Cosmology challenges brane scenarios in AdS$_5$

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Outline

introduction

at the brane: effective Einstein-like equation

AdS_5 bulk: large-scale structure at the brane?

conclusions
beyond General Relativity

- ongoing search for a unified description of
  - gravity
  - gauge interactions of the Standard Model
  - string theories amongst the most promising proposals

- low-energy effective action in string theories (only massless modes)
  - depending on the string theory type: includes terms for various particles
  - dilaton ($\phi$): scalar field accompanying gravity (common for all string theories)
  - at the leading order (when restricted to gravity and the dilaton)
  - Einstein's gravity coupled to the dilaton
  - in a spacetime with additional spatial dimensions
  - i.e. an extended, higher-dimensional theory of gravity: dilaton gravity

- dilaton gravity in a 5D brane scenario
  - Standard Model localized on a 4D brane embedded in a 5D spacetime
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scalor-tensor theories of gravity & conformal frames

- **dilaton gravity**: a scalar-tensor theory of gravity
  - can be formulated in various conformally-related frames
    - $g_{\mu\nu}$ & $\tilde{g}_{\mu\nu}$ related by a Weyl (conformal) transformation: $g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu}$
    - gravitational Lagrangians differ e.g. in the coefficient of the Ricci scalar
      - (generically) scalar field dependent coefficients
        - Einstein frame: $\mathcal{L} = \frac{1}{2\kappa} \mathcal{R} + \cdots$
          (natural in standard gravity & cosmology; coefficient: a constant)
        - Jordan frame: e.g. $\mathcal{L} = \frac{1}{16\pi} \phi \mathcal{R} + \cdots$
          ("traditional" frame in scalar-tensor theories of gravity; coefficient: a polynomial function of the scalar field)
        - string frame: e.g. $\mathcal{L} = e^{-\phi} \frac{\alpha'}{2} \mathcal{R} + \cdots$
          (natural in string theories; coefficient: an exponential function of the dilaton)
non-minimal matter-dilaton coupling

- If a matter term $\mathcal{L}_m$ is included into the Lagrangian in one frame:
  - Conformal transformation to another frame will change its coefficient.
  - If constant in one frame, it will become dilaton dependent in others.

- Which conformal frame is the natural physical frame?
  - No clear consensus in the literature.
  - In which frame the matter-dilaton coupling should be minimal?

- Thus: a general non-minimal coupling $f(\phi) \mathcal{L}_m$
  - Of the dilaton.
  - To the matter content of the universe $\mathcal{L}_m$
  - (Localized on the brane)
the aim of the game

- framework:
  - dilaton gravity in a 5D brane scenario
  - non-minimal matter-dilaton coupling $f(\phi) \mathcal{L}_m$
    ($\mathcal{L}_m$: matter content of the universe localized on the brane)

- take assumptions crucial to many models in the modern literature:
  - bulk: exact anti de Sitter type spacetime (AdS$_5$)
    (vital for many higher-dimensional scenarios, one of the simplest types of spacetimes)
  - brane: matter content of the universe
    (as in cosmological considerations)
    described by an inhomogeneous perfect fluid

- and answer the question:

  *can the observed large-scale structure of the universe exist on the brane in an AdS$_5$ bulk?*

  i.e.: is a “sufficiently” inhomogeneous perfect fluid (and thus the large-scale structure) permitted on the brane?
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dilaton gravity at the brane with general matter-dilaton coupling

- dilaton gravity in a 5D brane scenario: (Einstein frame)

\[ \mathcal{L} = \frac{\alpha_1}{2} \left[ \mathcal{R} - \frac{2}{3} \nabla^{\sigma} \partial_{(5)}^{(5)} \phi - \frac{1}{3} (\partial_{(5)}^{(5)} \phi)^2 \right] - V(\phi) + [f(\phi) L_m + \lambda(\phi)] \delta_B \]

- \( [\mathcal{R} - \frac{2}{3} \nabla^{\sigma} \partial_{(5)}^{(5)} \phi - \frac{1}{3} (\partial_{(5)}^{(5)} \phi)^2] \): 5D dilaton gravity

- \( \mathcal{L}_m \): (brane localized) matter content of the universe

- \( \lambda(\phi) \): ‘cosmological constant’-type term on the brane

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- \( \delta_B \): position of the co-dimension 1 brane: Dirac delta type distribution

- \( f(\phi) L_m \): (non-minimal) coupling of the dilaton \( \phi \) to brane localized matter \( L_m \)

- how does the gravity look like on the brane? (i.e. for us)

- its effective 4D description has to be established
derivation of the effective gravitational equations at the brane

- induced (projected) brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ (covariant approach)
  - $n^\mu$: vector field orthonormal to the brane at its position
  - $g_{\mu\nu}: R_{\mu\nu}{}^{\rho\sigma}$ & $\nabla_\mu$ vs $h_{\mu\nu}: R_{\mu\nu}{}^{\rho\sigma}$ & $D_\mu$

- assume a $\mathbb{Z}_2$ symmetry for the bulk (with its fixed point at the brane’s position)
  - usually imposed ‘automatically’
  - crucial for the existence of the effective gravitational equations at the brane

- in the absence of the bulk $\mathbb{Z}_2$ symmetry
  - only consistency condition (on the brane sources): $D_\lambda (f(\phi) \tau^\lambda_\mu) = f(\phi) \tau_\phi (\partial_\mu \phi)$
  - (on the brane: “generalized” covariant conservation of the energy-momentum tensor)
derivation of the effective gravitational equations at the brane

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at the brane: effective Einstein-like equation

▶ consequently, the effective Einstein-like equation at the brane reads

\[
R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \bar{G}(\phi) \tau_{\mu\nu} - h_{\mu\nu} \bar{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial \phi)^2 - E_{\mu\nu}
\]

\(\bar{G}(\phi) = \frac{-1}{48\pi\alpha_1^2} f(\phi)\lambda(\phi)\)

(\(\bar{\Lambda}(\phi) = \frac{1}{2\alpha_1} V - \frac{f^2}{4\alpha_1^2} \left[ \frac{3}{4} \tau^2 + \frac{3\lambda'}{2f} \tau_\phi - \frac{\lambda^2}{3f^2} + \frac{3\lambda'^2}{4f^2} \right]\) (effective brane Newton’s constant)

(\(\tau_{\mu\nu} = h_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta h_{\mu\nu}}, \tau_\phi = \frac{f'(\phi)}{f(\phi)} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta \phi}\)

(brane localized sources)

\(\tau_\phi = f'(\phi) f(\phi) \mathcal{L}_m + \delta \mathcal{L}_m \delta \phi\) (eff. brane cosmol. const.)

▶ three types of ‘corrections’ to the standard Einstein equation

\(\pi_{\mu\nu} = -\tau_{\mu\rho} \tau^\rho_\nu + \frac{1}{3} \tau \tau_{\mu\nu} + \frac{1}{2} h_{\mu\nu} \tau^\rho_\sigma \tau^\sigma_\rho - \frac{1}{6} h_{\mu\nu} \tau^2\)

(typical of brane gravity theories)

kinetic terms for the dilaton

(explicit bulk’s influence on the brane gravity: \(E_{\mu\nu} = n^\alpha h^\beta_\mu n^\gamma h^\delta_\nu C_{\alpha\beta\gamma\delta}\)

(bulk Weyl tensor projected on the brane: a single term, but generically non-vanishing)

\(\rightarrow\) to describe gravity induced on the brane: solution of the e.o.m.’s for the bulk gravity necessary
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conclusions
at the brane:

constraint on the spatial derivative of the matter energy density

▶ OR: “sufficiently” inhomogeneous perfect fluid on the brane in AdS$_5$ bulk?

▶ calculus ingredients:

- effective gravitational (Einstein-like) equation at the brane:
  \[
  R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \bar{G}(\phi) \tau_{\mu\nu} - h_{\mu\nu} \bar{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} \\
  + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial \phi)^2 - E_{\mu\nu}
  \]

- consistency condition (on the brane sources): \[D_\lambda (f(\phi) \tau_\lambda) = f(\phi) \tau_\phi (\partial_\mu \phi)\]

- 4D Bianchi identity: \[D_\nu (R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R) = 0\]

▶ assumptions:

- bulk: exact anti de Sitter type spacetime: \(\text{AdS}_5 \rightarrow E_{\mu\nu} = 0\)
  (no bulk influence on the brane gravity)

- brane (matter content of the universe): perfect fluid \(\rightarrow \tau_{\mu\nu} = \rho_m t_\mu t_\nu + \rho_m \gamma_{\mu\nu}\)
  \((\gamma_{\mu\nu}: 3d \text{ spatial metric}, \rho_m: \text{(dark) matter & radiation})\)
on the brane: late universe

- thus, spatial derivative of the matter energy density on the brane reads

\[ \rho_{m,i} = -\left( \frac{f'}{f} \rho_m - \frac{\lambda'}{f} \right) \phi, i + \frac{\alpha^2}{3f^2(\rho_m + \rho_m)} \left[ D^\nu \partial_i \phi - \dot{\phi}^{-1} \dot{\phi}_i D^\nu \partial_t \phi \right] (\partial_\nu \phi) \]

- imposes a strict condition on the matter content of the universe
  (potentially: a strong constraint on how inhomogeneous the matter distribution can be
  - and thus on the cosmological large-scale structure as is observed today)

- (at least) late universe: terms \( O((\partial \phi)(D \partial \phi)) \) can be neglected, as

\[ \dot{\phi}_0 \lesssim 2.4 H_0 \approx 1.8 \left( 10^{10} \text{ yr} \right)^{-1} \]

(derived: model-independent bound set by current observational data)

\[ |\ddot{\phi}_0| \ll \dot{\phi}_0^2 \]

(can be assumed / expected

(otherwise: currently observed \( \phi_0 \approx \text{const} \) would be yet another coincidence problem)

- typical models: \( |\phi, i| \lesssim c_1 |\dot{\phi}| \)

\( (c_1 > 0 \text{ and of order } 1) \)

(any initial inhomogeneities of the dilaton washed out by inflation)

- hereafter: \( \lambda \neq \lambda(\phi) \)

('cosmological constant'-type term in the energy-momentum tensor on the brane)

(only a contribution to the effective brane cosmological constant \( \bar{\lambda}(\phi) \) )
thus, spatial derivative of the matter energy density on the brane reads
\[
\rho_{m,i} = - \left( \frac{f'}{f} \rho_m - \frac{x'}{f} \right) \phi_i + \frac{\alpha^2}{3f^2(\rho_m + p_m)} \left[ D^\nu \partial_i \phi - \phi^{-1} \phi_{,i} D^\nu \partial_t \phi \right] (\partial_\nu \phi)
\]

\(\leftrightarrow\) imposes a strict condition on the matter content of the universe
(potentially: a strong constraint on how inhomogeneous the matter distribution can be
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\(\uparrow\) (at least) **late universe**: terms \(\mathcal{O}((\partial \phi)(D \partial \phi))\) can be neglected, as
\[
\dot{\phi}_0 \lesssim 2.4 H_0 \approx 1.8 \times 10^{10} \text{ yr}^{-1}
\]
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\(\Rightarrow\) \(|\ddot{\phi}_0| \ll \dot{\phi}_0^2\) can be assumed / expected
(otherwise: currently observed \(\phi_0 \approx \text{const}\) would be yet another coincidence problem)

\(\Rightarrow\) typical models: \(|\phi_{,i}| \lesssim c_1 |\dot{\phi}|\)
\((c_1 > 0 \text{ and of order } 1)\)
(any initial inhomogeneities of the dilaton washed out by inflation)

\(\Rightarrow\) hereafter: \(\lambda \neq \lambda(\phi)\) ('cosmological constant'-type term in the energy-momentum tensor on the brane)
(only a contribution to the effective brane cosmological constant \(\bar{\Lambda}(\phi)\)
late universe: spatial derivative of the energy density - quantify it!

- hence for the late universe we obtain

\[ \rho_{m0,i} \simeq -\frac{f'}{f} \rho_{m0} \phi_{0,i} \]

highly constrained: how inhomogeneous the matter energy density can be

for the common assumptions of AdS\(_5\) bulk and perfect fluid on the brane

- inhomogeneous perfect fluid (\(\rho_{m,i} \neq 0\)) on the brane? only if:

  matter content of the universe coupled non-minimally (\(f' \neq 0\)) to the dilaton

- if dilaton spatially homogeneous: no inhomogeneous matter energy density

  already: \(\dot{\phi}_0 \lesssim 2.4 H_0 \simeq 1.8 \times 10^{10} \text{ yr}^{-1}\)

  let’s quantify the implications of the constraint on \(\rho_{m0,i}\):

  current observational limits: \(|\ddot{G}_0/G_0| < (10^{11} \text{ yr})^{-1}\)

  (searches for time variation of the Newton’s constant:
  pulsar timing, solar system, stellar, cosmological constraints)

  \(\left|\frac{f'}{f} \phi_{0,i}\right| \lesssim 3.3 c_1 \left(10^5 \text{ Mpc}\right)^{-1}\) (for \(|\phi_{i,j}| \lesssim c_1 |\dot{\phi}|\)

  resulting in

  \(|\rho_{m0,i}| \lesssim 3.3 c_1 \rho_{m0} \left(10^5 \text{ Mpc}\right)^{-1}\) a stringent constraint!
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- current observational limits:
  \[ \left| \frac{\ddot{G}_0}{G_0} \right| < \left(10^{11} \text{ yr}\right)^{-1} \]
  \((\ddot{G} = G(\phi)!))

(sources for time variation of the Newton’s constant:
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  \((\text{for } |\phi_i| \lesssim c_1 |\dot{\phi}|)\)

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a stringent constraint!
cosmological large-scale structure (LSS) data

- LSS of the universe: spatial distribution of galaxies, their groups and clusters
- galaxy distribution: probed by galaxy redshift surveys
  (addressed: content and statistical properties of the LSS)
  (e.g. Sloan Digital Sky Survey (SDSS))
  characterized statistically through the two-point correlation function $\xi(x)$
  (excess number of galaxy pairs separated by $x$ relative to that expected for a random distribution)
- galaxy surveys $\rightsquigarrow$ estimation of the two-point correlation function of galaxies
  data usually presented in the form of the power spectrum of the perturbation field
  (i.e. Fourier transformation of the two-point correlation function)
**confrontation** with LSS data

- Time to *compare*
  - Upper limit on \( \rho_{m0,i} \) as *predicted* by the model (dilaton gravity, brane, AdS\(_5\))
    - (constraint on the allowed values of the spatial derivative of the matter energy density on the brane)
  - With \( \rho_{m0,i} \) estimation from the *observational data* on the LSS of the universe

- Approximations: (aim: just an estimation - allowing to compare orders of magnitude)
  - For the spatial derivative: \( \rho_{m,i} \simeq \frac{\rho_m(x_1) - \rho_m(x_2)}{|x_1 - x_2|} \)
  - LSS surveys probe the overall baryonic matter distribution
  - Spatial distributions of baryonic and dark matter similar
    - (typical of most dark matter models)
confrontation with LSS data

time to compare

- upper limit on $\rho_{m0,i}$ as predicted by the model (dilaton gravity, brane, AdS$_5$) (constraint on the allowed values of the spatial derivative of the matter energy density on the brane)

- with $\rho_{m0,i}$ estimation from the observational data on the LSS of the universe

approximations: (aim: just an estimation - allowing to compare orders of magnitude)

- for the spatial derivative: $\rho_{m,i} \simeq \frac{\rho_m(x_1) - \rho_m(x_2)}{|x_1 - x_2|}$

- LSS surveys probe the overall baryonic matter distribution

- spatial distributions of baryonic and dark matter similar (typical of most dark matter models)
confrontation with LSS data

and the outcome is...

\[ \rho_{m0,i} \text{ [model’s prediction - upper limit]} \ll \rho_{m0,i} \text{ [LSS data estimation]} \]

(within the entire range of measured scales)

i.e. \textbf{brane scenario of dilaton gravity with AdS$_5$ bulk}

(or any other spacetime with vanishing Weyl tensor)

(and \textbf{matter} content of the universe described by a \textbf{perfect fluid})

\textbf{means NO large scale structure as is observed today}
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- **dilaton gravity** studied in a 5D brane scenario
  - non-minimal coupling $f(\phi)L_m$ of dilaton to matter content of the universe localized on the brane
  - derived: effective gravitational equations at the brane

- **can large-scale structure of the universe exist on the brane?**
  - investigated for AdS$_5$ bulk & perfect fluid on the brane (matter content of the universe)
  - employed: effective gravitational eqs. at the brane, 4D Bianchi identity

  → **spatial derivative** of the brane matter energy density strongly constrained
  - non-minimal dilaton-matter coupling essential (in the Einstein frame!)
  - result quantified with current limits from the searches for time variation of the Newton’s constant
  - and confronted with the observational data on LSS from galaxy surveys

  → **NO large-scale structure** as is observed today

  dilaton gravity brane scenario ruled out? of course **NOT**: for exact AdS$_5$ bulk (or $E_{\mu\nu} = 0$) only!