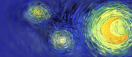


# Effective quantum gravity from the point of view of perturbative algebraic QFT

Kasia Rejzner

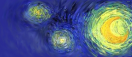
University of York

Marcel Grossmann Meeting MG14,  
13.07.2015



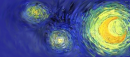
# Outline of the talk

- 1 Algebraic approach to QFT
  - AQFT
  - LCQFT
  
- 2 Quantum gravity
  - Effective quantum gravity
  - Symmetries
  - Background independence



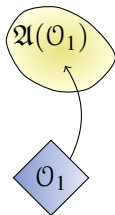
# Algebraic quantum field theory

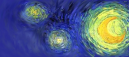
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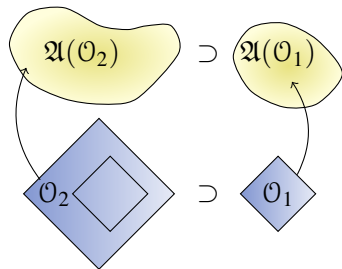
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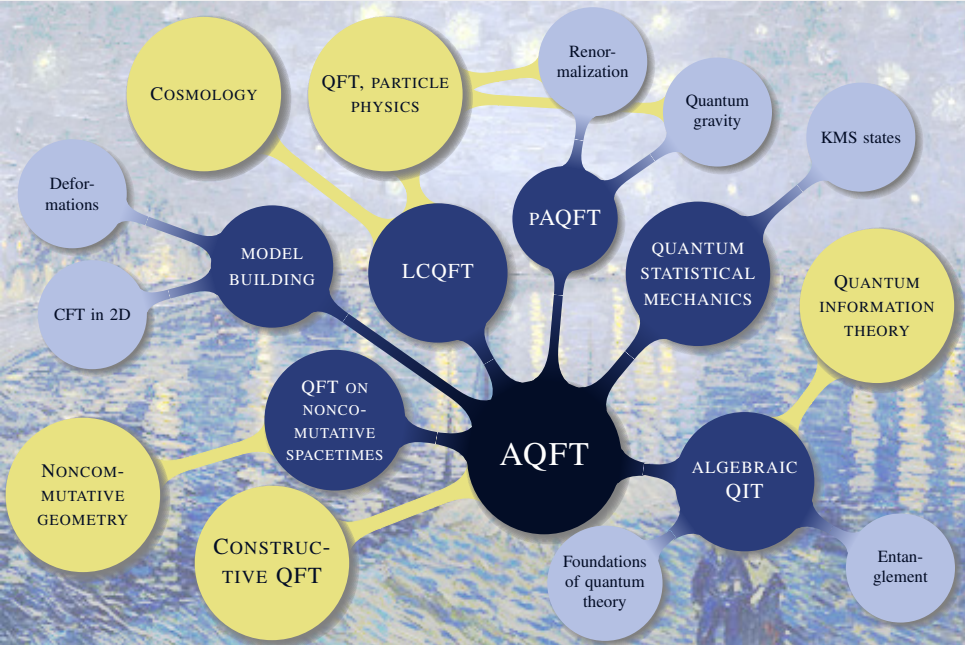


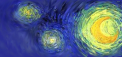
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- The physical notion of subsystems is realized by the condition of **isotony**, i.e.:  $\mathcal{O}_2 \supset \mathcal{O}_1 \Rightarrow \mathfrak{A}(\mathcal{O}_2) \supset \mathfrak{A}(\mathcal{O}_1)$ . We obtain a **net of algebras**.



# Different aspects of AQFT and relations to physics

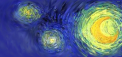




## Difficulties in QFT on curved spacetimes

To include effects of general relativity into QFT, one has to be able to describe quantum fields on a general class of spacetimes.

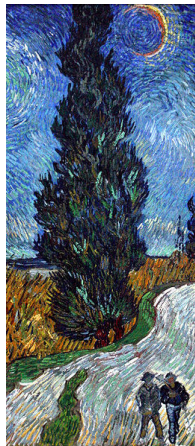




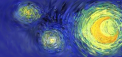
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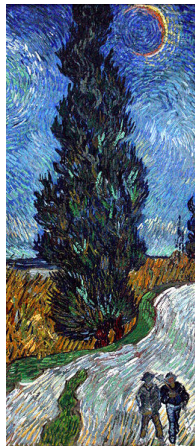


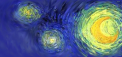


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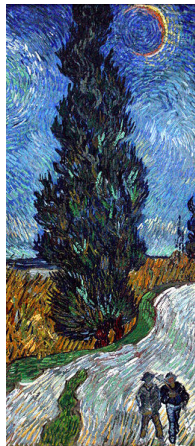


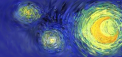


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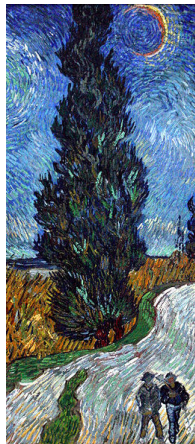


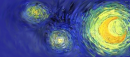


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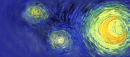
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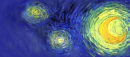
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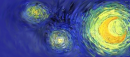
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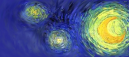
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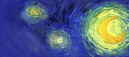
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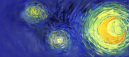


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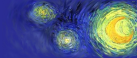


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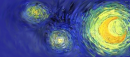
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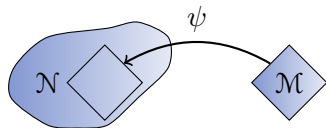
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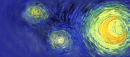
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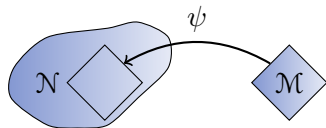
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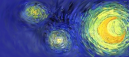




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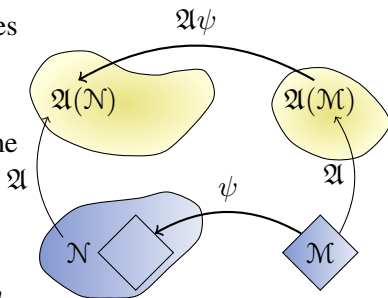
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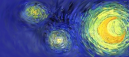




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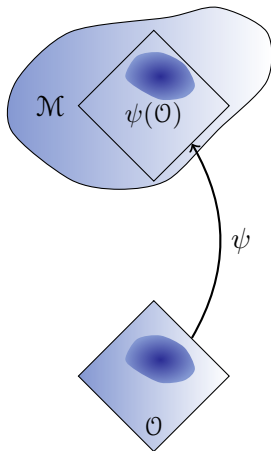
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- Assign to each spacetime  $\mathcal{M}$  an algebra  $\mathfrak{A}(\mathcal{M})$  and to each admissible embedding  $\psi$  a homomorphism of algebras  $\mathfrak{A}\psi$  (notion of subsystems). This has to be done **covariantly**.

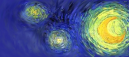




## Locally covariant fields

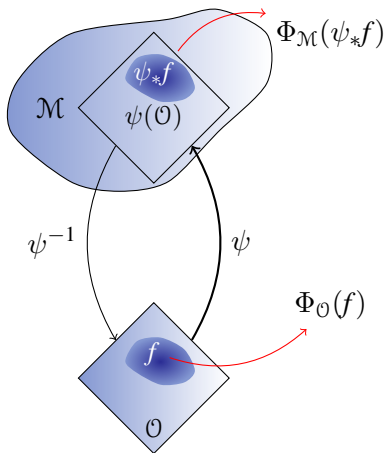
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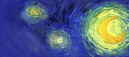


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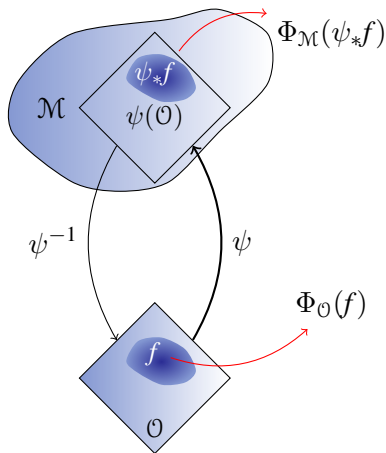


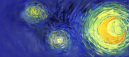




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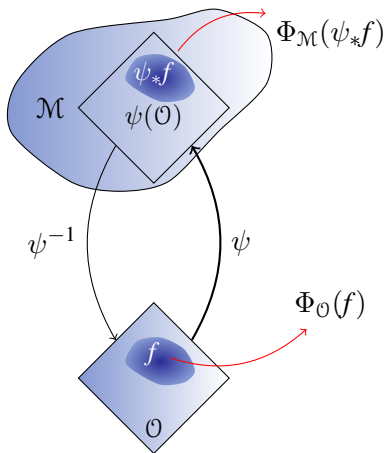
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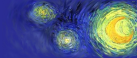




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- Locally covariant fields are candidates for observables in GR.

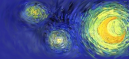




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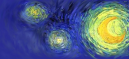




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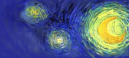




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- As a QFT, quantum gravity is **power counting non-renormalizable**.

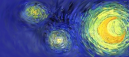




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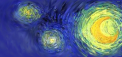


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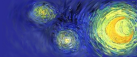
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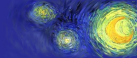


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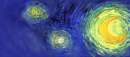


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- **Diffeomorphism invariance:** use the BV formalism to do the gauge fixing. Possible difficulties: base manifold is Lorentzian and non-compact, symmetry group is infinite dimensional, so is the space of metrics.



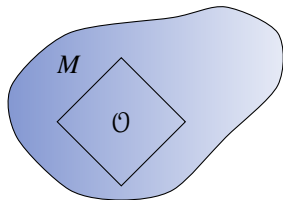
## Intuitive idea

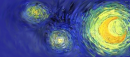
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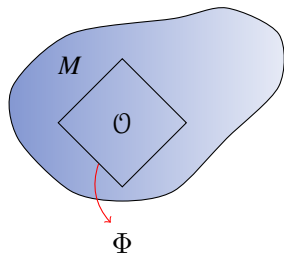
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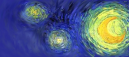




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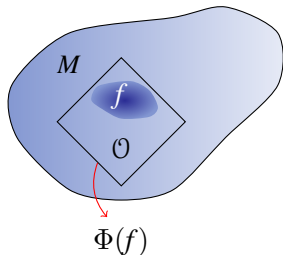


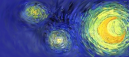


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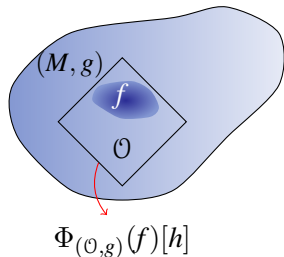
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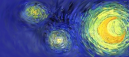
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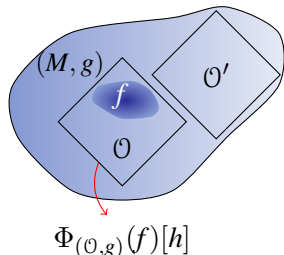
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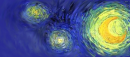
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- Diffeomorphism transformation: move our experimental setup to a different region  $\mathcal{O}'$ .



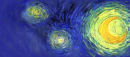
$$\Phi_{(\mathcal{O},g)}(f)[h]$$



## Building models in LCQFT

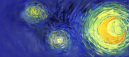
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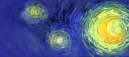
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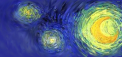
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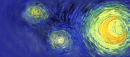
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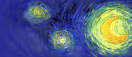
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- We work all the time on the **same set of functionals**, but we equip it with different algebraic structures (i.e. Poisson bracket, non-commutative  $\star$  product).



## Diffeomorphism invariance

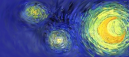
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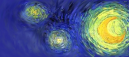


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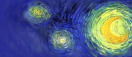
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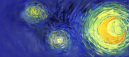
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- Example:  $\int R[g+h]f d\mu_{g+h}$  is diffeomorphism invariant, but  $\int R[g+h]f d\mu_g$  is not.





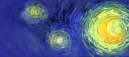
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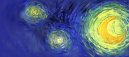
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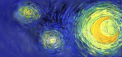


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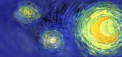


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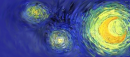


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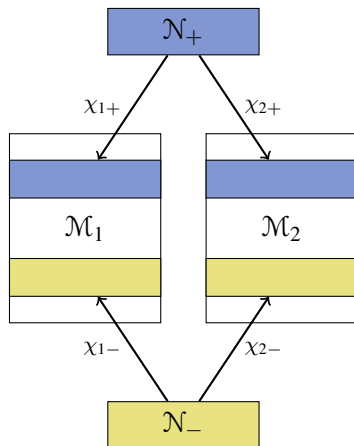
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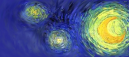
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- Each  $\Phi_f$  induces a relational observable  $g \mapsto \Phi_f(h, X_{g+h})$ .



# Relative Cauchy evolution

- Let  $\mathcal{N}_+$  and  $\mathcal{N}_-$  be two spacetimes that embed into two other spacetimes  $\mathcal{M}_1$  and  $\mathcal{M}_2$  around Cauchy surfaces, via admissible embeddings  $\chi_{k,\pm}$ ,  $k = 1, 2$ .



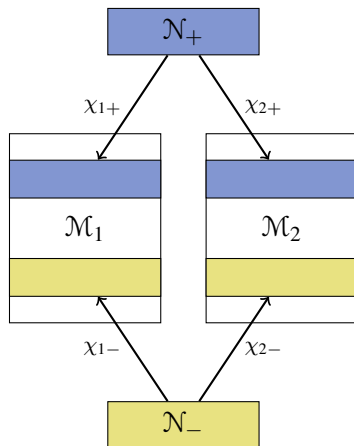


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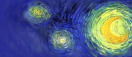
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- Then

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is an automorphism of  $\mathfrak{A}(\mathcal{M}_1)$ . This is the consequence of the **Time-slice axiom** of LCQFT.

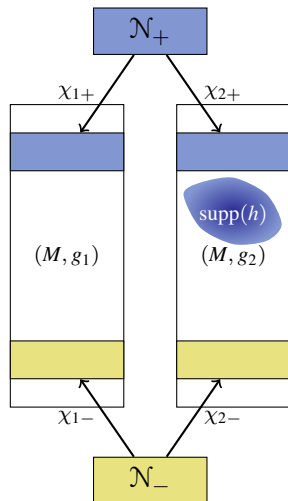


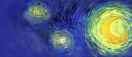




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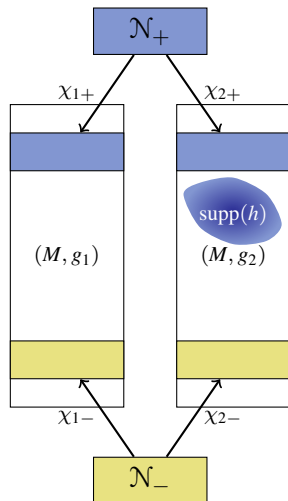
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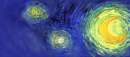




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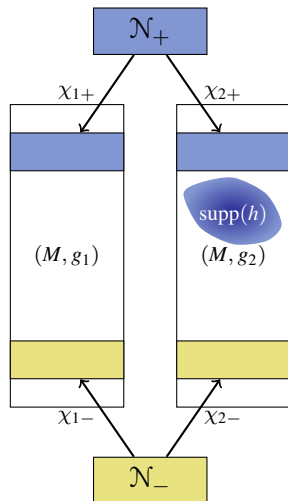
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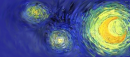




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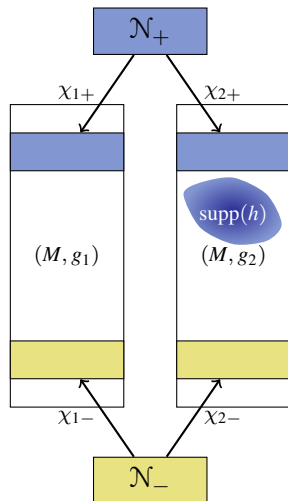
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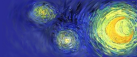




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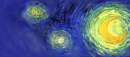
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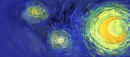
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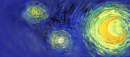
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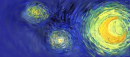
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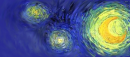
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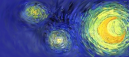
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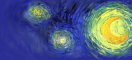
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Thank you for your attention!