String-like structures in the real and complex Kerr geometry.

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Outline:

i – The real Kerr geometry, Kerr theorem, twistors, problem of the source
ii – Kerr singular ring as a fundamental heterotic string
iii – Complex Kerr geometry, complex string, M2-brane source
iv – Calabi-Yau twofold from the Kerr theorem

based on:

A.B., *Calabi-Yau twofold from the Kerr theorem*, [arXiv:1203.4210].
KERR-NEWMAN (KN) SPINNING PARTICLE


The Kerr-Newman solution plays in this respect especial role:
– it is a model of extended spinning object consistent with Gravity,
– it has gyromagnetic ratio \( g = 2 \) as that of the Dirac electron (Carter, 1968).

Parameters of electron: mass, spin, charge and magnetic moment determine unambiguously that its background should be the Kerr-Newman solution! Because of the large spin of the electron, \( a = J/m >> m \), the black hole horizons disappear:

NAKED SINGULAR RING IS A SOURCE OF THE KERR SPINNING PARTICLE

– the spacetime has a topological peculiarity at the Compton distance \( r_c = a = \frac{\hbar}{2m} \), which may be interpreted as a closed string.

Second stringy structure appears in the complex Kerr geometry.
REAL structure of the Kerr-Newman solution: Metric

\[ g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \] (1)

and electromagnetic (EM) vector potential is

\[ A_{KN}^{\mu} = Re \frac{e}{r + ia \cos \theta} k^{\mu}. \] (2)

Gravitational and EM fields are concentrated near the Kerr singular ring.

The Kerr ring forms a branch line of space. The KN geometry is TWOSHEETED! Vector field \( k_\mu(x) \) is tangent to Principal Null Congruence (PNC),

\[ k_\mu dx^\mu = P^{-1}(du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y \bar{Y} dv), \quad Y(x) = e^{i\phi} \tan \frac{\theta}{2}, \] (3)
where \( Y(x) \) is projective angular coordinate, and
\[
\zeta = (x + iy)/\sqrt{2}, \quad \bar{\zeta} = (x - iy)/\sqrt{2}, \quad u = (z - t)/\sqrt{2}, \quad v = (z + t)/\sqrt{2}
\]
are the null Cartesian coordinates.

Kerr congruence is controlled by the

**KERR THEOREM:**

The geodesic and shear-free Principal null congruences (type D metrics) are determined by holomorphic function \( Y(x) \) which is analytic solution of the equation
\[
F(T^a) = 0,
\]
where \( F \) is an arbitrary analytic function of the

*projective twistor coordinates*

\[
T^a = \{ Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta} \}.
\]

The Kerr theorem is a practical tool for obtaining exact solutions:

\[
F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x)
\]

For the Kerr-Newman solution function \( F \) is quadratic in \( Y \), which yields TWO roots \( Y^\pm(x) \) \( \Rightarrow \) two congruences!
KN solution brings in the **NEW DIMENSIONAL PARAMETER** \( a = J/m \), which is the Compton length for \( J \sim \hbar \).

Kerr’s ring is a branch line of space on two sheets: “negative (–)” and “positive (+)” where the fields change their directions. In particular,

\[
k_{\mu}(+) \neq k_{\mu}(-) \quad \Rightarrow \quad g_{\mu\nu}(+) \neq g_{\mu\nu}(-).
\]

(6)

**Twosheeted mystery creates the problem of the source of the KN solution.**

Kerr’s oblate spheroidal coordinates \( x + iy = (r + ia)e^{i\phi}\sin \theta, \quad z = r \cos \theta \), cover spacetime twice: disk \( r = 0 \) separates the ‘out’-sheet \( r > 0 \), from the ‘in’-sheet \( r < 0 \).


(d) **Gravitating soliton: vacuum bubble bounded by membrane**, AB (2010).

(e) **Complex KN source as a COMPLEX STRING**, AB (1993-2012).

**Pointlike ‘IMAGE’ of the electron in Quantum theory is created by the relativistically rotating heterotic string on the boundary of M2-brane.**
GRAVITATING SOLITON (AB, 2010) – chiral Higgs model. Supersymmetric phase transition from external KN solution to a ‘false vacuum’ bubble bounded by the domain wall M2-brane.

Perspective goal – description of the Weinberg-Salam model.

Peculiarities of the KN soliton model:

(i) the Kerr ring is regularized, forming a closed relativistically rotating string of the Compton radius \( r_c \) on the border of disklike membrane,

(ii) the KN electromagnetic potential forms a quantized loop \( \oint eA_\varphi d\varphi = -4\pi ma \), which results in quantization of the soliton spin, \( J = ma = nh/2, \ n = 1, 2, 3, \ldots \),

(iii) the Higgs condensate forms a coherent vacuum state oscillating with the frequency \( \omega = 2m - \text{oscillon} \),
FUNDAMENTAL STRINGS IN GRAVITY as soliton like classical solutions in the effective field theory. Dabholkar at.al (NPB 1990).

Classical solutions in the effective string field theory may correspond to fields around a HETEROTIC STRING E. Witten (Phys.Lett.B 1985).

MACROSCOPIC CHARGED HETEROTIC STRING, A. Sen (NPB 1992-1993): bosonic zero modes of the four dimensional solutions in the effective field theory are in one to one correspondence to the bosonic degrees of freedom of heterotic string moving in four dimensions. PP-wave solutions. In particular, critical heterotic string theory in four dimensions with the extra six dimensions compactified.

Solutions to Einstein’s eqs. are solutions of (super)string theory. PP-WAVES, Horowitz & Steif (PRL 1990), A. Tseytlin (PRD 1993).

Strings as Solitons & Black Holes as Strings Dabholkar at.al (NPB 1995).


The Kerr SINGULAR RING is a ‘closed’ heterotic string. The field around Kerr-Sen solution to low energy string theory is similar to the Sen solution for HETEROTIC STRING. AB (PRD 1995) (Lightlike circular currents.)
Complex Structure of the Kerr geometry.

Complex Shift. Appel solution 1887!

A point-like charge \( e \), placed on the complex z-axis \( (x_0, y_0, z_0) = (0, 0, -ia) \), gives a real potential

\[
\phi_a = \text{Re} \frac{e}{r + ia \cos \theta},
\]

where \( r \) and \( \theta \) are the Kerr oblate spheroidal coordinates.

There is an exact correspondence between Appel’s complex shift and Kerr’s geometry.

New objects: Complex light cones with the vertexes on the complex world-line \( x^\mu_0 \in CM^4 \): \( (x^\mu - x^\mu_0)(x^\mu - x^\mu_0) = 0 \), split into two families of the ”left” and ”right” complex null planes: \( x^\mu_L = x^\mu_0(\tau) + \alpha e^{1\mu} + \beta e^{3\mu} \) spanned by \( e^1 \) and \( e^3 \), and \( x^\mu_R = x^\mu_0(\tau) + \alpha e^{2\mu} + \beta e^{3\mu} \), spanned by null vectors \( e^2 \) and \( e^3 \).

The Kerr congruence \( \mathcal{K} \) arises as a real slice of the family of the ”left” null planes \( (Y = \text{const.}) \) of the complex light cones whose vertices lie at a complex world-line \( x^\mu_0(\tau) \).

Complex string as source of the Kerr geometry. AB, [arXiv: gr-qc/9303003, 1203.4210].

Kerr’s source can be considered as a mysterious ”particle” propagating along a complex world-line \( x^\mu_0(\tau) \) in \( CM^4 \), parametrized by a complex time \( \tau \). There appears a Newman’s complex retarded-time construction. E.T.Newman
(1973). The Left and Right retarded times as intersections with the Left and Right complex world-lines.

**Real Kerr’s geometry appears as real slice of this complex structure.**

Along with the considered complex world-line (say ‘Left’), there is a complex conjugate world-line, $X_L(\tau_L)$ and $X_R(\tau_R)$.

![Diagram](image)

Figure 1: Complex light cone at a real point $x$. The adjoined to congruence Left and Right complex null planes. Four roots: $X_L^{adv}$, $X_L^{ret}$ and $X_R^{adv}$, $X_R^{ret}$ which are related by crossing symmetry.

Complex world-line forms a world-sheet. [Earlier discussion of the complex world-line as a string by Ooguri and Vaffa.] The open Euclidean string $X_L(\tau_L) \equiv X_L^{\mu}(t_L + i\sigma_L)$ with the ends at $\sigma = \pm a$. Left and Right complex structures form an

**Orientifold: projection** $\Omega = \text{Antipodal map} + \text{Compl. Conj.} + \text{Revers of time}$. 


The Left and Right structures by excitations should be considered as independent and generated by different KN sources \( \Rightarrow \), which corresponds to two-particle KN system with quadratic generating functions of the Kerr theorem \( F_1(T) \) and \( F_2(T) \), determined on the projective twistor space \( CP^3 \). The joint twistor system is described by the equation

\[
F_{12}(T) = F_1(T) \cdot F_2(T) = 0,
\]

which turns out to be \textit{QUARTIC} in the projective twistor space, and therefore, it is the \textit{Calabi-Yau twofold} \cite{arXiv:1203.4210}.

Product of the KN closed heterotic string on the KN complex string creates the M2-brane – which corresponds to the relativistically rotating BUBBLE source of KN spinning particle.
Striking parallelizm with the superstring theory.
In the same time there are very essential differences:

- the space-time is four-dimensional – a ”compactification without compactification”,
- a natural consistency with gravity,
- characteristic parameter of the Kerr strings $a = \frac{\hbar}{m}$ corresponds to Compton scale, which is closer to particle physics vs. the Planck scale of superstring theory.

SHAPE OF THE REGULARIZED KN SOLUTION is RELATIVISTICALLY ROTATING M2-brane OF COMPTON SIZE!
THE POINT-LIKE IMAGE appears due to the LORENTZ CONTRACTION by relativistic rotation!!!
THANK YOU FOR YOUR ATTENTION!