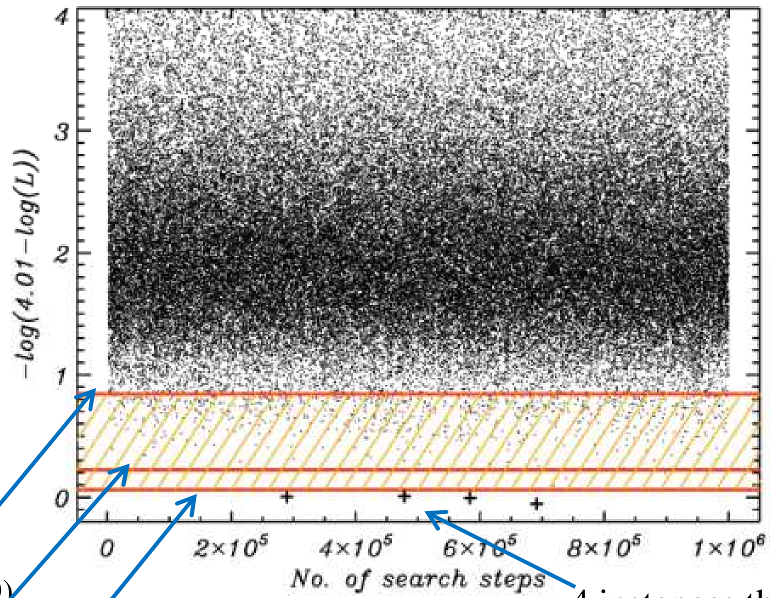
Does the GW template lie inside this envelope?

What is the likelihood of the GW150914 template?

Evolution of the likelihood when use the LOSC GW150914 template as the initial guess



Where is the likelihood of GW150914 template in the set of solutions?



Max. likelihood obtained from the "template bank"(40 & 38 ⊙)

A template with larger total mass (48 & 38 ⊙)

GW150914 LOSC template (36 & 29 ⊙)

Due to degeneracy of the template bank (See also James Creswell's talk), the result is insensitive to how to go through the template bank

4 instances that fall below the GW-template

The probability that the best common signal is in the GW template bank is p = 0.008!

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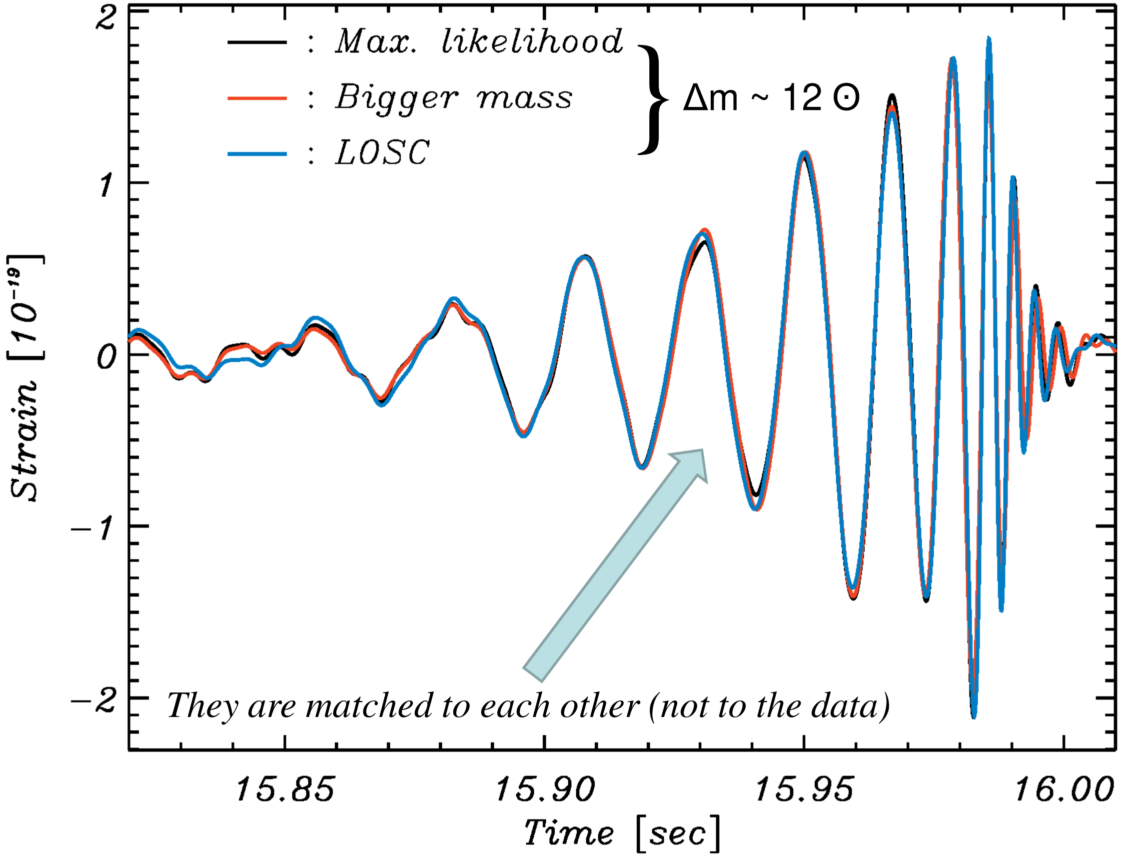
How much are those three example GW templates different?



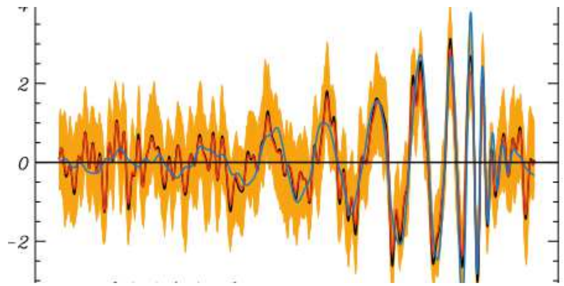
Working within the template bank greatly limits the morphology of the template.

For a residual analysis, one has to go beyond the template bank.

Stay in the bank = reject all other physical possibilities from the start!

Some observations:

- As the SNR decreases, the magnitude of the best common solution becomes smaller. The result is that the uncertainties in its determination grow until they include the possibility that the best common solution is consistent with zero everywhere (by range of fluctuation). At that point, no meaningful conclusions can be drawn.
- We have restricted our attention to GW150914 since it is the only GW candidate for which a non-trivial best common signal can be found.
  - This includes the proposed NS-NS merger. (We also note that the template for this event is not publicly available.)
- More detectors in the network would greatly improve the residual analysis.



Conclusions

- There are significant residual correlations between the Hanford and Livingston detectors for GW150914.

- It seems unlikely ($p = 0.008$) that the best common signal for GW150914 can be described by *any* template in the GW template bank.

- Comparisons of the best common signal with physical models can rule out some mechanisms; they *cannot* confirm mechanisms with certainty.

- Without residual analysis, the analysis is incomplete in many aspects
  - Covariance matrix and likelihood
  - Feedback cycle
  - Other possibilities and new proposals
  - Full power of the network

- A greater focus on residual correlations can open the door to a more refined understanding of LIGO's results.