Supersymmetry and Inflation

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Contents

We describe approaches to inflaton dynamics based on **Supergravity**, the combination of **Supersymmetry** with **General Relativity (GR)**.

Nowadays it is well established that inflationary Cosmology is accurately explained studying the evolution of a single real scalar field, the inflaton, in a **Friedmann, Lemaître, Robertson, Walker** geometry. A fundamental scalar field, which described the Higgs particle, was also recently discovered at **LHC**, confirming the interpretation of the **Standard Model** as a spontaneously broken phase (**BEH** mechanism) of a non-abelian Yang-Mills theory ***(Brout, Englert, Higgs, 1964).***

There is then some evidence that **Nature** is inclined to favor, both in Cosmology and in Particle Physics, theories which use scalar degrees of freedom, even if in diverse ranges of energy scales. Interestingly, there is a cosmological model where the two degrees of freedom, **inflaton** and **Higgs**, are identified, the **Higgs inflation** model **(Bezrukov, Shaposhnikov, 2008)**, where a non-minimal coupling $\mathbf{h^2 R}$ of the Higgs field $\mathbf{h}$ to gravity is introduced.

Another model based on a **$R + R^2$** extension of General Relativity is the **Starobinsky** model ***(Starobinsky, 1980; Chibisov, Mukhanov, 1981)***, which is also conformally equivalent to GR coupled to a scalar field, the **scalaron** ***(Whitt, 1984)***, with a specific form of the scalar potential which drives the inflation:

$$V = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\,\varphi}\right)^2 \qquad V_0 \sim 10^{-9} \text{ in Planck units}$$

These two models (and also a more general class) give the same prediction for the **slow-roll parameters**.

Slow-roll Parameters

- Spectral index of scalar perturbations (scalar tilt):

$$n_s = 1 - 6\,\epsilon + 2\,\eta = 1 - \frac{2}{N}$$

- Tensor-to-scalar ratio:

$$r = 16\,\epsilon = \frac{12}{N^2}$$

- ε, η, N:

$$\epsilon = \frac{{M_P}^2}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta = M_P^2 \frac{V''}{V} , \quad N = \frac{1}{M_P^2}\int_{\varphi_{\text{end}}}^{\varphi} \frac{V'}{V}\, d\varphi$$

- N is the number of *e*-folds at the end of inflation

An interesting modification of the Starobinsky potential, suggested by its embedding in **R + R²** Supergravity ***(SF, Kallosh, Linde, Porrati),*** involves and α-deformed potential

$$V_\alpha = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\,\alpha}}\,\varphi}\right)^2$$

It gives the same result for n_s but now

$$r = \frac{12\,\alpha}{N^2}$$

This family of models provides an interpolation between the **Starobinsky model** (for α=1) and the chaotic inflation model ***(Linde)*** with quadratic potential (for α→∞)

$$V_\alpha \;\to\; m^2\,\varphi^2 \quad (V_0/\alpha \text{ fixed}) \quad \text{same } n_s \text{ but } r = \frac{8}{N}$$

The recent **2015** data analysis from Planck-BICEP2 favors the previous **(Starobinsky)** model with **$n_s \cong 0.97$, $r < 0.1$**.

The previous expression for V_α can be further generalized by introducing an arbitrary monotonically increasing function $f\left(\tanh\frac{\varphi}{\sqrt{6\,\alpha}}\right)$ so that

$$V_\alpha(\varphi) \;=\; f^2\left(\tanh\frac{\varphi}{\sqrt{6\,\alpha}}\right)\;,\; \varphi\rightarrow\infty \;:\; f\left(\tanh\frac{\varphi}{\sqrt{6\,\alpha}}\right)\rightarrow 1\;-e^{-\sqrt{\frac{2}{3\,\alpha}}\,\varphi}+\ldots$$

These modifications led to introduce the concept of **α -** attractors ***(Kallosh, Linde, Roest)***

In the sequel we report on the extension of these “single field” inflationary models in the framework of **(N=1) Supergravity**, where the problem of embedding the inflaton φ in a supermultiplet and the role of its superpartners will arise.

Inflationary models, in a supersymmetric context, must be embedded in a general Supergravity theory coupled to matter in a **FLRW** geometry.

Under the assumption that no additional Supersymmetry **(N ≥ 2)** is restored in the Early Universe, the most general N=1 extension of **GR** is obtained coupling the graviton multiplet (2,3/2) to a certain number of chiral multiplets (1/2,0,0), whose complex scalar fields are denoted by z^i, $i=1\ldots N_s/2$ and to (gauge) vector multiplets (1,1/2), whose vector fields are denoted by A_μ^Λ ($\Lambda=1,..,N_V$).

These multiplets can acquire supersymmetric masses, and in this case the massive vector multiplet becomes (1,2(1/2),0) eating a chiral multiplet in the supersymmetric version of the BEH mechanism.

For Cosmology, the most relevant part of the Lagrangian ***(Cremmer, SF, Girardello, Van Proeyen; Bagger, Witten)*** is the sector which contains the scalar fields coupled to the **Einstein-Hilbert** action

$$\mathcal{L} \;=\; -\,R \;-\; \partial_i\,\partial_{\bar{j}}\,K\,D_\mu z^i\,D_\nu \bar{z}^{\bar{j}}\,g^{\mu\nu} \;-\; V(z,\bar{z}) \;+\; \ldots$$

K is the Kahler potential of the σ-model scalar geometry and the “dots” stand for fermionic terms and gauge interactions.

The scalar covariant derivative is $D_\mu z^i \;=\; \partial_\mu z^i \;+\delta_\Lambda z^i\,A_\mu^\Lambda$, where $\delta_\Lambda z^i$ are Killing vectors. This term allows to write massive vector multiplets ***à la* Stueckelberg**.

The scalar potential is

$$V(z^i,\bar{z}^{\bar{i}}) \;=\; e^G \left[G_i \, G_{\bar{j}} \left(G^{-1}\right)^{i\,\bar{j}} \;-\; 3 \right] \;+\; \frac{1}{2} \left(Ref_{\Lambda\Sigma}\right)^{-1} D_\Lambda \, D_\Sigma$$

$$G \;=\; K \;+\; \log |W|^2 \;, W(z^i) \;\text{ superpotential} \;, G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$$

The first and third non-negative terms are referred to as “F” and “D” term contributions: they explain the possibility of having unbroken Supersymmetry in **Anti-deSitter** space.

The potential can be recast in the more compact form

$$V(z^i,\bar{z}^{\bar{j}}) \;=\; F_i \, F^i \;+\; D_\Lambda \, D^\Lambda \;-\; 3 \, |W|^2 \, e^K$$

with

$$F_i \;=\; e^{\frac{K}{2}} \left(W \, K_{,i} \;+\; W_{,i}\right) \;,\; D_\Lambda = G_{,i} \, \delta_\Lambda z^i$$

The **D** term potential can provide a supersymmetric mass term to a vector multiplet and also a **deSitter** phase, since its contribution to the potential is non negative. Only **F** breaking terms can give AdS phases.

The (**field dependent**) matrices $Ref_{\Lambda\Sigma}\ ,\ Imf_{\Lambda\Sigma}$ provide the normalizations of the terms quadratic in Yang-Mills curvatures. They could also be of interest for Cosmology, since they give direct couplings of the inflaton to matter.

In a given phase (it could be the inflationary phase of the exit from inflation) unbroken Supersymmetry requires

$$F_i = D^\Lambda = 0 \,, \text{ so that } V = -\,3\,|W|^2\,e^K$$

These are **Minkowski** or **AdS** phases **depending on whether** W vanishes or not. Supersymmetry is broken if at least one of the **F_i**, **D^Λ** does not vanish. Hence in phases with broken Supersymmetry one can have **AdS**, **dS** or **Minkowski**. Therefore one can accommodate both the inflationary phase (dS) and the Particle Physics phase (Minkowski). However, it is not trivial to construct corresponding models, since the two scales are very different if Supersymmetry is at least partly related to the Hierarchy problem.

In view of the negative term $-3\,e^{G}$ present in the scalar potential it may seem impossible (or at least not natural) to retrieve a scalar potential exhibiting a **de Sitter** phase for large values of a scalar field to be identified with the inflaton. The supersymmetric versions of the **R+R^2** (Starobinsky) model show how this puzzle is resolved: either the theory has (with F-terms) a **no-scale structure**, which makes the potential positive along the inflationary trajectory ***(Cecotti)*** or the potential is a **pure D-term** and therefore positive ***(Cecotti, SF, Porrati, Sabharwal)*** .

These models contain two chiral superfields **(T,S)** ***(Ellis, Nanopoulos, Olive; Kallosh, Linde)*** , as in the **old minimal** version of **R+R^2** Supergravity ***(Cecotti),*** or one massive vector multiplet ***(SF, Kallosh, Linde, Porrati; Farakos, Kehagias, Riotto)***, as in the **new minimal** version.

These models have unbroken Supersymmetry in Minkowski vacuum at the end of inflation. Recently progress was made ***(Kallosh, Linde; Dall'Agata, Zwirner)*** to embed two different supersymmetry breaking scales in the inflationary potential in the framework of **nilpotent Superfield inflation.**

The multiplet S, which does not contain the inflaton (T multiplet), is replaced by a **nilpotent superfield** **($S^2=0$)**: this eliminates the sgoldstino scalar from the theory but still its F-component drives inflation or at least participates in it.

This mechanism was first applied to the Starobinsky model, replacing the S field by a **Volkov-Akulov** nilpotent field ***(Antoniadis, Dudas, SF, Sagnotti)*** and then to general F-term induced inflationary models ***(Kallosh, Linde, SF)***.

Minimal models for inflation in Supergravity

This class includes models in which the inflaton is identified with the sgoldstino and only one chiral multiplet T is used. However, the **f(R)** Supergravity models ***(Ketov)*** yield potentials that either have no plateau or, when they do, lead to AdS rather than dS phases. This reflects a no-go theorem ***(Ellis, Nanopoulos, Olive)***.

A way out of this situation was recently found with **"α-scale Supergravity"** ***(Roest, Scalisi)*** : adding two superpotentials $\mathbf{W_{+} + W_{-}}$

which separately give a flat potential along the inflaton **(ReT)** direction gives rise to a **de Sitter plateau** for large ReT. The problem with these models is that **the inflaton trajectory is unstable in the ImT direction**, but only for small inflaton field: modifications to the superpotential are advocated to cure this illness.

R+R^2 Supergravity, D-term inflation ***(SF, Kallosh, Linde, Porrati; SF, Fré, Sorin)***, α-attractor scenarios ***(Kallosh, Linde, Roest, Carrasco),*** no-scale inflationary models ***(Ellis, Nanopoulos, Olive)*** and α-scale models ***(Roest, Scalisi)*** have a nice SU(1,1)/U(1) hyperbolic geometry for the inflaton superfield, with $R_\alpha = -\frac{2}{3\alpha}\ ,\ n_s \approx 1 - \frac{2}{N}\ ,\ r = \frac{12\alpha}{N^2}$

D-term inflation

An appealing and economical class of models allows to describe any potential of a single scalar field which is the square of a real function *(SF, Kallosh, Linde, Porrati)*: $V(\varphi) = \frac{g^2}{2} P^2(\varphi)$

These are the **D-term models**, which describe the self-interactions of a massive vector multiplet whose scalar component is the inflaton. Up to an integration constant (the Fayet-Iliopoulos term), the potential is fixed by the geometry, since the Kahler metric is

$$ds^2 = (d\varphi)^2 + \left(P'(\varphi)\right)^2 da^2$$

After gauging the field a is absorbed by the vector, via da+gA, giving rise to a mass term $\frac{g^2}{2}\left(P'(\varphi)\right)^2\ A_\mu^2$ (BEH mechanism).

The Starobinsky model corresponds to

$$P(\varphi)\ =\ 1\ -\ e^{-\sqrt{\frac{2}{3}}\,\varphi}$$

In these models there is no superpotential and only a de Sitter plateau is possible. At the end of inflation φ=0, D=0 and Supersymmetry is recovered in Minkowski space, V=0.

$R + R^2$ Supergravity

There are two distinct models depending on the choice of auxiliary fields: **old** and **new** minimal.

Off-shell degrees of freedom: **$g_{\mu\nu}$: 6(10-4)** , **ψ_μ: 12(16-4)**

$n_B = n_F$ off shell requires six extra bosons:

- old minimal: A_μ , S, P (6 DOF's)
- new minimal: A_μ , $B_{\mu\nu}$, (6 DOF's, due to gauge invariance)

The $12_B + 12_F$ DOF must fill massive multiplets:

$$\mathrm{Weyl}^2 \ : (2, 2(3/2), 1) \ , \quad R^2_{old} \ : \ 2(1/2, 0, 0) \ , \quad R^2_{new} \ : \ (1, 2(1/2), 0)$$

After **Superconformal** manipulations these two theories are equivalent to standard Supergravity coupled to matter. The new minimal gives D-term inflation as described before, while the old minimal gives F-term inflation with the two chiral superfields T (inflaton multiplet) and S (sgoldstino multiplet).

The T submanifold is SU(1,1)/U(1) with R = -2/3, and the no-scale structure of the Kahler potential is responsible for the universal expression

$$V = M^2 M_{Pl}^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\varphi}\right)^2$$

along the inflationary trajectory where $F_S \neq 0$, $F_T = 0$ → S: sgoldstino

Other models

Several examples exist with two chiral multiplets of the same sort, for which F_S → dS plateau , $F_T=0$, while at the end of inflation $F_S=F_T=0$ and Supersymmetry is recovered.

A class of models **(α attractors)** modify the superpotential but not the Kahler geometry of the original $R+R^2$ theory ***(Kallosh, Linde, Roest)*** :

$$W(S,T) \;=\; S\, f(T)\;, \quad R \;=\; -\,\frac{2}{3\,\alpha}$$

Along the inflationary trajectory the potential $V \;\sim |f|^2 \;\geq\; 0$

An alternative class of models with opposite role for the Kahler potential and the superpotential obtain with $W(S,T) \;=\; S\,F(T)$ but with a trivial Kahler geometry,

$$K \;=\; \frac{1}{2}\left(\Phi + \overline{\Phi}\right)^2 \;+\; S\,\overline{S}$$

The inflaton is now identified with $\varphi \;=\; Im\Phi$, which avoids the dangerous exponential factor in e^K . Along the inflationary trajectory

$$V(\varphi) \;\sim\; \left|F(\varphi)\right|^2$$

With a trivial Kahler geometry, the inflaton potential is fully encoded in the superpotential shape.

Nilpotent superfields and Cosmology

(sgoldstinoless models)

The problem with the models presented so far resides in the difficulty in obtaining an exit from inflation with broken Supersymmetry much lower than the deSitter plateau scale (Hubble scale during inflation).

A way to solve this problem is to introduce a **nilpotent sgldstino multiplet S (S^2=0)**, so that the goldstino lacks a scalar partner. S is the **Volkov-Akulov** superfield. In this way the stabilization problem is overcome and a deSitter plateau is obtained.

The first examples of cosmological models with a **nilpotent sgoldstino multiplet** was a generalization of the Volkov-Akulov-Starobinsky supergravity ***(Antoniadis, Dudas, SF, Sagnotti)***, with ***(SF, Kallosh, Linde)***

$$W(S,T) \;=\; S f(T)\ , \qquad V \;=\; e^{K(T)}\, K^{-1}_{S\bar{S}}\, |f(T)|^2$$

Models which incorporate separate scales of Supersymmetry breaking during and at the exit of inflation have a trivial (flat) Kahler geometry

$$K(\Phi, S) \;=\; \frac{1}{2}\left(\Phi \;+\; \overline{\Phi}\right)^2 \;+\; S\,\overline{S}$$

These models differ in the supersymmetry breaking patterns during and after inflation.

- **in the first class of models** ***(Kallosh, Linde)***

$$W(\Phi, S) \; = \; M^2 S \left(1 \; + \; g^2(\Phi)\right) \; + \; W_0$$

with $g(\Phi)$ vanishing at $\Phi = 0$and the inflaton φ identified with its imaginary part. Along the inflaton trajectory $Re(\Phi) = 0$ is then

$$V \; = \; M^4 \; |g(\varphi)|^2 \left(2 \; + \; |g(\varphi)|^2\right) \; + \; V_0 \; , \quad V_0 = M^4 - 3\, W_0^2$$

Assuming $V_0 \simeq 0$, one finds $m_{\frac{3}{2}} = \frac{1}{\sqrt{3}}\, H \quad E_{SB} = |F_S|^{\frac{1}{2}} = \sqrt{H M_P} > H$

$$V \; = \; F_S\, F^S \; - \; 3\, W_0^2 \; , F_\Phi = 0 \text{ during inflation } (Re\Phi = 0)$$

- in the **second class of models** ***(Dall'Agata, Zwirner)***

$$W(\Phi, S) \;=\; f(\Phi)\left(1 \;+\; \sqrt{3}\, S\right)$$

which combines **nilpotency** and **no-scale structure**. Here:

$$\overline{f}(\Phi) \;=\; f(-\overline{\Phi}) \quad f'(0) = 0 \;\; f(0) \neq 0$$

The scalar potential is of **no-scale** type $\left(\Phi = \frac{1}{\sqrt{2}}\,(a + i\,\varphi\right)$

$$F^S\, F_S \;=\; 3\, e^G \;=\; 3\, e^{a^2}\, |f(\Phi)|^2$$

$$V(a, \varphi) \;=\; F^\Phi\, F_\Phi \;=\; e^{a^2}\left|f'(\Phi) \;+\; a\sqrt{2}\, f(\Phi)\right|^2$$

a is stabilized at 0 since f is even in a. During inflation a gets a mass O(H) without mass mixing with Φ and is rapidly driven to a=0.

The inflationary potential is $V(a=0,\varphi) \;=\; \left|f'\left(\frac{i\varphi}{\sqrt{2}}\right)\right|^2 \;,\quad V(0,0)=0$

These **models lack the fine-tuning** of the previous class (V_0=0).

It is interesting to compare the supersymmetry breaking patterns.

Here F_S never vanishes, and at the end of inflation

$$F^S\,F_S \;=\; 3\,e^{G(0,0)} \;=\; 3\,m^2_{\frac{3}{2}}$$

More in detail, $\langle F^S\rangle_{\Phi=0} \;=\; \sqrt{3}\,\overline{f}(0)\;,\; m_{\frac{3}{2}} \;=\; |f(0)|$

and the inflaton potential vanishes at the end of inflation. A choice that reproduces the **Starobinsky potential** is

$$f(\Phi) \;=\; \lambda \;-\; i\,\mu_1\,\Phi \;+\; \mu_2\,e^{i\,\frac{2}{\sqrt{3}}\,\Phi}$$

Interestingly, $m_a, m_{\frac{3}{2}}$ depend on the integration constant λ but m_φ does not, since V is independent of λ.