Spontaneous Pair Production in Reissner-Nordström Black Holes

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Outline

- Overview
- Pair Production in Reissner-Nordström Black Holes
- Conclusion
Overview

- **Quantum Vacuum:** virtual particles
- **Spontaneous Pair Production: from virtual to real**
  - Schwinger mechanism: electric field
    - Schwinger, 1951
  - Hawking radiation: causal horizon (tunneling)
    - Parikh, Wilczek, [hep-th/9907001]
- **Purpose:** spontaneous pair production in black holes
  - technical simplicity: constant electric field (exactly solvable)
  - holographic duality: anti de Sitter
- **Reissner-Nordström (RN) Black Holes:** near extremal
  - near horizon region: where the production occurs

\[ AdS_2 \times S^2 + \text{constant electric field} \]
Overview

- **Pair Production**: probe massive charged scalar
  

- **Boundary condition I**: particle view point

- incident: virtual particles
- reflected: re-annihilated
- transmitted: pair produced “particles”
Overview

- **Boundary condition II: antiparticle view point**

  - $D_H^{(\text{in})}$ (transmitted)
  - $D_H^{(\text{out})} = 0$ (boundary condition)
  - $D_B^{(\text{in})}$ (incident)
  - $D_B^{(\text{out})}$ (reflected)

- **inner boundary (horizon)**
  - $\rho = 0$

- **outer boundary (asymptotic)**
  - $\rho = \infty$

- **incident**: virtual particles
- **reflected**: re-annihilated
- **transmitted**: pair produced “antiparticles”

**Equivalence:**
- particles and antiparticles should always appear in pairs
Overview

- **vacuum persistence amplitude** $|\alpha|^2$:  
  mean number of produced pairs $|\beta|^2$:

  $$ |\alpha|^2 \equiv \frac{D_{\text{incident}}}{D_{\text{reflected}}}, \quad |\beta|^2 \equiv \frac{D_{\text{transmitted}}}{D_{\text{reflected}}} $$

- **flux conservation and Bogoliubov relation**

  $$ |D_{\text{incident}}| = |D_{\text{reflected}}| + |D_{\text{transmitted}}| \Leftrightarrow |\alpha|^2 - |\beta|^2 = 1 $$

- **absorption cross section** $\sigma_{\text{abs}}$:

  $$ \sigma_{\text{abs}} \equiv \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{|\beta|^2}{|\alpha|^2} $$
Pair Production in RN Back Holes

- Near horizon geometry of near-extremal RN

\[ ds^2 = -\frac{\rho^2 - B^2}{Q^2} d\tau^2 + \frac{Q^2}{\rho^2 - B^2} d\rho^2 + Q^2 d\Omega_2^2, \]

\[ A_{[1]} = -\frac{\rho}{Q} d\tau; \quad F_{[2]} = \frac{1}{Q} d\tau \wedge d\rho \]

- probe massive charged scalar \( \Phi \)

\[(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi - m^2 \Phi = 0\]

- Ansatz: \( \Phi(\tau, \rho, \theta, \phi) = e^{-i\omega \tau + in\phi} R(\rho) S(\theta) \)
  - \( S(\theta) \) is spherical harmonics with the eigenvalue \( l(l + 1) \)
  - Presence of Schwinger mechanism and/or Hawking radiation

\[
(m^2 - q^2)Q^2 + (l + 1/2)^2 < 0
\]

- Violation of Breitenlohner-Freedman (BF) bound in AdS\(_2\)
Bogoliubov coefficients

\[
|\alpha|^2 = \frac{\cosh(\pi a - \pi b) \cosh(\pi \tilde{a} + \pi b)}{\cosh(\pi a + \pi b) \cosh(\pi \tilde{a} - \pi b)},
\]

\[
|\beta|^2 = \frac{\sinh(2\pi b) \sinh(\pi \tilde{a} - \pi a)}{\cosh(\pi a + \pi b) \cosh(\pi \tilde{a} - \pi b)}
\]

\[a \equiv qQ, \quad b \equiv \sqrt{(q^2 - m^2)Q^2 - (l + 1/2)^2}, \quad \tilde{a} \equiv \frac{\omega Q^2}{B}\]

Leading term of $|\beta|^2$ leads to the Schwinger formula

\[
|\beta|^2 \approx e^{-\frac{\pi m^2 q}{Q}} \approx e^{-\frac{\pi m^2 r_H^2}{qQ}}
\]

Absorption cross section:

\[
\sigma_{\text{abs}} = \frac{\sinh(2\pi b) \sinh(\pi \tilde{a} - \pi a)}{\cosh(\pi a - \pi b) \cosh(\pi \tilde{a} + \pi b)}
\]
An interesting observation:

\[
\frac{|\beta(B = 0)|^2}{|\beta(B \neq 0)|^2} = \frac{\cosh(\pi \tilde{a} - \pi b)}{\sinh(\pi \tilde{a} - \pi a)} e^{\pi b - \pi a} = \frac{1 + e^{2\pi(b - \tilde{a})}}{1 - e^{2\pi(a - \tilde{a})}} \geq 1
\]

Production rate in the extremal limit (Schwinger) is greater than in the near extremal case (Schwinger + Hawking).

From the extremal to near extremal black holes, the increasing attractive gravitational force will reduce the electromagnetic repulsive force for the Schwinger mechanism.

Production rate of the Schwinger mechanism is suppressed faster than the increasing part from the Hawking thermal radiation.

Such kind of interaction generically prohibits to distinguish the Schwinger mechanism from the Hawking radiation.
Holographic Dual Description

- CFT absorption cross section:
  \[
  \sigma_{abs} \sim \left(\frac{2\pi T_L}{\Gamma(2h_L)}\right)^{2h_L-1} \left(\frac{2\pi T_R}{\Gamma(2h_R)}\right)^{2h_R-1} \sinh\left(\frac{\omega_L - q_L \Omega_L}{2T_L} + \frac{\omega_R - q_R \Omega_R}{2T_R}\right)
  \times \left|\Gamma\left(h_L + i\frac{\omega_L - q_L \Omega_L}{2\pi T_L}\right)\right|^2 \left|\Gamma\left(h_R + i\frac{\omega_R - q_R \Omega_R}{2\pi T_R}\right)\right|^2
  \]

- CFT dual to RN black holes
  \[
  c_L = c_R = \frac{6Q^3}{\ell}, \quad T_L = \frac{\ell}{2\pi Q}, \quad T_R = \frac{\ell B}{\pi Q^2}
  \]

- Free parameter \(\ell\) is related to measure of U(1) bundle.
- Conformal weights of dual operator
  \[h_L = h_R = 1/2 \pm ib\]

- The complex conformal weight means that the dual operator is unstable.
Holographic Dual Description

- Identification by thermodynamics:

\[
\frac{\delta M}{T_H} - \frac{\Omega_H \delta Q}{T_H} = \frac{\omega_L - q_L \Omega_L}{T_L} + \frac{\omega_R - q_R \Omega_R}{T_R}
\]

- Hawking temperature: \( T_H = \frac{B}{2\pi Q^2} \)
- Chemical potential: \( \Omega_H = A_T(B) = -B/Q \)
- \( \delta M = \omega \) and \( \delta Q = -q \)

\[
\tilde{\omega}_L \equiv \omega_L - q_L \Omega_L = -q \ell \quad \text{and} \quad \tilde{\omega}_R \equiv \omega_R - q_R \Omega_R = 2\omega \ell
\]

- The absorption cross section agrees with the CFT’s result only up to some numerical factors.
We discuss the spontaneous pair production of charged scalar field in RN black holes (both extremal and near extremal limits).

This is one of the few examples that we exactly know the Bogoliubov coefficients.

The pair production (tachyon) is holographically dual to an operator with complex conformal weight.

The production rate is suppressed when the black hole temperature is turned on. (It should be enhanced in dS space.)

The effects of the Schwinger mechanism and the Hawking radiation generically cannot be distinguished by imposing different boundary conditions.