
Réjean Plamondon and Claudéric Ouellet-Plamondon
Département de Génie Électrique
École Polytechnique de Montréal
The authors
Basic questions

• Could there be another starting point to redirect our long-term quest for bridging the gap between General Relativity and Quantum Mechanics?
• Can a modification to Newton’s law of gravitation be derived from such a model?
• What would be the main resulting predictions of this modified Newton’s law?
• A statistical pattern recognition approach
• Putting general relativity into a probabilistic context (The interdependence principle)
• The emergence of a quasi-Newton’s law
• A symmetric geometry
• An axisymmetric geometry
• Take home messages
Statistical Pattern Recognition

Patterns are generated by a probabilistic system
Statistical Pattern Recognition

• REPRESENTATION
  scaled features ⇔ N-dimensional space
  object ⇔ random vector

• INTERPRETATION
  class ⇔ density function

• MAPPING
  class delimitation ⇔ discriminating function
• A statistical pattern recognition approach
• **Putting general relativity into a probabilistic context** (The interdependence principle)
• The emergence of a quasi-Newton’s law
• A symmetric geometry
• An axisymmetric geometry
• Discussion
Putting general relativity into a probabilistic context

Two information spaces must be analyzed and compared.
First information space: the structure of a manifold

- **REPRESENTATION**
  - coordinates + metrics $\leftrightarrow$ 4-dimensional space
  - metric quantified coordinate $\leftrightarrow$ arbitrary specific feature of the manifold

- **MAPPING**
  - Einstein tensor: $G$

- **INTERPRETATION**
  - curvature space
Second information space: the content of a manifold

- **REPRESENTATION**
  - coordinates + metrics $\Leftrightarrow$ 4-dimensional space
  - metric quantified coordinates $\Leftrightarrow$ localization of events

- **MAPPING**
  - Momentum-Energy tensor: $T$

- **INTERPRETATION**
  - Mass-energy density, energy flux, momentum density and stress components
Einstein’s equation

\[ G = KT \]

- The Einstein’s equation can be seen as making a link between the two interpretation spaces.
- **BUT**, according to the statistical pattern recognition paradigm, these interpretation spaces could be given a probabilistic meaning...How?
Topic

• A statistical pattern recognition approach
• Putting general relativity into a probabilistic context (The interdependence principle)
• The emergence of a quasi-Newton’s law
• A symmetric geometry
• An axisymmetric geometry
• Discussion
Interdependence principle

Spacetime curvature ($S$) and matter-energy density ($E$) are two inextricable information spaces defining the physically observable universe ($U$); they must be mutually exploited to describe any subset $U_i$ of this universe. In terms of expectations, the probability of observing a subset ($U_i$) is:

$$P(U_i) = P(S_i, E_i) < 1$$
Corrolary

The probability of observing and describing a given subset of the universe $P(U_i)$, that is the joint probability of $P(S_i, E_i)$, can be studied from two equivalent *modi operandi*: either by analyzing the structure of the spacetime as an interpretation space associated with an *a priori* given matter-energy density or by analyzing the matter-energy density as an interpretation space associated with an *a priori* given spacetime structure.
In terms of Bayes’ law...
(conditionnal probabilities)

\[ P(U_i) = P(S_i, E_i) = P(S_i / E_i)P(E_i) = P(E_i / S_i)P(S_i) \]

\[ f(S_i / E_i)f(E_i) = f(E_i / S_i)f(S_i) \]

\[ f(S_i / E_i) = f(E_i / S_i) \frac{f(S_i)}{f(E_i)} \]
A link with Einstein’s law?

\[ f(\frac{S_i}{E_i}) = f(\frac{E_i}{S_i}) \frac{f(S_i)}{f(E_i)} \iff G = KT \]

\[ f(\frac{S_i}{E_i}) = k_1 trG \]
\[ \frac{f(S_i)}{f(E_i)} = k_2 trT \]

\[ f(\frac{E_i}{S_i}) = ? \]
Introducing quantum mechanical concepts?

\[ f(E_i/S_i) = k_3 \psi^*_E \psi_E = k_3 f_\psi(S_i) \]

\[ G = \frac{k_2 k_3}{k_1} f_\psi(S_i) T \]

\[ f_\psi(S_i) ? \]
• A statistical pattern recognition approach
• Putting general relativity into a probabilistic context (The interdependence principle)
• The emergence of a quasi-Newton’s law
• A symmetric geometry
• An axisymmetric geometry
• Discussion
A statically spherically symmetric system
low speed and weak field condition

\[ R_{00} \approx \frac{1}{c^2} \nabla^2 \Phi = \frac{1}{2} K T_{00} f_\psi (S_i) \]

\[ f_\psi (S_i) = ? \]

\[ T_{00} = ? \]
Estimating the probability of presence

• Building a star from scratch by adding numerous identical particles \((N \to \infty)\), each one with its own wave function, density function and associated space-time, as seen from a locally flat tangent space.

• Making the convolution of their corresponding density functions.

• The central limit theorem predicts that the ideal form of the global probability density \(f(x)\) will be a Gaussian multivariate function.
Emergence of Newton’s law of gravitation: the density factor

\[ f(x) = \frac{1}{4\pi^2 \sigma^4} \exp \left( -\frac{x^2}{2\sigma^2} \right) \]

\[ \frac{x^2}{\sigma^2} = \frac{\sigma^2}{r^2} \]

\[ f(E_i / S_i) = k_3 f_\psi (S_i) = k_3 f(r) = \frac{1}{4\pi^2 \sigma^2 r^2} \exp \left( -\frac{\sigma^2}{2r^2} \right) \]
Estimating the momentum-energy tensor component

\[ 2\sigma = \frac{p}{\pi} \Rightarrow \sigma = \frac{p}{2\pi} \Rightarrow \sigma = \frac{c\delta \tau}{4\pi} \]

\[ T_{00(r)} = \frac{Mc^2}{4\pi\sigma^2} \left( \frac{\sigma}{r} \right)^3 = \frac{M\sigma c^2}{4\pi r^3} \]
Emergence of Newton’s law of gravitation: the Sun’s Laplacian

\[ R_{00} \approx \frac{1}{c^2} \nabla^2 \Phi = \frac{1}{2} KT_{00} f_\psi (S_i) \]

\[ \nabla^2 \Phi = \frac{2KMc^4 \sigma^2}{(4\pi \sigma)^3 r^5} \exp \left( -\frac{\sigma^2}{2r^2} \right) \]
Emergence of Newton’s law of gravitation: the field

$$g(r) = -\left| \nabla \Phi(r) \right| = -\frac{2KMc^4}{(4\pi\sigma)^3 r^2} \exp\left( -\frac{\sigma^2}{2r^2} \right)$$

$$g(r) \cong -\frac{2KMc^4}{(4\pi\sigma)^3 r^2} = -\frac{GM}{r^2}$$
Emergence of Newton’s law of gravitation: the potential

\[ \Phi_{\text{erfc}}(r) = \frac{2KMc^4}{(4\pi\sigma)^3} \left( \frac{\sqrt{\pi}}{\sqrt{2}\sigma} \right) \text{erfc}\left( \frac{\sigma}{\sqrt{2}r} \right) = \Phi_{\text{erfc}}(r) \]

\[ \Phi_{\text{erf}}(r) = -\frac{2KMc^4}{(4\pi\sigma)^3} \left( \frac{1}{r} - \frac{1}{6r^3} + \frac{1}{40r^5} - \ldots \right) \equiv - \frac{GM}{r} \]
• According to the present paradigm, the Newton’s law is not empirical. It is an approximation of a more general law. It can be seen as an emerging phenomenon when the proper representation and interpretation spaces are used to describe a physical manifold.

• The resulting \( \text{erfc} \) potential can be incorporated in a metric to study statically symmetric system.
• A statistical pattern recognition approach
• Putting general relativity into a probabilistic context (The interdependence principle)
• The emergence of a quasi-Newton’s law
• A symmetric geometry
• An axisymmetric geometry
• Discussion
The symmetric metric and the field equation

\[ ds^2 = \left[ 1 + \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sigma \sqrt{2}} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2r}} \right) \right] c^2 dt^2 \]

\[ -\left[ 1 + \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sigma \sqrt{2}} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2r}} \right) \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \]

- No coordinate singularity
- No intrinsic singularity
- Temporal offset at infinity
- Radial delays
Exact analytical solutions: Ricci components

\[
R_{00} = -\frac{\alpha c^2 \beta u^2}{16\pi^2 r^3}
\]

\[
R_{11} = \frac{\beta u^2}{16\pi^2 \alpha r^3}
\]

\[
R_{22} = (\alpha - 1) + 2r\beta
\]

\[
R_{33} = [(\alpha - 1) + 2r\beta] \sin^2 \theta
\]

\[
\alpha = 1 + \frac{2\Phi}{c^2} = 1 + \frac{2GM}{c^2} \left( \frac{4\pi \sqrt{\pi}}{\sqrt{2u}} \right) \text{erfc} \left( \frac{u}{4\pi \sqrt{2r}} \right)
\]

\[
\beta = \frac{GM}{c^2 r^2} \exp \left( \frac{u}{4\pi \sqrt{2r}} \right)
\]
Exact analytical solution: Einstein components

\[ G_{00} = \alpha c^2 \left[ \frac{2\beta}{r} + \frac{(\alpha - 1)}{r^2} \right] = KT_{00} f_\psi (S_i) \]

\[ G_{11} = -\frac{1}{\alpha} \left[ \frac{2\beta}{r} + \frac{(\alpha - 1)}{r^2} \right] = KT_{11} f_\psi (S_i) \]

\[ G_{22} = -\frac{u^2 \beta}{16\pi^2 r} = KT_{22} f_\psi (S_i) \]

\[ G_{33} = -\frac{u^2 \beta \sin^2 \theta}{16\pi^2 r} = KT_{33} f_\psi (S_i) \]
Exact analytical solutions: curvature scalars

\[ K_r = \frac{\beta^2 [-u^2 + 32\pi^2 r^2]^2}{\pi^4 r^6} + \frac{16\beta^2}{r^2} + \frac{4(\alpha - 1)^2}{r^4} \]

\[ R_s = -\left[ \frac{\beta u^2}{8\pi^2 r^3} + \frac{4\beta}{r} + \frac{2(\alpha - 1)}{r^2} \right] \]
The second order geodesics

\[ \ddot{r} + \frac{2\beta \dot{r}}{\alpha} = 0 \]

\[ \ddot{r} + \alpha \beta c^2 \dot{r}^2 - \frac{\beta}{\alpha} \dot{r}^2 - \alpha r \left( \dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2 \right) = 0 \]

\[ \ddot{\theta} + \frac{2\dot{r} \dot{\theta}}{r} - \left( \sin \theta \cos \theta \right) \dot{\phi}^2 = 0 \]

\[ \ddot{\phi} + \frac{2\dot{r} \dot{\phi}}{r} + \left( 2 \cot \theta \right) \dot{\theta} \dot{\phi} = 0 \]
The first order equatorial geodesics

\[ \alpha \dot{t} = k \]

\[ \dot{r}^2 + \frac{h^2 \alpha}{r^2} + 2\Phi = c^2 (k^2 - 1) \]

\[ r^2 \dot{\phi} = -h \]
The geodesics: a modified Kepler’s law

\[ \dot{\phi}^2 = \frac{GM \exp\left(\frac{-u^2}{32\pi^2 r^2}\right)}{r^4 \left[ \frac{1}{r^2} \left(1 + \frac{2K}{c^2}\right) - \frac{GM}{c^2 r^2} \exp\left(\frac{-u^2}{32\pi^2 r^2}\right) - \frac{2K}{c^2 r} \text{erf}\left(\frac{u}{4\pi \sqrt{2r}}\right) \right]} \]
Massive particle effective potential

\[ V_{\text{eff}} = \text{Kerfc}\left(\frac{3c\delta\tau}{16\pi^2\sqrt{\pi}r}\right) + \frac{h^2}{2r^2} + \frac{h^2}{c^2r^2}\left[\text{Kerfc}\left(\frac{3c\delta\tau}{16\pi^2\sqrt{\pi}r}\right)\right] \]

Converges toward Einstein’s predictions at large \( r \) and when the constant term of the \( \text{erfc} \) potential is neglected.
Most striking properties

- New set of exact analytical solutions
- Converge towards Schwarzchild’s predictions at large distance
- Differs from Einstein’s predictions at small distance
- No singularities
- No unstable inner circular orbits
- No gravitational collapse
- Gauge dependent?
$erfc(z) = 1 - erf(z)$
• A statistical pattern recognition approach
• Putting general relativity into a probabilistic context (The interdependence principle)
• The emergence of a quasi-Newton’s law
• A symmetric geometry
• An axisymmetric geometry
• Discussion
A rotation term

\[
\left[ 1 + \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sigma \sqrt{2}} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2r}} \right) \right] c^2 dt^2 = ?
\]

\[
= \left[ 1 + \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sigma \sqrt{2}} \right) \text{erf} \left( \frac{\sigma}{\sqrt{2r}} \right) \right] c^2 dt^2 + \frac{2GM}{\omega_{st} c^2} \left( \frac{\sqrt{\pi}}{\sigma \sqrt{2}} \right) d\phi dt
\]
An expansion term

\[-\left[1 + \frac{2}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma \sqrt{2}}\right) \text{erfc}\left(\frac{\sigma}{\sqrt{2r}}\right)\right]^{-1} dr^2 = ?\]

\[= -\left[1 - \frac{2}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma \sqrt{2}}\right) \text{erf}\left(\frac{\sigma}{\sqrt{2r}}\right)\right]^{-1} dr^2\]

\[+ \frac{2v_{st}}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma \sqrt{2}}\right) \left[1 - \frac{2}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma \sqrt{2}}\right) \text{erf}\left(\frac{\sigma}{\sqrt{2r}}\right)\right]^{-1} \times\]

\[\left[1 + \frac{2}{c^2} GM \left(\frac{\sqrt{\pi}}{\sigma \sqrt{2}}\right) \text{erfc}\left(\frac{\sigma}{\sqrt{2r}}\right)\right]^{-1} drdt\]
Two new components

• A rotation term:

\[ \omega_{st} = \frac{d\phi}{dt} \Rightarrow + \frac{2K}{\omega_{st}} d\phi dt \]

• An expansion term:

\[ \nu_{st} = \frac{dr}{dt} \Rightarrow + \frac{2Kv_{st}}{c^2} \left[ 1 - \frac{2K}{c^2} \text{erf} \left( \frac{u}{4\pi\sqrt{2r}} \right) \right]^{-1} \]
In other words...

The axisymmetric metric can be seen as explaining why any massive body in the universe is rotating and its associated space-time looks like expanding.
An axisymmetric solution

\[ ds^2 = g_{00} dt^2 + 2g_{03} d\phi dt + 2g_{01} dR dt + g_{11} dR^2 + g_{22} d\theta^2 + g_{33} d\phi^2 \]

\[ ds^2 = \left[ 1 - \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sqrt{2}} \right) \text{erf} \left( \frac{\sigma}{\sqrt{2}r} \right) \right] c^2 dt^2 + \frac{2}{\omega_{st}} GM \left( \frac{\sqrt{\pi}}{\sqrt{2}} \right) d\phi dt + \frac{2\nu_{st}}{c^2} GM \left( \frac{\sqrt{\pi}}{\sqrt{2}} \right) \left[ 1 - \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sqrt{2}} \right) \text{erf} \left( \frac{\sigma}{\sqrt{2}r} \right) \right]^{-1} \times \]

\[ \left[ 1 + \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sqrt{2}} \right) \text{erfc} \left( \frac{\sigma}{\sqrt{2}r} \right) \right]^{-1} d\phi dt \]

\[ -\left[ 1 - \frac{2}{c^2} GM \left( \frac{\sqrt{\pi}}{\sqrt{2}} \right) \text{erf} \left( \frac{\sigma}{\sqrt{2}r} \right) \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \]
Very complex exact analytical solutions

- 6 different covariant metric components
- 7 non-null contravariant metric components
- 10 non-zero analytical expressions for the Christoffel symbols of the first kind
- 21 coefficients of the second kind
- 16 Riemann tensor covariant components
- 9 different Ricci tensor components
- 9 different Einstein tensor components.
The second order geodesics

\[(\alpha c^2 - 2K)i' + 2\beta c^2r'i + \frac{K}{\omega_{st}} \phi' - \frac{4K\beta v_s(\alpha c^2 - K)r^2}{\alpha^2(\alpha c^2 - 2K)^2} + \frac{Kv_{st}i''}{\alpha(\alpha c^2 - 2K)} = 0\]

\[\frac{Kv_{st}i''}{\alpha(\alpha c^2 - 2K)} + \frac{\beta c^4 r^2}{(\alpha c^2 - 2K)^2} - \frac{c^2 \phi'}{(\alpha c^2 - 2K)} - \beta c^2 i^2 + r\dot{\theta}^2 + (r \sin^2 \theta)\dot{\phi}^2 = 0\]

\[\ddot{\theta} + \frac{2i\dot{\theta}}{r} - (\sin \theta \cos \theta)\dot{\phi}^2 = 0\]

\[\frac{Ki'}{\omega_{st}} - (2r \sin^2 \theta)i'\phi' - (2r^2 \sin \theta \cos \theta)\dot{\phi} - (r^2 \sin^2 \theta)\ddot{\phi} = 0\]

\[\alpha = 1 + \frac{2\Phi}{c^2} = 1 + \frac{8\pi \sqrt{\pi} GM}{\sqrt{2uc^2}} \text{erfc} \left( \frac{u}{4\pi \sqrt{2r}} \right)\]

\[\beta = \frac{GM}{c^2 r^2} \exp \left( \frac{-u^2}{32\pi^2 r^2} \right)\]
The axisymmetric metric can be seen as explaining why any massive body in the universe is rotating and its associated space-time looks like expanding.

There will be no gravitational collapse in systems described by such a metric.

Black holes without any intrinsic singularity

Dark matter might not be necessary to explain the orbital rotation velocity at and beyond the visible outer edge of distant galaxies
Predicted and measured values of the orbital rotation velocity of the NGC 801 galaxy as a function of its radius.

The experimental data are from Table 4, chapter 7 of Broeils (1992).

\[ \nu_{\text{exp}} = A \exp \left( \frac{-B}{r^2} \right) \]
Dark Matter: rotational velocity of galaxies

Predicted vs measured values of the orbital rotation velocity of some distant galaxies. from table 1 and table 4 of Blok and McGaugh (1997)
Take home messages

• The present paradigm provides partial answers to the initial questions.
• Two new metrics that will require further investigations.
• But the whole approach raises more numerous interrogations and point out many challenges.
• In the long run, it might provide a new pathway to bridge the gap between General Relativity and Quantum Mechanics...
• More investigations, theoretical and experimental, will be required to survive the Occam’s razor.
• This is the tip of an iceberg.
To investigate further...

Why are there four basic laws of nature and where do they come from? Why does any massive body in the universe experience an intrinsic rotation? What is the link between the speed of light and the gravitational constant? What are the relationships between electron mass, the Avogadro number, vacuum permittivity, and the masses of the Sun and the Earth? Are dark matter and dark energy necessary to explain the observable Universe? Can the lepton family be reduced to two members? These are just a few of the many questions that this scientific work addresses and to which it provides potential answers.

When we apply various pattern analysis methods to study the Universe, this leads us to considering the physical laws of Nature as emerging blueprints, and the fundamental constants as numerical primitives. Starting from two basic premises, the principles of interdependence and of asymptotic convergence, and using an actual pattern recognition paradigm based on Bayes’ law and the central limit theorem, Einstein’s global field equation is generalized to incorporate a probabilistic factor that further reflects the interconnected role of space-time, momentum and mass-energy density, with the aim of bridging the gap between quantum mechanics and general relativity. The whole concept predicts the emergence of the elementary interactions and the numerical value of the fundamental constants. To accomplish this, many notions and concepts are revisited, from the origin of the electron charge to the existence of black holes and the singularity Big Bang, providing a novel starting point to redirect our long-term quest for the unification of physics.
Final take home message

• The main message conveyed throughout the chapters of this book is that the four basic interactive forces of physics, which are considered to be empirical facts, can be seen as emergent phenomena described by specific mathematical patterns, when seen through the appropriate representation and interpretation schemes.
Final take home message

• Similarly, in such a model, once a coherent set of physical units is defined, the values of the fundamental constants can be seen as numerical parametric patterns that can be predicted after taking into account the various projections that are required to perform these measurements as well as the physical environment and the specific context in which these estimates are made.
Thank you for your attention!