Black hole perturbation in modified gravity theories

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References:
HM & Suyama, PRD 84, 084041 (2011) [arXiv:1107.3705]

2012.07.05 13th Marcel Grossmann Meeting, Stockholm
Alternative theories of gravity

• GR has been well tested both observationally and experimentally in the weak gravitational field regime, such as Solar System or on the Earth.

• In the near future, we shall be capable to test GR in the strong gravity regime, such as the vicinity of BH.

• It is interesting to consider alternative theories of gravity and see what happens.
Modified gravity theories

We studied BH perturbation in the following theories:

1. Chern-Simons gravity
   Parity violation
   \[ C \equiv R \tilde{R} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} R^{\mu\nu}_{\alpha\beta} R^{\rho\sigma\alpha\beta} \]

2. Generalized Galileon theory
   The most general scalar-tensor theory with second order field equation

   \[ L_2 = K(\phi, X), \]
   \[ L_3 = -G_3(\phi, X) \Box \phi, \]
   \[ L_4 = G_4(\phi, X) R + G_{4X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \]
   \[ L_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X} \left[ (\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right] \]

   Horndeski (1974)
   Deffayet et al (2011)
   KobaGayashi et al (2011)
Regge-Wheeler-Zerilli formalism

• Background:
  Static, spherically symmetric spacetime

\[ ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) \]

• Odd type perturbations

\[
\begin{align*}
  h_{tt} &= 0, \quad h_{tr} = 0, \quad h_{rr} = 0, \\
  h_{ta} &= \sum_{\ell,m} h_{0,\ell m}(t, r) E_{ab} \partial^b Y_{\ell m}(\theta, \varphi), \\
  h_{ra} &= \sum_{\ell,m} h_{1,\ell m}(t, r) E_{ab} \partial^b Y_{\ell m}(\theta, \varphi), \\
  h_{ab} &= \frac{1}{2} \sum_{\ell,m} h_{2,\ell m}(t, r) [E_a^c \nabla_c \nabla_b Y_{\ell m}(\theta, \varphi) + E_b^c \nabla_c \nabla_a Y_{\ell m}(\theta, \varphi)]
\end{align*}
\]

\[ a, b, c \cdots = \theta, \varphi \]

\[ E_{ab} = \sqrt{\det \gamma} \, \epsilon_{ab} \]

gauge fixing

Regge & Wheeler (1957)
Zerilli (1970)
Regge-Wheeler-Zerilli formalism

- Even type perturbations

\[ h_{tt} = A(r) \sum_{\ell,m} H_{0,\ell m}(t, r) Y_{\ell m}(\theta, \varphi), \]

\[ h_{tr} = \sum_{\ell,m} H_{1,\ell m}(t, r) Y_{\ell m}(\theta, \varphi), \]

\[ h_{rr} = \frac{1}{B(r)} \sum_{\ell,m} H_{2,\ell m}(t, r) Y_{\ell m}(\theta, \varphi), \]

\[ h_{ta} = \sum_{\ell,m} \zeta_{\ell m}(t, r) \partial_a Y_{\ell m}(\theta, \varphi), \]

\[ h_{ra} = \sum_{\ell,m} \alpha_{\ell m}(t, r) \partial_a Y_{\ell m}(\theta, \varphi), \]

\[ h_{ab} = \sum_{\ell,m} K_{\ell m}(t, r) g_{ab} Y_{\ell m}(\theta, \varphi) + \sum_{\ell,m} G_{\ell m}(t, r) \nabla_a \nabla_b Y_{\ell m}(\theta, \varphi), \]
Reduce second order action

Regge, Wheeler and Zerilli did
1. Calculate second order action
2. Derive field equations
3. Find master variables
4. Check stability
Reduce second order action

We did
1. Calculate second order action
2. Derive field equations for nondynamical modes
3. Erase nondynamical variables
4. Obtain action for master variables
5. Check stability

\[ \mathcal{L} = \frac{1}{2} \kappa \phi^2 - \frac{1}{2} \, g \phi'^2 - \frac{1}{2} \, \mathcal{M} \phi^2 \]

\[ \implies \kappa > 0, \; g > 0, \; \mathcal{M} > 0 \]

De Felice, Suyama, Tanaka (2011)
Reduce second order action

• (Part of) Ostrogradski theorem

\[ \mathcal{L} \supset a \dddot{\phi}^2 \implies \text{ghost} \]

• Proof

\[ \mathcal{L} \supset a \dddot{\phi}^2 = a(-q^2 + 2q\ddot{\phi}) = a(-q^2 - 2\dot{q}\dot{\phi}) \]

\[
\begin{vmatrix}
\frac{\partial^2 \mathcal{L}}{\partial \dddot{\phi}^2} & \frac{\partial^2 \mathcal{L}}{\partial \dddot{\phi} \partial \dot{\phi}} \\
\frac{\partial^2 \mathcal{L}}{\partial \dddot{\phi} \partial \ddot{q}} & \frac{\partial^2 \mathcal{L}}{\partial \dddot{\phi} \partial \ddot{q}} \\
\frac{\partial^2 \mathcal{L}}{\partial \ddot{\phi} \partial \dot{\phi}} & \frac{\partial^2 \mathcal{L}}{\partial \ddot{\phi} \partial \ddot{q}} \\
\frac{\partial^2 \mathcal{L}}{\partial \ddot{\phi} \partial \ddot{q}} & \frac{\partial^2 \mathcal{L}}{\partial \ddot{\phi} \partial \ddot{q}}
\end{vmatrix} = -4a^2 < 0 \implies \text{ghost}
\]

\[ \mathcal{L} \supset a \dddot{\phi}^2 \implies \text{The stability condition is } a = 0 . \]
1. Chern-Simons gravity

\[ C \equiv R \tilde{R} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} R^{\mu\nu}_{\alpha\beta} R^{\rho\sigma}_{\alpha\beta} \]

- Background:
  \[ C = 0 \] for spherically symmetric spacetime.
  We cannot distinguish CS gravity from GR.

- Perturbation:
  Due to the parity violation by \( \epsilon_{\mu\nu\rho\sigma} \),
  the odd and even modes couple each other.
  Thus, it is difficult to solve the system of equations.
  (Although some previous works partially studied BH perturbation,
   e.g., the odd mode metric perturbation only, dipole only, etc,
   full perturbation analysis has not been done so far.)
1. Chern-Simons gravity

We investigated the following models:

a. \( f(R, C) \) theory

\[
S = \frac{M_P^2}{2} \int d^4 x \sqrt{-g} f(R, C)
\]

b. Nondynamical Chern-Simons theory

\[
S = \frac{M_P^2}{2} \int d^4 x \sqrt{-g} \left( R - \frac{1}{4} \psi C \right)
\]

Model: Jackiw & Pi (2003)
BH pert.: Yunes & Sopuerta (2007)

ψ : External field

ψ : Dynamical field

c. Dynamical Chern-Simons theory

\[
S = \frac{M_P^2}{2} \int d^4 x \sqrt{-g} \left( R - \frac{\xi}{4} \psi C - \frac{1}{2} (\nabla \psi)^2 - V(\psi) \right)
\]

Model: Smith et al. (2007)
BH pert.: Molina et al. (2010)
1a. $f(R, C)$ theory

- Rewrite the action

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( RF(\lambda, s) + W(\lambda, s)C - V(\lambda, s) \right)$$

$$F(\lambda, s) = \frac{\partial f(\lambda, s)}{\partial \lambda}, \quad W(\lambda, s) = \frac{\partial f(\lambda, s)}{\partial s}, \quad V(\lambda, s) = \lambda F(\lambda, s) + s W(\lambda, s) - f(\lambda, s)$$

- Second order action

$$\mathcal{L} = p_1 \hat{h}_1^2 + p_2 \hat{h}_1(r\hat{h}_0 - 2\dot{h}_0) + p_3 \dot{h}_0^2 + p_4 \dot{h}_1^2 + p_5 h_1^2 + p_6 \delta F^2 + p_7 \beta^2 + p_8 \dot{h}_0 \delta F + p_9 \dot{h}_0 \beta + p_{10} \beta \delta F + p_{11} h_0^2$$

$$+ p_{12} \delta F'^2 + p_{13} \beta'^2 + \left[ p_{14} h_0' \delta F' + p_{15} h_0' \beta' \right] + p_{16} \beta' \delta F' + p_{17} h_0' \dot{h}_1 + p_{18} \dot{h}_0 \dot{h}_1 + \left[ p_{19} h_0' \delta F + p_{20} h_0' \beta \right]$$

$$+ p_{21} \dot{h}_1 \delta F + p_{22} \dot{h}_1 \beta + p_{23} \delta F \beta' + p_{24} h_0^2 + \left[ h_0 (p_{25} \delta F + p_{26} \beta) \right] + p_{27} h_1^2 + p_{28} \delta F^2 + p_{29} \delta F \beta + p_{30} \beta^2.$$

- Odd-even coupling

- Stability conditions

$$\frac{\partial^2 f}{\partial R \partial C} R' = 0, \quad \frac{\partial f}{\partial R} > 0, \quad \frac{\partial^2 f}{\partial R^2} > 0$$

- Three modes propagate with the speed of light.
1b. Nondynamical CS theory

- Action

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{1}{4} \psi C \right) \]

\[ \psi = \mu t + \Psi(r) \]

- Second order action

\[ \mathcal{L} = p_1 h_0' \beta' + p_2 \beta' h_0 + p_3 h_0 \dot{\beta} + p_4 h'_1 \beta' + p_5 \beta^2 + p_6 \beta'^2 + p_7 h_0 h'_1 + p_8 h_0 \beta + p_9 h_0 \beta' + p_{10} h'_0 \delta \psi + p_{11} h_1 \beta' + p_{12} h_1 \delta \psi \]

\[ + p_{13} h_0^2 + p_{14} h_0 h'_1 + p_{15} h_0 \beta + p_{16} h_0 \dot{\beta} + p_{17} h'_1^2 + p_{18} h_1 \beta + p_{19} \beta^2. \]

\[ \beta \equiv \alpha + \frac{a_4}{a_5} H_2 \]

- After erasing \( h_1 \), it is revealed that ghost mode always exists.
1c. Dynamical CS theory

• Action

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{\xi}{4} \psi C - \frac{1}{2} (\nabla \psi)^2 - V(\psi) \right) \]

\[ \psi = \Psi(r) \]

• Second order action

\[ \mathcal{L} = \text{ghost} + \text{odd-even coupling} \]

• Stability condition

\[ \psi'(r) = 0, \quad |\xi| < \frac{r_g^2}{6} \sqrt{1 + \frac{r_g V_{\psi \psi}}{6}} \]

• Three modes propagate with the speed of light.
2. Generalized Galileon theory

\[
\begin{align*}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X) \Box \phi, \\
\mathcal{L}_4 &= G_4(\phi, X) R + G_{4X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \\
\mathcal{L}_5 &= G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X} \left[ (\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right]
\end{align*}
\]

- Perturbation:
  The odd and even modes decouple.
  We restrict ourselves to the odd mode only.
  (The even mode is a work in progress.)
2. Generalized Galileon theory

• Second order action for the odd mode

\[ \mathcal{L} \propto \frac{\mathcal{F}}{ABG} \dot{Q}^2 - Q'^2 - \frac{\ell(\ell + 1)\mathcal{F}}{r^2 B\mathcal{H}} Q^2 - V(r)Q^2 \]

\[ Q \propto (\mathcal{F})^{-1/2} q \]

\[ \mathcal{F} := 2 \left( G_4 + \frac{1}{2} B\phi' X' G_{5X} - X G_{5\phi} \right) \]

\[ G := 2 \left[ G_4 - 2XG_{4X} + X \left( \frac{A'}{2A} B\phi' G_{5X} + G_{5\phi} \right) \right] \]

\[ \mathcal{H} := 2 \left[ G_4 - 2XG_{4X} + X \left( \frac{B\phi'}{r} G_{5X} + G_{5\phi} \right) \right] \]

• Stability conditions for the odd mode

\[ \mathcal{F} > 0, \ G > 0, \ \mathcal{H} > 0 \]
Conclusion

• We studied BH perturbation in parity violating theories and the most general scalar-tensor theory with second order field equation.

• We derived the stability conditions and propagation speed for each mode.

References:
HM & Suyama, PRD 84, 084041 (2011) [arXiv:1107.3705]