Generating matter inhomogeneities in general relativity

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The current standard model of the universe is spatially homogeneous.

The explanation for the origin of galaxy clusters is that they are seeded by tiny quantum fluctuations, which are then magnified greatly during a period of cosmic inflation.

An alternative paradigm to the standard model is an inhomogeneous model. In such a model, gravitational distortion makes imprint on matter, thus marking out locations towards which matter gravitates.

How does the gravitational field behave at the beginning of the universe? A possible scenario is the BKL singularity.

Belinski, Khalatnikov and Lifshitz made substantial progress using approximate analyses.
Q: How does a general cosmological solution of Einstein’s equations behave near a spacelike initial singularity?

A generic singularity is
1. **Vacuum-dominated** ("matter doesn’t matter")
2. **Local** (spatial derivatives are negligible)
3. **Oscillatory** (an infinite sequence of Kasner states)
In particular, BKL conjectured that local gravitational distortion is negligible near the singularity.

Surprisingly, Berger & Moncrief 1993 found otherwise. In their numerical simulations, they found thin, sheet-like gravitational distortion (now called spikes).

Early numerical work on spikes were crude. It was not until 2003 that Garfinkle & Weaver produced qualitatively correct simulations of spikes.

Earlier in 2001, Rendall & Weaver found a solution-generating transformation.

In 2008, I found the explicit spike solution by repeated applications of the Rendall-Weaver transformation.

\[
\text{Kasner (Bianchi I)} \xrightarrow{\text{RW}} \text{Taub (Bianchi II)} \xrightarrow{\text{RW}} \text{Spike solution}
\]
With the guidance of the explicit solution, my collaborators and I were finally able to develop a numerical zooming technique as a way of maintaining high numerical resolution, and produced quantitatively correct simulations of spikes [Lim, Andersson, Garfinkle, Pretorius 2009].

Simulations show that spikes recur, and are described by the spike solution.
Numerical confirmation: recurring spikes and matching each of them to an exact solution

[Lim, Andersson, Garfinkle & Pretorius 2009]
Spike imprints

- So far, spikes have generally only been studied in a vacuum.
- Spikes are expected to leave sheet-like imprints on matter, thus providing a way of seeding galaxy clusters.
- Spikes are also expected to leave imprints on the cosmic microwave background.
- Spikes provide a purely classical (non-quantum) relativistic way of forming large-scale structures.
We study the imprint of spikes on a tilted perfect fluid in spacetimes with 2 spacelike Killing vector fields [Coley & Lim 2012].

We use perturbations to show that imprint is present.

We use numerical simulations to see how large the imprint is.

We found that spikes (sheet-like gravitational distortions) leave an imprint on the tilted perfect fluid, causing a delay in the evolution of the fluid.

This delay creates a sheet-like overdensity in the fluid.
Figure shows the time evolution of 7 separate simulations centred on 7 worldlines

\[ x = 0, 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2} \]

(labeled worldline number 6 to 0). The spike worldline \((x = 0)\) is labeled worldline number 6.

\[ T \to \infty \] towards singularity.

The ratio of \(\Omega\) is taken to be \(\Omega/\Omega|_{\text{worldline number 0}}\).

The spike leaves an imprint on the tilted perfect fluid, causing a delay in the evolution of the fluid.
In more general models without Killing vector fields, spikes can intersect and interact.

Similarly, sheets of overdensity in the fluid can intersect in filaments and points to make even more pronounced overdensity in the fluid – a web of large scale structures form.

The space between the sheets are filled with underdensed fluid, and the underdensity becomes more pronounced – voids form.
Open questions

- Spike intersections/interactions are open questions. No explicit solutions have been found. Numerical solutions are costly. Need new methods. e.g. transform a simple simulation.
- How does the model evolve from highly inhomogeneous and anisotropic BKL singularity to highly homogeneous and isotropic state by the time of CMB last scattering?
- Is the future attractor (of voids) homogeneous and isotropic?
- It is clear that we are just starting to understand the rich dynamics of the inhomogeneous universe.
References

- W.C. Lim, Classical and Quantum Gravity 25, 045014 (2008)