

Dark energy from cosmic structure

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DLW: New J. Phys. 9 (2007) 377

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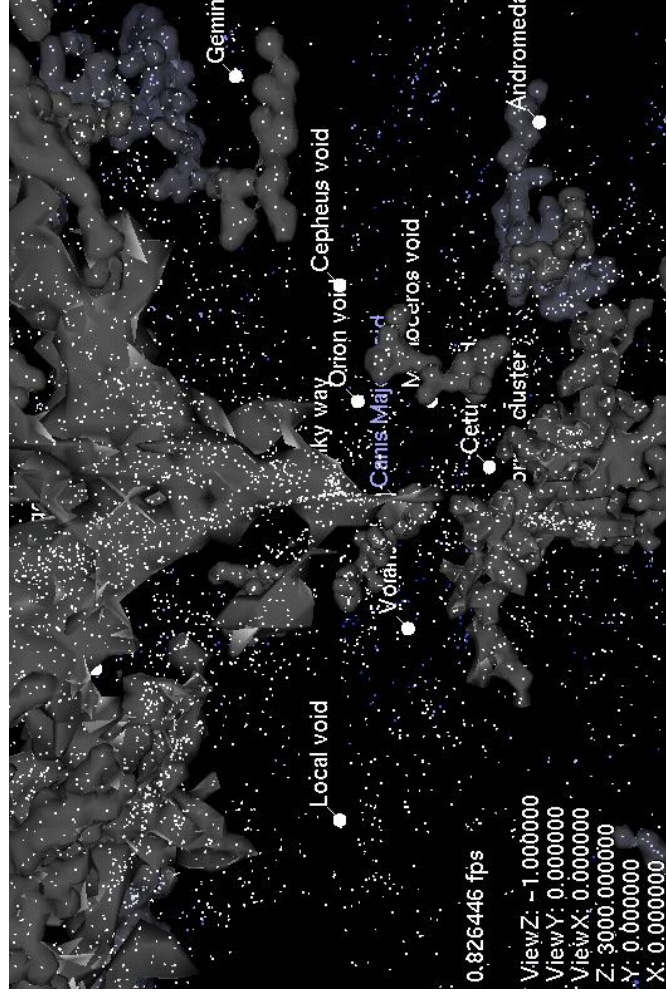
arXiv:0712.3984

Phys. Rev. D78 (2008) 084032

new results, to appear

B.M. Leith, S.C.C. Ng and DLW:

ApJ 672 (2008) L91



What is “dark energy”?

- Usual explanation: a homogeneous isotropic form of “stuff” which violates the strong energy condition.
(Locally pressure $P = w\rho c^2$, $w < -\frac{1}{3}$.)
Best-fit close to cosmological constant, Λ , $w = -1$.
- *Cosmic coincidence*: Why now? Why $\Omega_{\Lambda 0} \sim 2\Omega_{M 0}$, so that a universe which has been decelerating for much of its history began accelerating only at $z \sim 0.7$?
- Onset of acceleration coincides also with the nonlinear growth of large structures
- Are we oversimplifying the geometry?
- Hypothesis: must understand nonlinear evolution with backreaction, AND gravitational energy gradients within the inhomogeneous geometry

From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ($\delta\rho/\rho \sim 10^{-5}$ in photons and baryons; $\sim 10^{-4}, 10^{-3}$ in non-baryonic dark matter).
- FLRW approximation very good early on.
- Universe is very lumpy or inhomogeneous today.
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter $30h^{-1}$ Mpc. [Hubble constant $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void-dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

The Sandage-de Vaucouleurs paradox...

- Matter homogeneity only observed at $\gtrsim 200$ Mpc scales
- If “the coins on the balloon” are galaxies, their peculiar velocities should show great statistical scatter on scale much smaller than ~ 200 Mpc
- However, a nearly linear Hubble law flow begins at scales above 1.5–2 Mpc from barycentre of local group.
- Moreover, the local flow is statistically “quiet”; despite a possible 100–200 Mpc Hubble bubble feature.
- Peculiar velocities are isotropized in FLRW universes which expand forever (regardless of dark energy); but attempts to explain the paradox not a good fit to Λ CDM parameters (Axenides & Perivolaropoulos 2002).

Approach 1: Exact solutions

- Much work has been done on spherically symmetric dust Lemaitre–Tolman–Bondi models (Célérier 2000, Tomita 2001, . . . MANY researchers. . .)
- Violates Copernican principle.
- With 2 free functions can fit any luminosity distance relation, at cost of introducing very large voids or humps
- Distinguishable from Λ CDM, e.g., inhomogeneity test function (Clarkson, Bassett & Lu 2008), cosmic shear (Garcia-Bellido & Haugboelle 2008), . . .
- Unlikely symmetry for whole universe; difficult to reconcile with initial conditions at last scattering and structure formation
- Cut-and-paste exact solutions – *Swiss cheese* – invariably retain FLRW evolution at some level

Approach 2: Weak backreaction

- Much argument about backreaction from perturbation theory near a FLRW background (e.g., Kolb *et al.* 2006)
- Debate is largely about mathematical consistency, and conclusions vary with assumptions made
- Debate shows a potential problem – perturbation theory does not converge – and if backreaction changes the background, then any FLRW model may simply be the wrong background at present epoch
- Noh, Jeong & Hwang [PRL 103, 021301 (2009)] find the pure Einstein gravity contribution appearing in the 3rd order perturbation leads to an infrared divergence

Approach 3: Strong backreaction

- Nonlinear averaging of full inhomogeneous Einstein equations leads to modified evolution equations
- Many approaches, with different assumptions; generally $\bar{G}_{\mu\nu}(\bar{g}_{\alpha\beta}) \neq G_{\mu\nu}(\bar{g}_{\alpha\beta})$; and $\rho_{\text{cr}} \neq 3H_{\text{av}}^2 / (8\pi G)$
- Do we average tensors on curves of observers? (Zalaletdinov 1992, 1993) ... (Also studied by Coley, Pelavas, van den Hoogen, Paranjape, Singh...)
- Can we get away with averaging scalars (density, pressure, shear...)? (Buchert 2000, 2001) ... (MANY papers, e.g. Buchert, Carfora, Räsänen, Schwarz ...)
- Other new and interesting approaches (e.g., Larena 2009; Brown, Behrend & Malik 2009; Korzynski, 2009)
- Interpretation of observations is an additional problem. I address this using Buchert formalism...

Buchert-Ehlers-Carfora-Piotrkowska -Russ-Soffel-Kasai-Börner equations

For irrotational dust cosmologies, with energy density, $\rho(t, \mathbf{x})$, expansion, $\theta(t, \mathbf{x})$, and shear, $\sigma(t, \mathbf{x})$, on a compact domain, \mathcal{D} , of a suitably defined spatial hypersurface of constant average time, t , and spatial 3-metric, average cosmic evolution described by exact equations

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\langle\rho\rangle - \frac{1}{2}\langle\mathcal{R}\rangle - \frac{1}{2}\mathcal{Q}$$

$$3\frac{\ddot{a}}{a} = -4\pi G\langle\rho\rangle + \mathcal{Q}$$

$$\partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0$$

$$\partial_t(\bar{a}^6\mathcal{Q}) + \bar{a}^4\partial_t(\bar{a}^2\langle\mathcal{R}\rangle) = 0.$$

$$\mathcal{Q} \equiv \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2) - 2\langle\sigma^2\rangle$$

Back-reaction

Angle brackets denote the spatial volume average, e.g.,

$$\langle \mathcal{R} \rangle \equiv \left(\int_{\mathcal{D}} d^3x \sqrt{\det^3 g} \mathcal{R}(t, \mathbf{x}) \right) / \mathcal{V}(t)$$

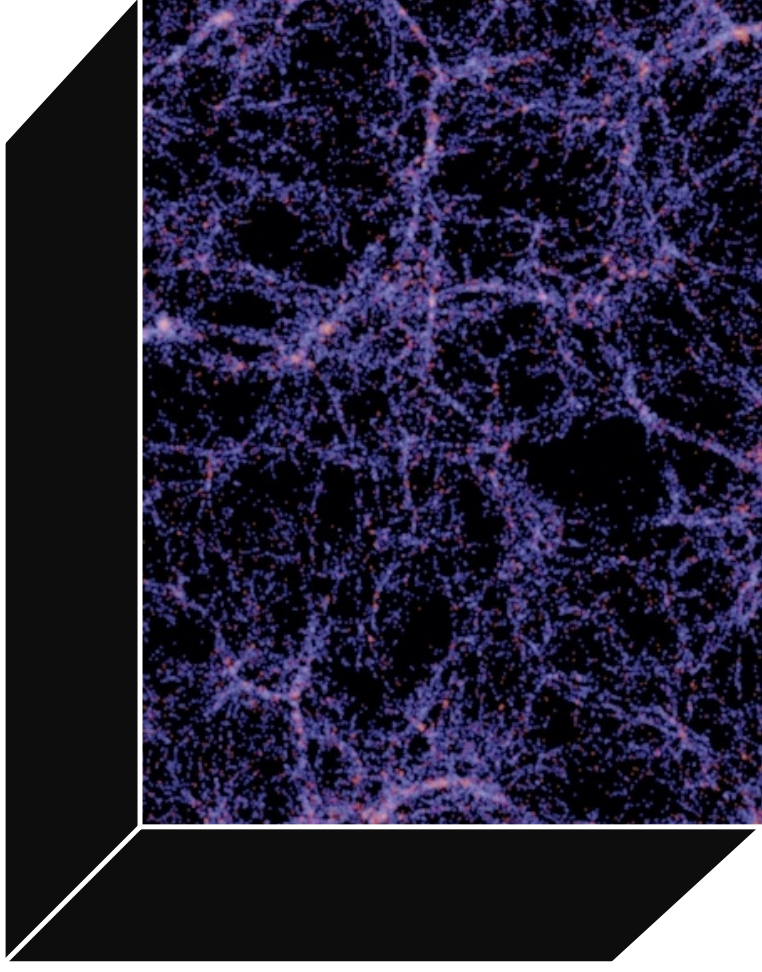
$$\langle \theta \rangle = 3 \frac{\dot{a}}{a}$$

Generally for any scalar Ψ ,

$$\frac{d}{dt} \langle \Psi \rangle - \left\langle \frac{d\Psi}{dt} \right\rangle = \langle \Psi \theta \rangle - \langle \theta \rangle \langle \Psi \rangle$$

- The extent to which the back-reaction, \mathcal{Q} , can lead to apparent cosmic acceleration or not has been the subject of much debate (e.g., Ishibashi & Wald 2006).

Within a statistically average cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rods and clocks and volume average ones

The Copernican principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume-average environment (void)

Dilemma of gravitational energy...

- In GR spacetime carries energy & angular momentum

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle, $T_{\mu\nu}$ contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in $G_{\mu\nu}$: variations are “quasilocal”!
- Newtonian version, $T - U = -V$, of Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

where $T = \frac{1}{2}m\dot{a}^2x^2$, $U = -\frac{1}{2}kmc^2x^2$, $V = -\frac{4}{3}\pi G\rho a^2x^2m$;
 $\mathbf{r} = a(t)\mathbf{x}$.

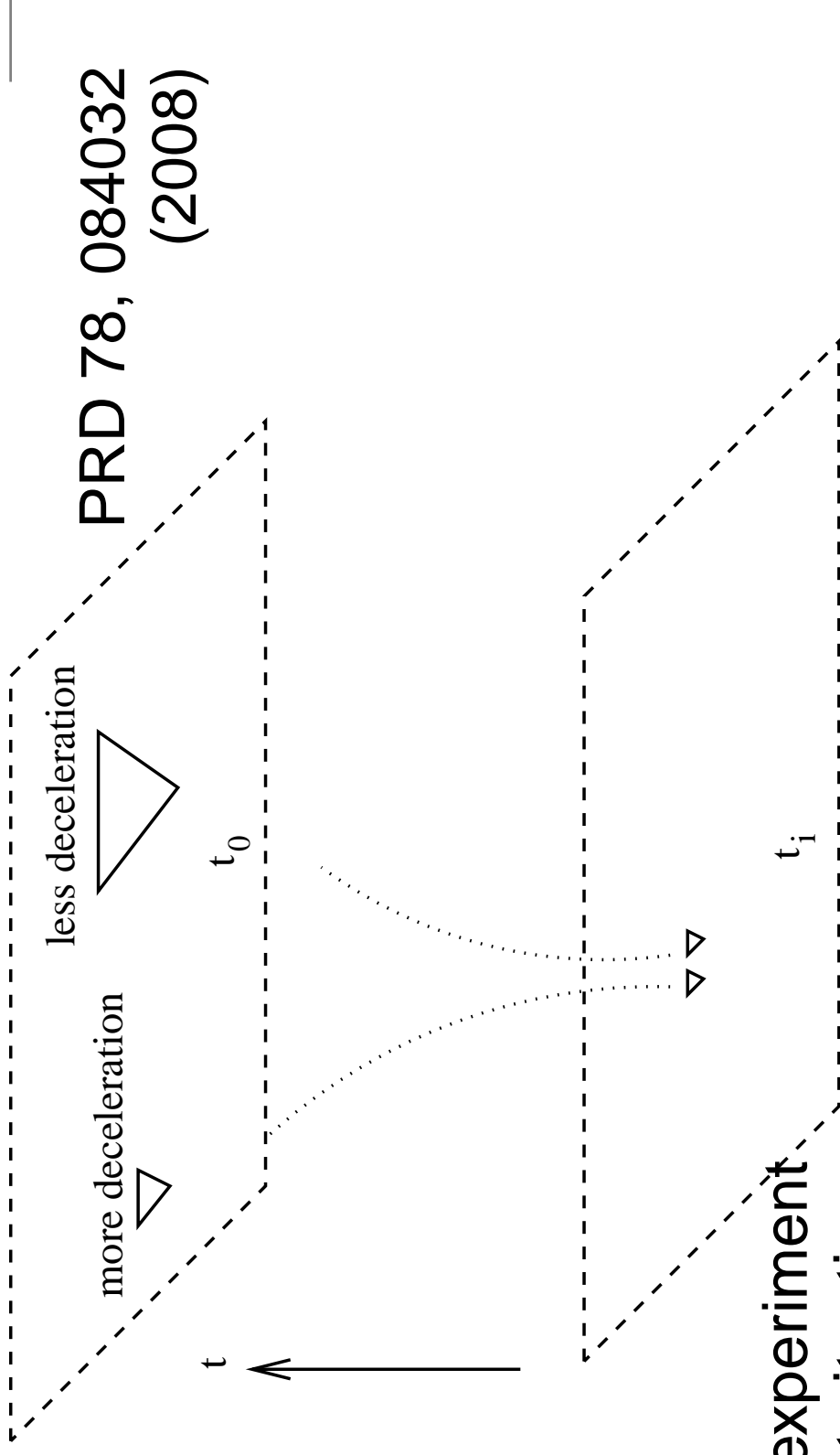
Cosmological Equivalence Principle

- *At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighbourhood, the cosmological inertial frame, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation,*

$$ds_{\text{CIF}}^2 = a^2(\eta) [-d\eta^2 + dr^2 + r^2 d\Omega^2],$$

- **Defines Cosmological Inertial Frame (CIF)**
- **Accounts for regional average effect of density in terms of frames for which the state of rest in an expanding space is indistinguishable from decelerating expansion of particles moving in a static space**

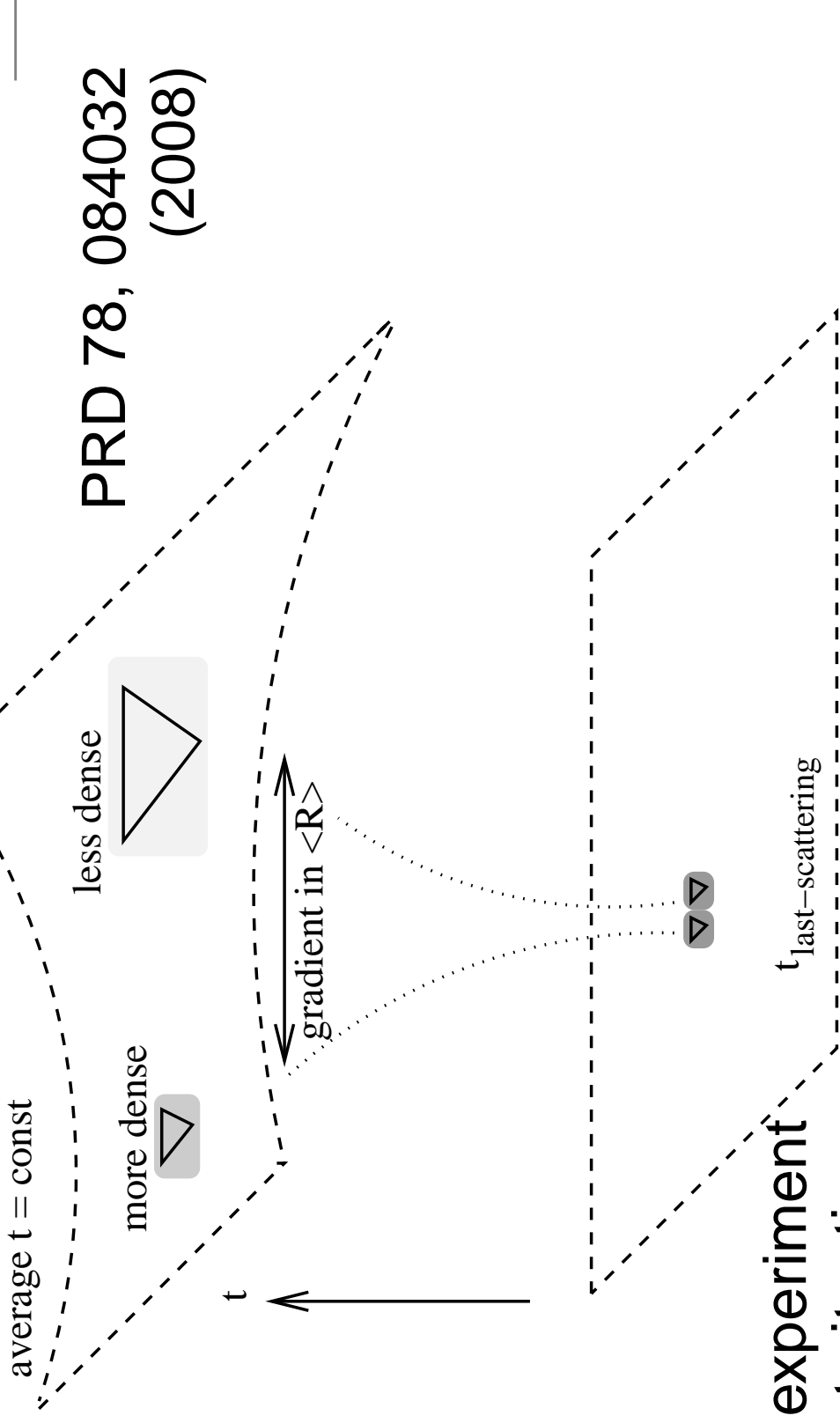
Cosmological Equivalence Principle



Thought experiment:
equivalent situations:

- SR: observers in disjoint regional semi-tethered lattices volume decelerate at different rates
- Those who decelerate more age less

Cosmological Equivalence Principle



Thought experiment equivalent situations:

- GR: regions of different density have different volume deceleration (for same initial conditions)
- Those in denser region age less

Cosmic rest frame

- Patch together CIFs for observers who see an isotropic CMB by taking surfaces of uniform volume expansion

$$\left\langle \frac{1}{l_r(\tau)} \frac{dl_r(\tau)}{d\tau} \right\rangle = \frac{1}{3} \langle \theta \rangle_1 = \frac{1}{3} \langle \theta \rangle_2 = \dots = \bar{H}(\tau)$$

- Average over regions in which (i) spatial curvature is zero or negative; (ii) space is expanding at the boundaries, at least marginally.
- Solves the Sandage–de Vaucouleurs paradox implicitly.
- Voids appear to expand faster; but their local clocks tick faster, locally measured expansion can still be uniform.
- Global average H_{av} on large scales with respect to *any* one set of clocks will differ from \bar{H}

Two/three scale ‘timescape’ model

$$\bar{a}^3 = f_{\text{wi}} a_{\text{w}}^3 + f_{\text{vi}} a_{\text{v}}^3 \equiv \bar{a}^3 (f_{\text{w}} + f_{\text{v}})$$

- Split statistical volume average into *spatially flat walls* (containing galaxies), f_{w} , and negatively curved voids with fraction, $f_{\text{v}} = 1 - f_{\text{w}}$.
- Solve Buchert equations, and construct a spherically symmetric average geometry (LTB metric NOT LTB solution) by radial null cone average
- Volume–average Buchert cosmological parameters, $\bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_Q = 1$, defined w.r.t. coarse–grained 100/h Mpc cell with average (negative) spatial curvature
- Match to local spatially flat geometry within expanding wall regions by conformally matching radial null geodesics with uniform bare Hubble flow condition

Dressed cosmological parameters

- Conventional parameters for “wall observers” in galaxies: defined by assumption (no longer true) that others in entire observable universe have synchronous clocks and same local spatial curvature

$$\begin{aligned} ds_{\mathcal{F}_I}^2 &= -d\tau_w^2 + a_w^2(\tau_w) [d\eta_w^2 + \eta_w^2 d\Omega^2] \\ \rightarrow ds^2 &= -d\tau_w^2 + a^2 [d\bar{\eta}^2 + r_w^2(\bar{\eta}, \tau_w) d\Omega^2] \end{aligned}$$

where $a \equiv \bar{a}/\bar{\gamma}_w$, $r_w \equiv \bar{\gamma}_w (1 - f_v)^{1/3} f_{wi}^{-1/3} \eta_w(\bar{\eta}, \tau_w)$, and volume-average conformal time $d\bar{\eta} = dt/\bar{a} = \bar{\gamma}_w d\tau_w/\bar{a}$.

- This leads to conventional dressed parameters which do not sum to 1, e.g.,

$$\Omega_M = \bar{\gamma}_w^3 \bar{\Omega}_M.$$

Tracker solution PRL 99, 251101

- General exact solution possesses a “tracker limit”

$$\bar{a} = \frac{\bar{a}_0(3\bar{H}_0 t)^{2/3}}{2 + f_{v0}} \left[3f_{v0}\bar{H}_0 t + (1 - f_{v0})(2 + f_{v0}) \right]^{1/3}$$

$$f_v = \frac{3f_{v0}\bar{H}_0 t}{3f_{v0}\bar{H}_0 t + (1 - f_{v0})(2 + f_{v0})},$$

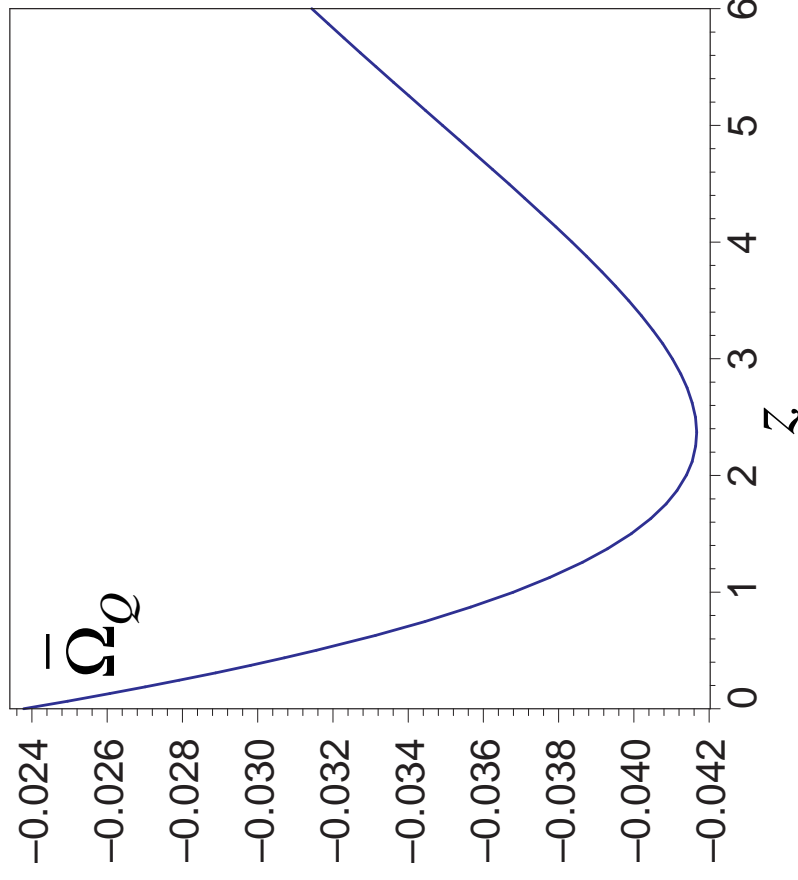
- Void fraction $f_v(t)$ determines many parameters:

$$\bar{\gamma}_w = 1 + \frac{1}{2}f_v = \frac{3}{2}\bar{H}t$$

$$\tau_w = \frac{2}{3}t + \frac{2(1 - f_{v0})(2 + f_{v0})}{27f_{v0}\bar{H}_0} \ln \left(1 + \frac{9f_{v0}\bar{H}_0 t}{2(1 - f_{v0})(2 + f_{v0})} \right)$$

$$H \equiv \frac{1}{a} \frac{da}{d\tau_w} = \bar{\gamma}_w \bar{H} - \dot{\bar{\gamma}}_w = \bar{\gamma}_w \bar{H} - \bar{\gamma}_w^{-1} \frac{d}{d\tau_w} \bar{\gamma}_w$$

Magnitude of backreaction



- Magnitude of $\bar{\Omega}_Q$ determines departure from FLRW evolution and variance of density of “statistically average” coarse-grained cells
- This of order 4–5% at most

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2(1 - f_v)^2}{(2 + f_v)^2}.$$

As $t \rightarrow \infty$, $f_v \rightarrow 1$ and $\bar{q} \rightarrow 0^+$.

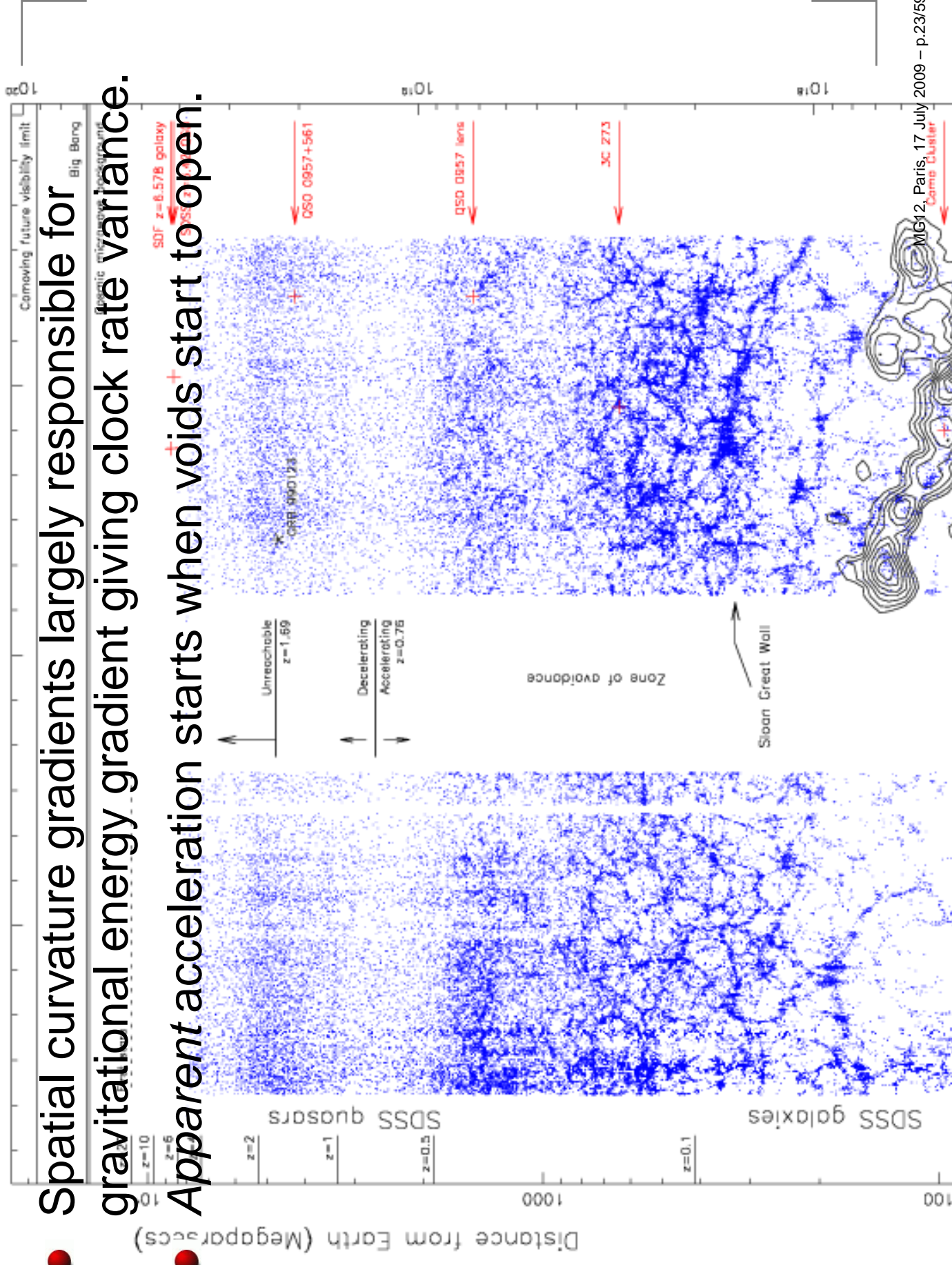
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-(1 - f_v)(8f_v^3 + 39f_v^2 - 12f_v - 8)}{(4 + f_v + 4f_v^2)^2},$$

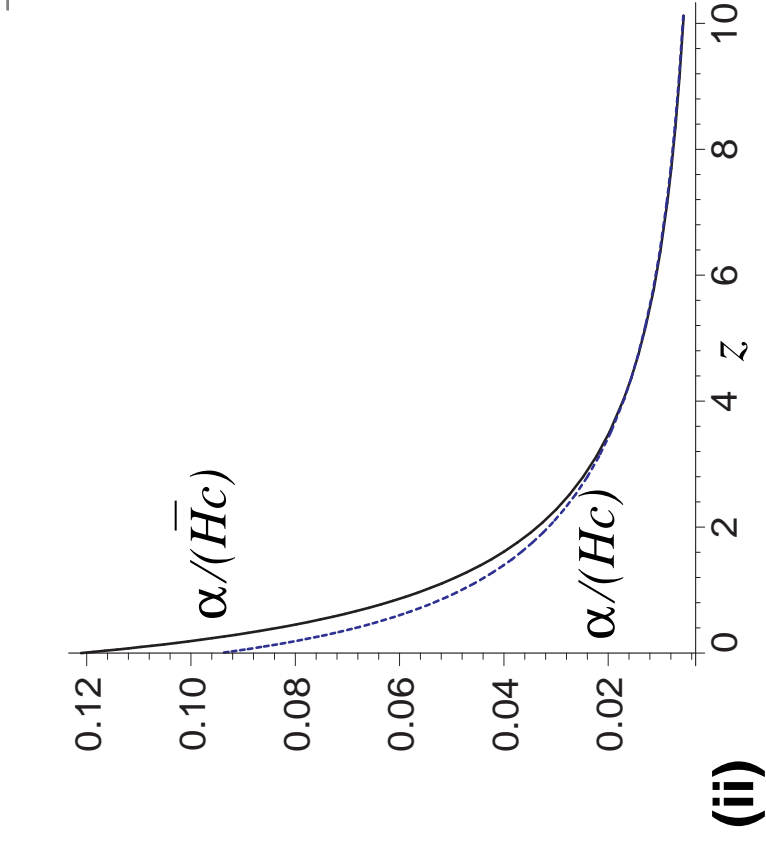
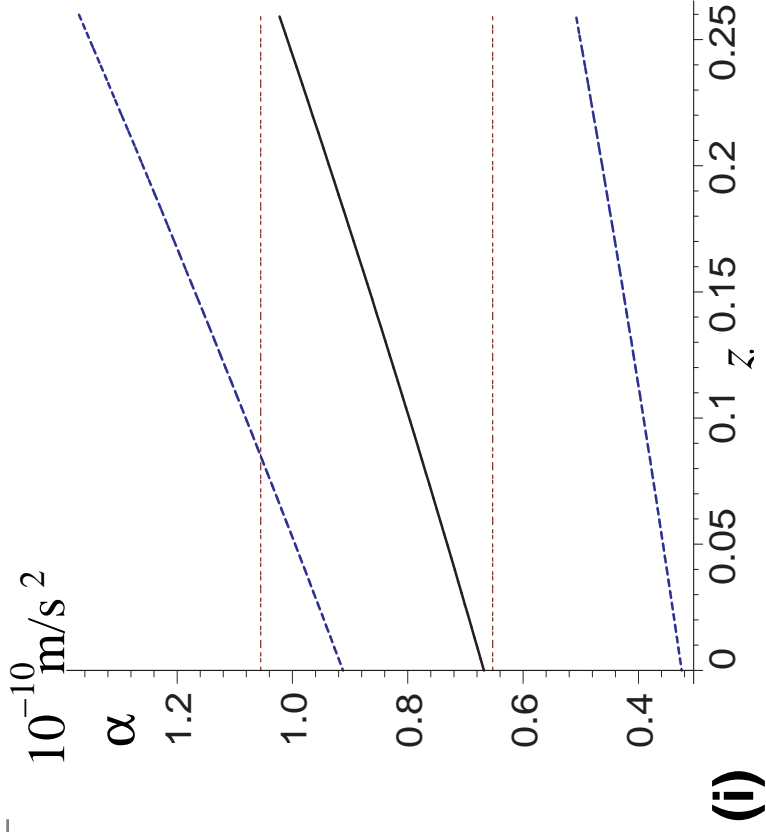
Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small f_v ; changes sign when $f_v = 0.58670773\dots$, and approaches $q \rightarrow 0^-$ at late times.

Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- Apparent acceleration starts when voids start to open.



CEP relative deceleration scale



By equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c \bar{\gamma}_w \dot{\gamma}_w / (\sqrt{\bar{\gamma}_w^2 - 1})$ beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large z .

● For $z \lesssim 0.25$, coincides with empirical MOND scale

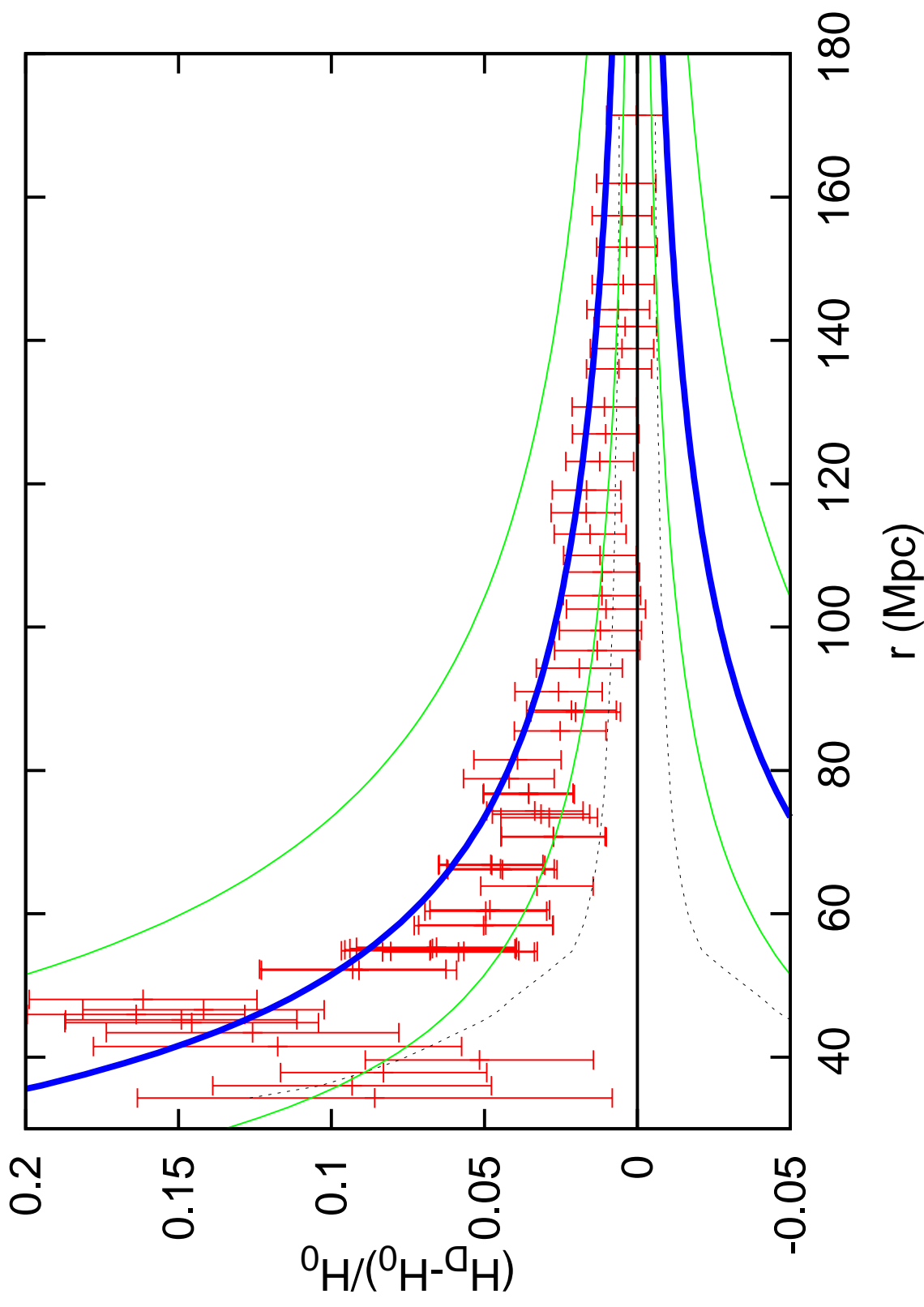
$$\alpha_0 = 1.2^{+0.3}_{-0.2} \times 10^{-10} \text{ ms}^{-2} h_{75}^2 = 8.1^{+2.5}_{-1.6} \times 10^{-11} \text{ ms}^{-2} \text{ for}$$

$$H_0 = 61.7 \text{ km s}^{-1} \text{ Mpc}^{-1} .$$

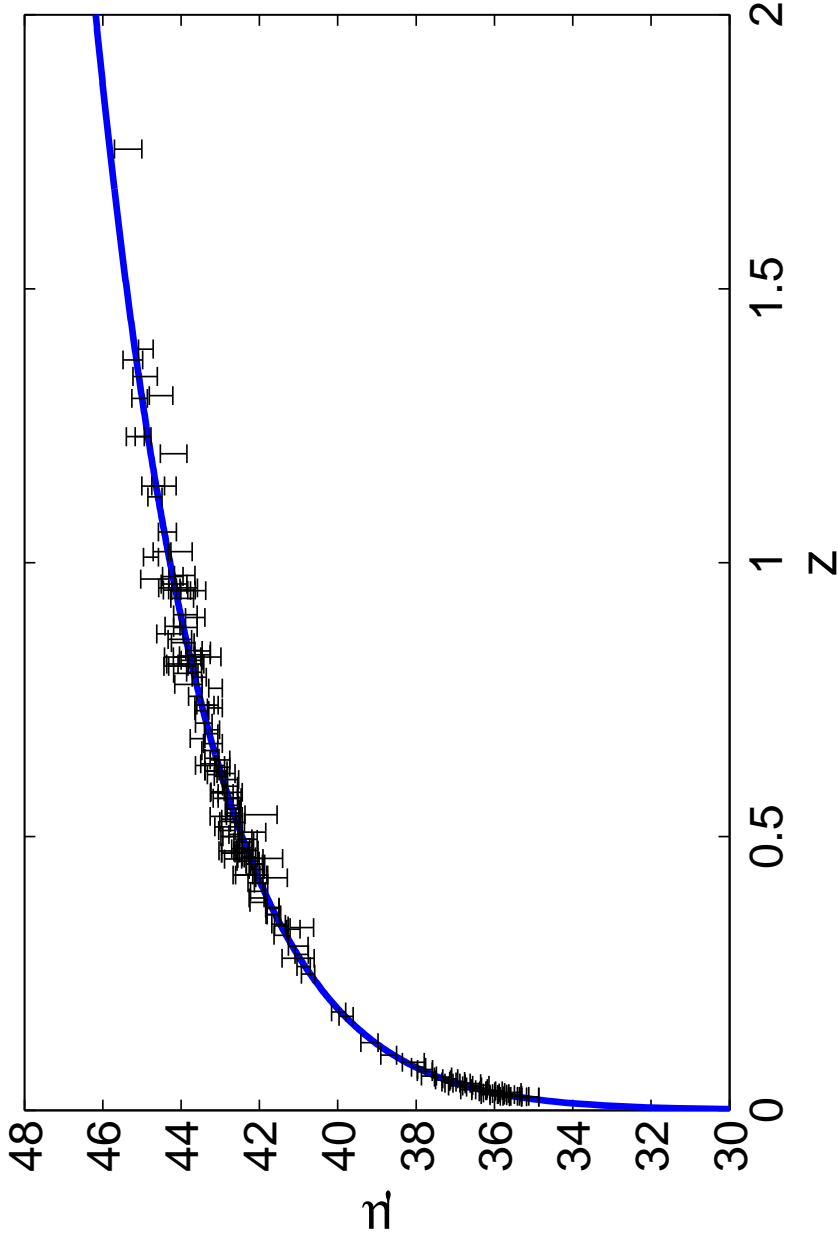
Apparent Hubble flow variance

- As voids occupy largest volume of space expect to measure higher average Hubble constant locally until the global average relative volumes of walls and voids are sampled at scale of homogeneity; thus expect maximum H_0 value for isotropic average on scale of dominant void diameter, $30h^{-1}\text{Mpc}$, then decreasing till levelling out by $100h^{-1}\text{Mpc}$.
- Consistent with observed Hubble bubble feature (Jha, Riess, Kirshner ApJ 659, 122 (2007)), which is unexplained (and problem for) ΛCDM .
- Intrinsic variance in apparent Hubble flow exposes a local scale dependence which may partly explain difficulties astronomers have had in converging on a value for H_0 .

N. Li & D. Schwarz, arxiv:0710.5073v1-2

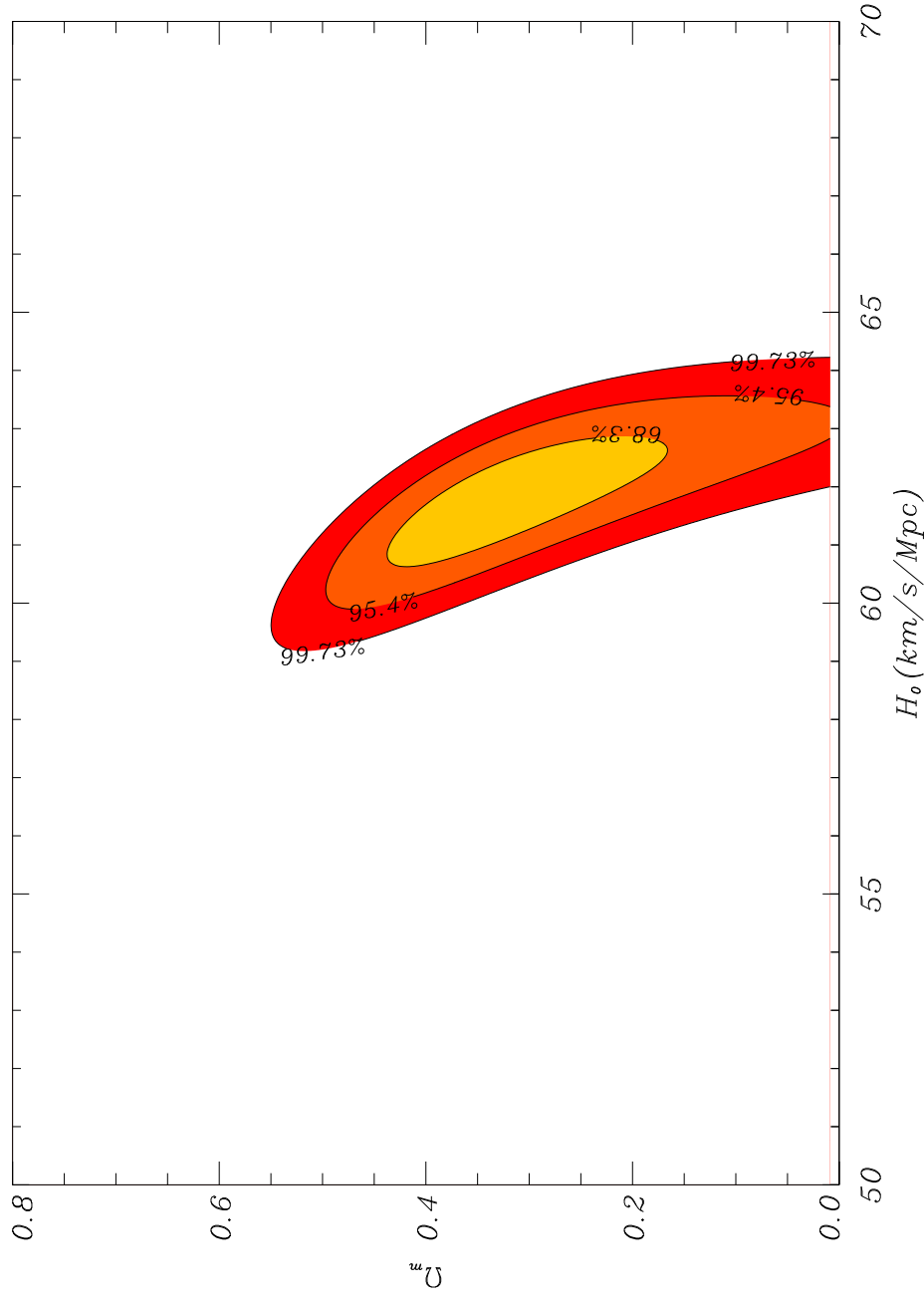


Test 1: Snela luminosity distances



- Type Ia supernovae of Riess07 Gold data set fit with χ^2 per degree of freedom = 0.9
- With $55 \leq H_0 \leq 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $0.01 \leq \Omega_{M0} \leq 0.5$, find Bayes factor $\ln B = 0.27$ in favour of TS model (marginally): statistically indistinguishable from Λ CDM.

Test 1: Snela luminosity distances



Best-fit H_0 agrees with HST key team, Sandage et al.,

$H_0 = 62.3 \pm 1.3$ (stat) ± 5.0 (syst) $\text{km s}^{-1} \text{Mpc}^{-1}$ [ApJ 653 (2006) 843].

Best fit parameters

- Hubble constant $H_0 = 61.7^{+1.2}_{-1.1} \text{ km s}^{-1} \text{ Mpc}^{-1}$
- present void volume fraction $f_{v0} = 0.76^{+0.12}_{-0.09}$
- bare density parameter $\bar{\Omega}_{M0} = 0.125^{+0.060}_{-0.069}$
- dressed density parameter $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$
- non-baryonic dark matter / baryonic matter mass ratio $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0}) / \bar{\Omega}_{B0} = 3.1^{+2.5}_{-2.4}$
- bare Hubble constant $\bar{H}_0 = 48.2^{+2.0}_{-2.4} \text{ km s}^{-1} \text{ Mpc}^{-1}$
- mean lapse function $\bar{\gamma}_0 = 1.381^{+0.061}_{-0.046}$
- deceleration parameter $q_0 = -0.0428^{+0.0120}_{-0.0002}$
- wall age universe $\tau_0 = 14.7^{+0.7}_{-0.5} \text{ Gyr}$

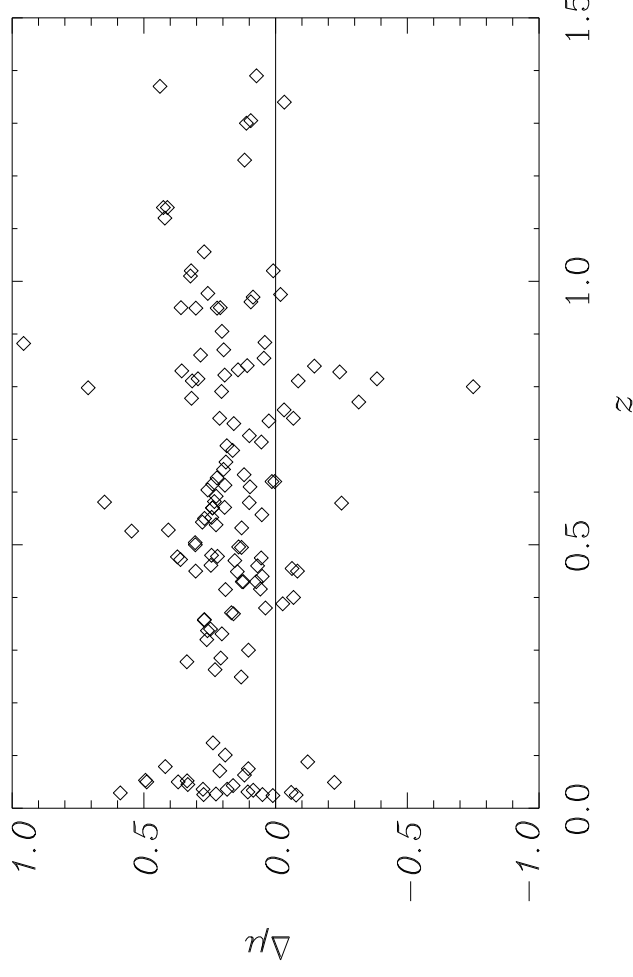
More recent results

- Kwan, Francis and Lewis arXiv:0902.4249 find that the Λ CDM model favoured over TS model using Union and Consitution datasets
- BUT data is reduced by SALT which combines empirical light curve parameters and cosmological parameters assuming the *Friedmann equation* with Λ CDM.

$$\mu_B = m_B^{\max} - M + \alpha(s - 1) - \beta c$$

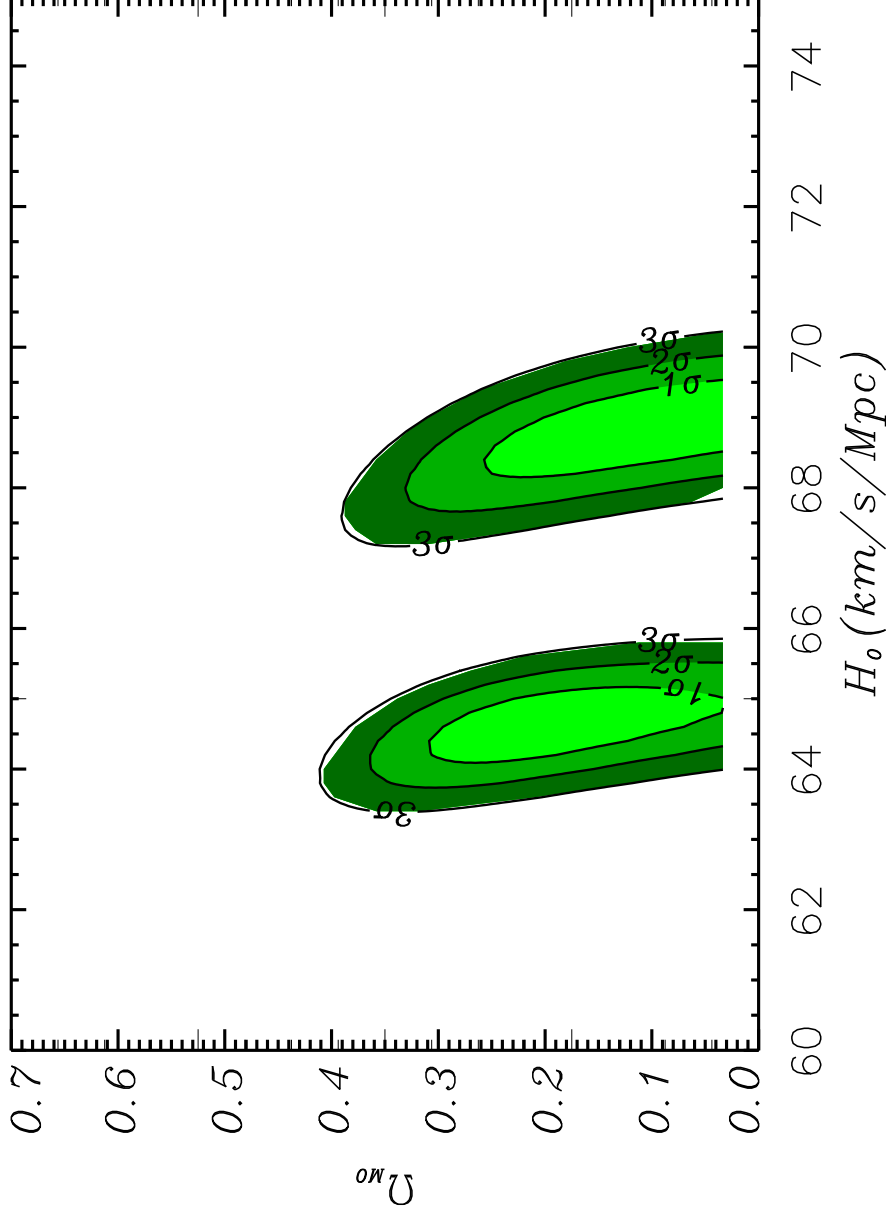
- When presenting Constitution set Hicken *et al.* (CfA) arXiv:0901.4804 find systematic issues between SALT, SALT2, MLC31 and MLC17 methods

More recent results



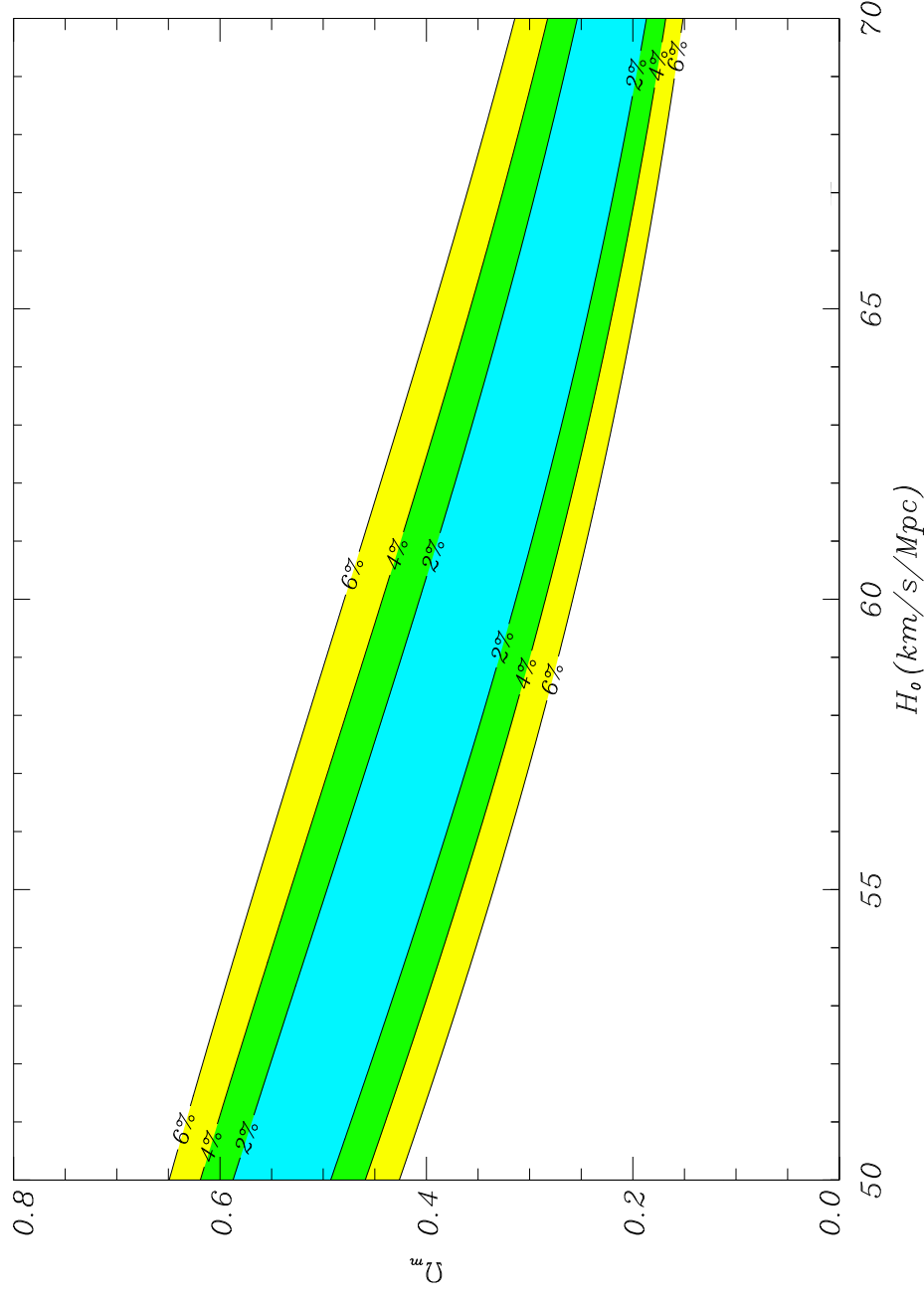
- Subset of 140 Snela common to Riess07 and Union have distance moduli giving $\mu_{\text{Riess}} - \mu_{\text{Union}}$ with considerable scatter
- Fitting the 377 Snela of the Hicken *et al.* MCLS17 reduction, gives a minimum χ^2 for the TS model comparable to the Λ CDM model

Test 1: Snela luminosity distances



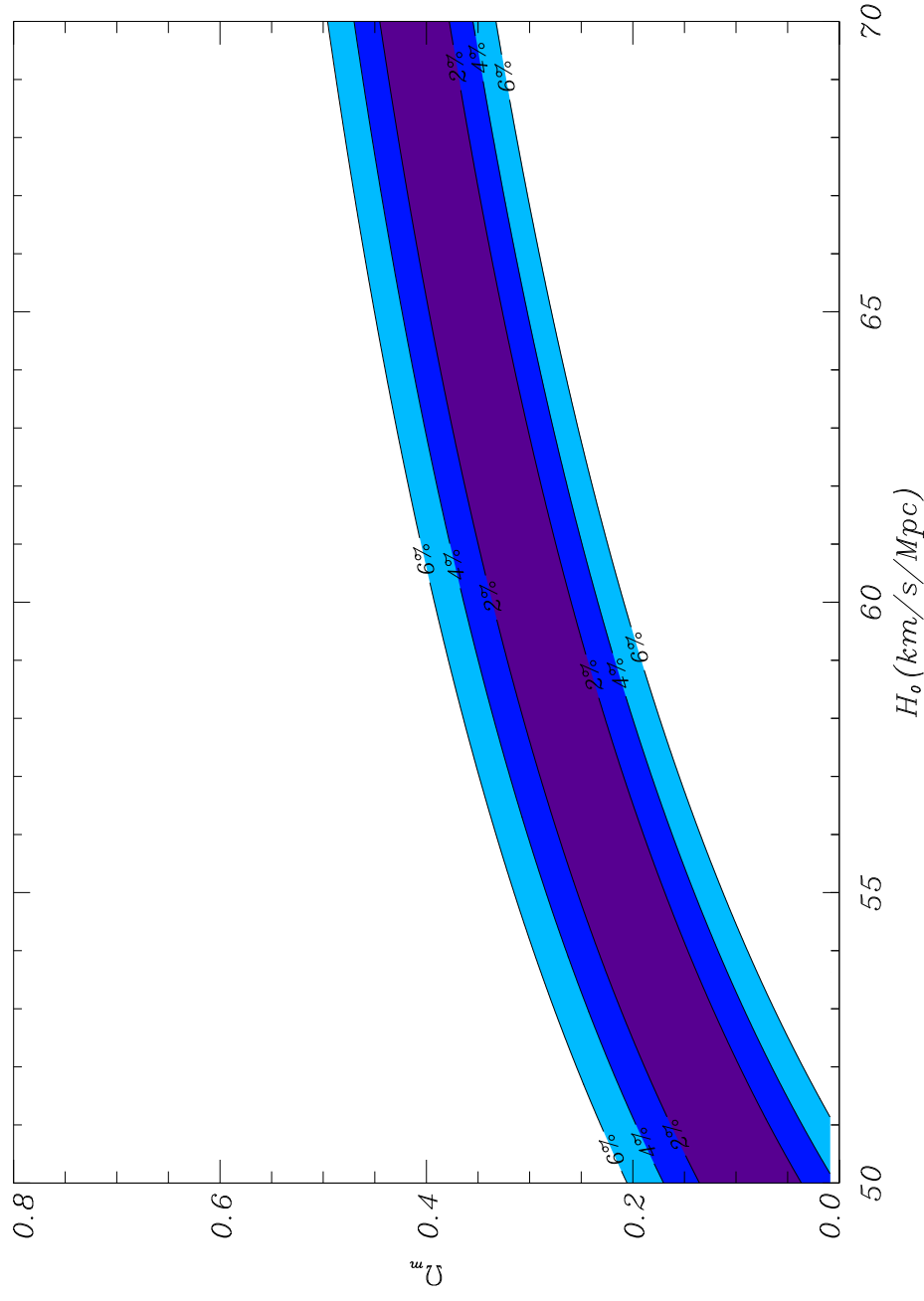
307 Union Snela (right hand) fits worse than Λ CDM; 372 MLC17 Snela of Hicken *et al.* (left hand) fits comparable to Λ CDM.

Test 2: Angular scale of CMB Doppler peaks



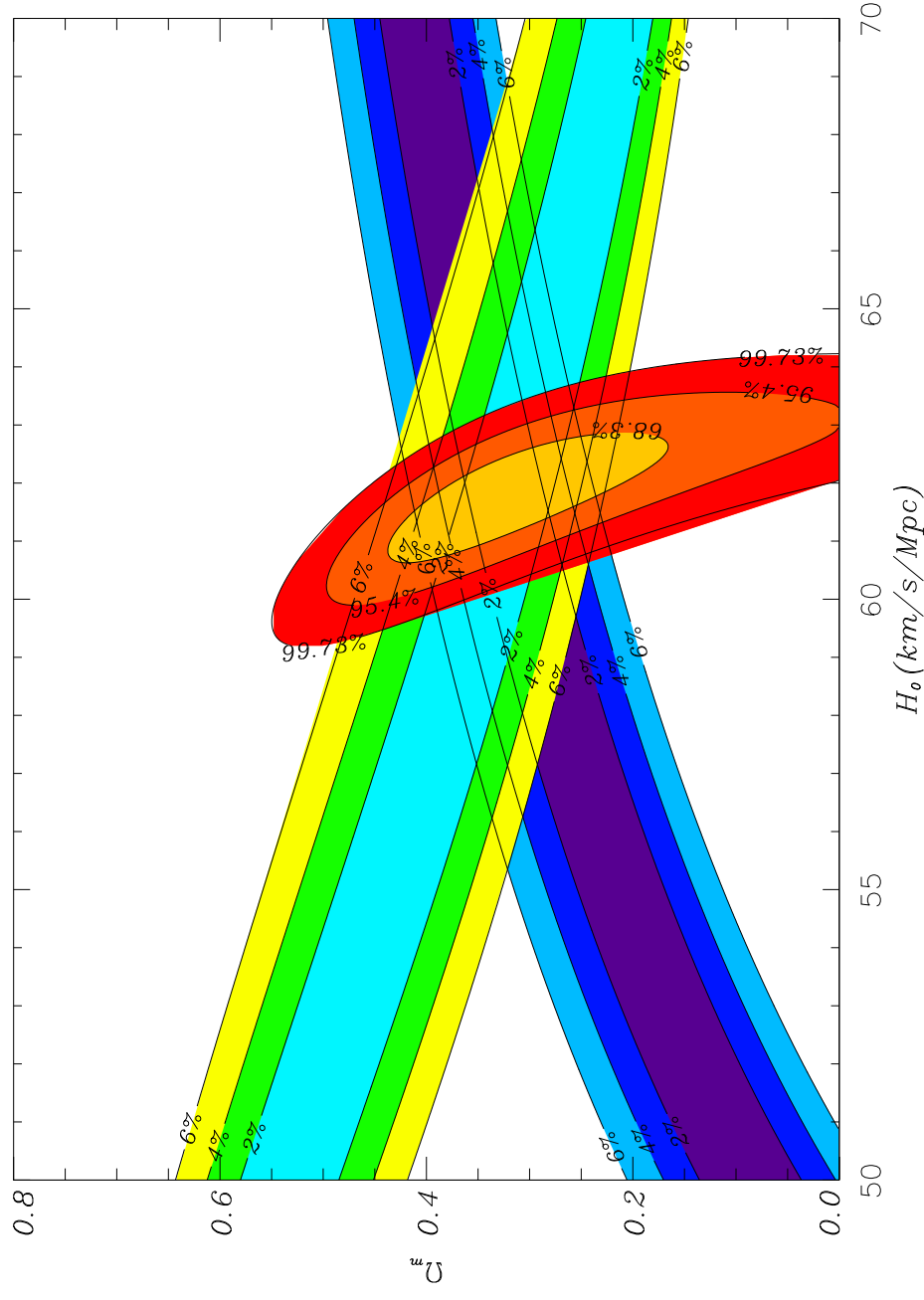
Parameters within the (Ω_m, H_0) plane which fit the angular scale of the sound horizon $\delta = 0.01$ rad deduced for WMAP, to within 2%, 4% and 6%.

Test 3: Baryon acoustic oscillation scale



Parameters within the (Ω_m, H_0) plane which fit the effective comoving baryon acoustic oscillation scale of $104h^{-1}$ Mpc, as seen in 2dF and SDSS.

Agreement of independent tests



Best-fit parameters: $H_0 = 61.7^{+1.2}_{-1.1} \text{ km s}^{-1} \text{ Mpc}^{-1}$,

$\Omega_m = 0.33^{+0.11}_{-0.16}$ (1σ errors for SnIa only) [Leith, Ng &

Wiltshire, ApJ 672 (2008) L91]

The value of H_0

Value of $H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ of S_{H_0} ES survey (Riess *et al.*, 2009) calibrated by NGC4258 maser distance at 7.5 Mpc is a challenge for the timescape model.

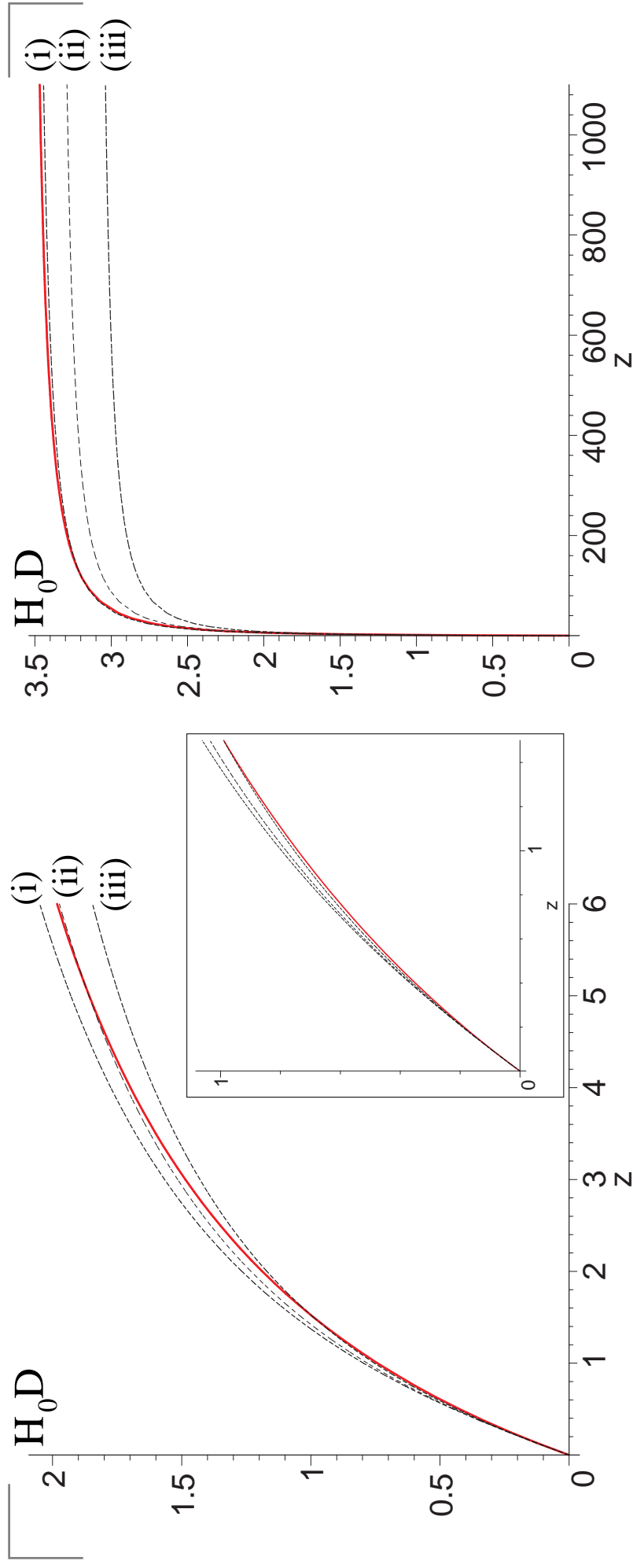
However,

- Expect variance in Hubble flow below scale of homogeneity with typical higher value

$$H_0 = 72.3 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ at } 30h^{-1} \text{ Mpc scale}$$

- Latest FLRW BAO value: $H_0 = 68.2 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (SDSS7: Percival *et al.*, arXiv:0907.1660)
- Megamaser project will give better geometric distances and independent H_0 measure. First more distant maser galaxy NGC3789 at 49.06 Mpc gives [by 8 June NRAO press release] $H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Dressed “comoving distance” $D(z)$



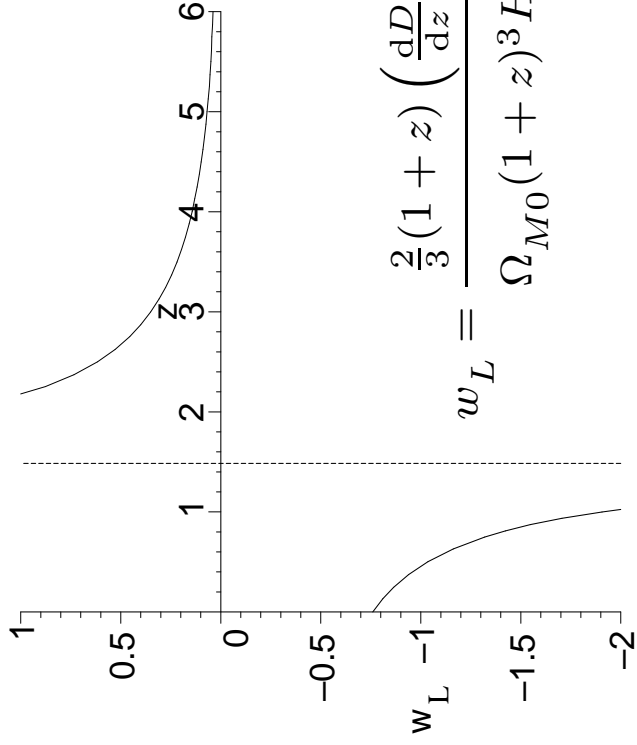
Best-fit timescape model (**red line**) compared to 3 spatially

flat Λ CDM models: **(i)** best-fit to WMAP5 only ($\Omega_\Lambda = 0.75$);

(ii) joint WMAP5 + BAO + SnIa fit ($\Omega_\Lambda = 0.72$);

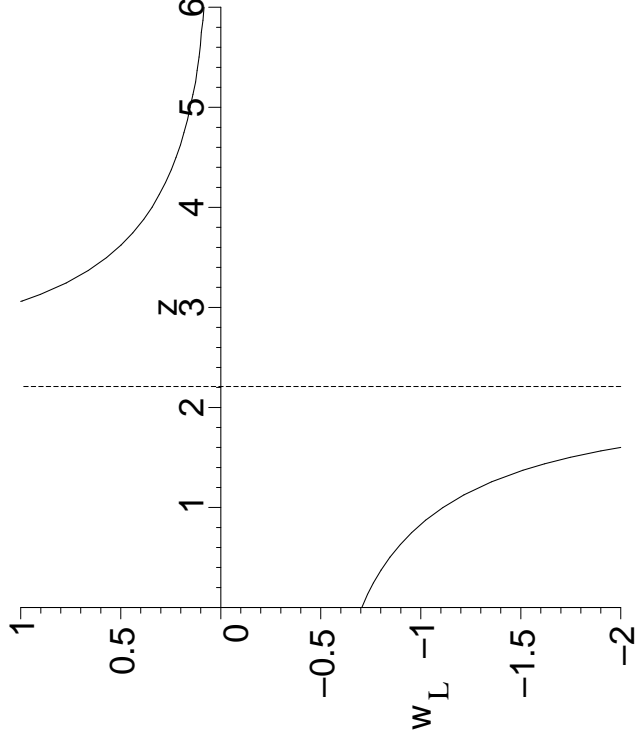
(iii) best flat fit to (Riess07) SnIa only ($\Omega_\Lambda = 0.66$).

Equivalent “equation of state”?



(i)

$$w_L = \frac{\frac{2}{3}(1+z) \left(\frac{dD}{dz}\right)^{-1} \frac{d^2D}{dz^2} + 1}{\Omega_{M0}(1+z)^3 H_0^2 \left(\frac{dD}{dz}\right)^2} - 1$$

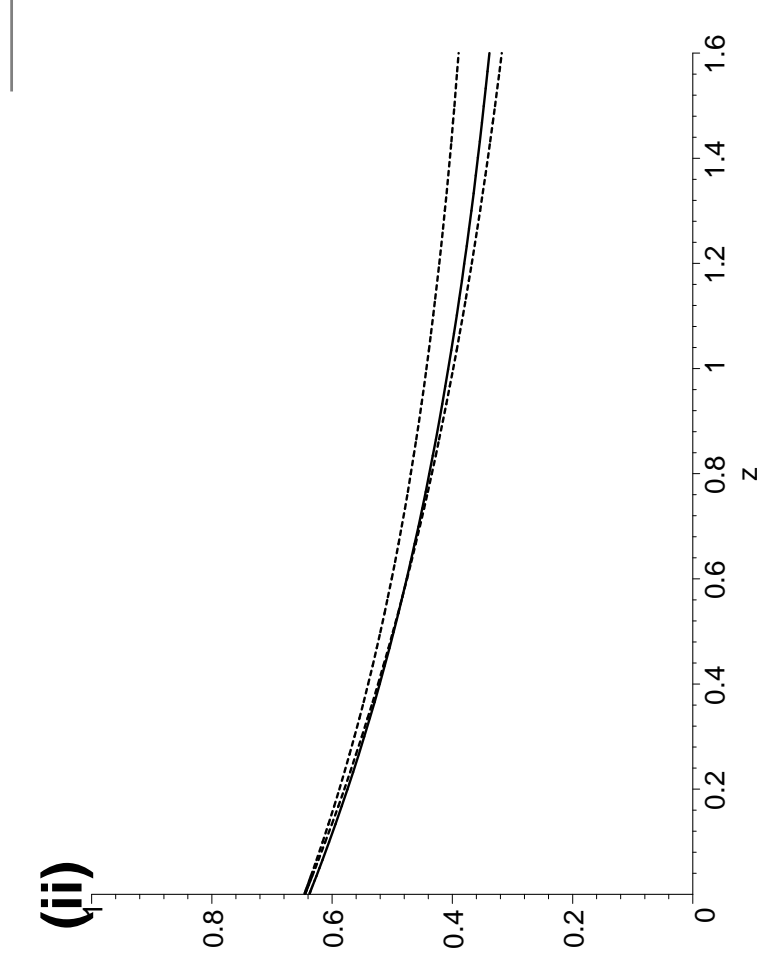
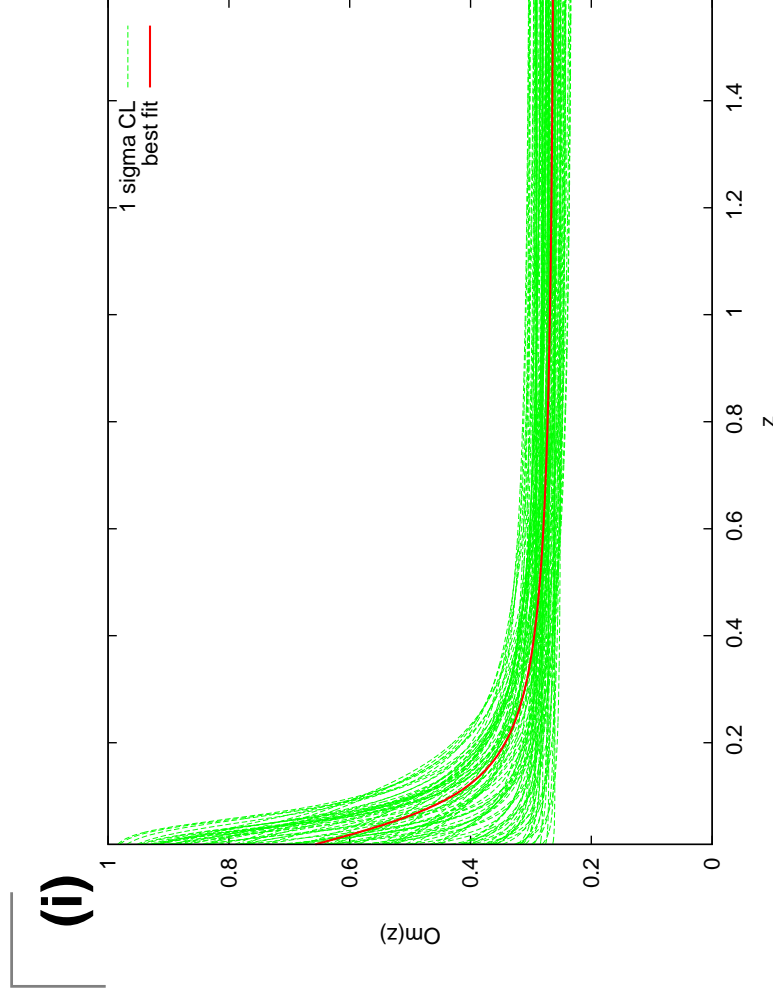


(ii)

A formal “dark energy equation of state” $w_L(z)$ for the best-fit timescape model, $f_{v0} = 0.76$, calculated directly from $r_w(z)$: **(i)** $\Omega_{M0} = 0.33$; **(ii)** $\Omega_{M0} = 0.279$.

- Description by a “dark energy equation of state” makes no sense when there’s no physics behind it; but average value $w_L \simeq -1$ for $z < 0.7$ makes empirical sense.

Sahni, Shafieloo and Starobinsky $\Omega_m(z)$



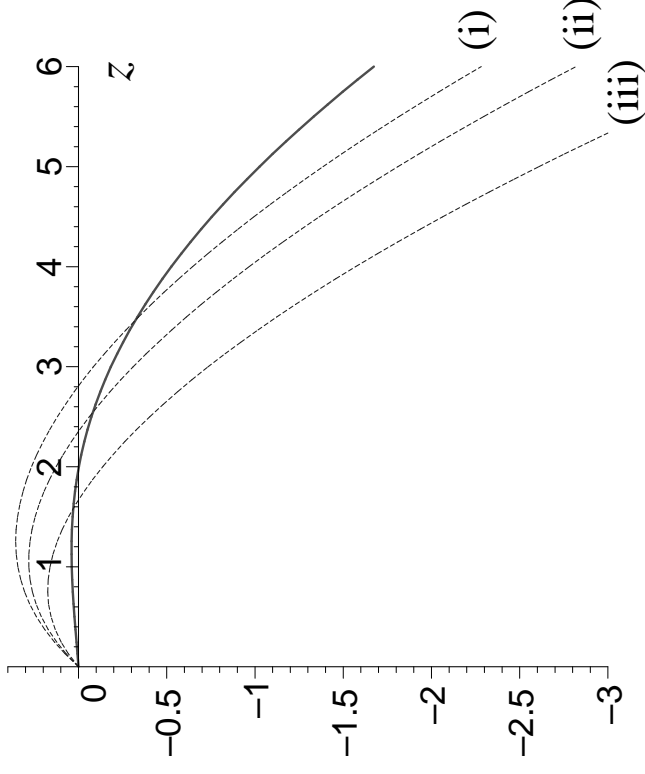
(i) $\Omega_m(z)$ fit by Shafieloo *et al.* to SN+BAO+CMB with $w(z) = -\frac{1}{2} [1 + \tanh((z - z_t) \Delta)]$;

(ii) TS model prediction for $\Omega_m(z)$ (NOT same $w(z)$) – best-fit and 1σ uncertainties

● Shafieloo *et al.*, arxiv:0903.5141, fit $\Omega_m(z)$ with hint that “dark energy is decaying” .

● Intercept $\Omega_m(0)$ agrees well with TS model expectation

Redshift time drift (Sandage–Loeb test)



$H_0^{-1} \frac{dz}{d\tau}$ for the timescape model with $f_{v0} = 0.762$ (solid line) is compared to three spatially flat Λ CDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda0})$ as in previous figures.

- Measurement is extremely challenging. May be feasible over a 10–20 year period by precision measurements of the Lyman- α forest over redshift $2 < z < 5$ with next generation of Extremely Large Telescopes

Clarkson, Bassett and Lu homogeneity test

- For FLRW equations, irrespective of dark energy model

$$\Omega_{k0} = \frac{\mathcal{B}}{[H_0 D(z)]^2} = \text{const}$$

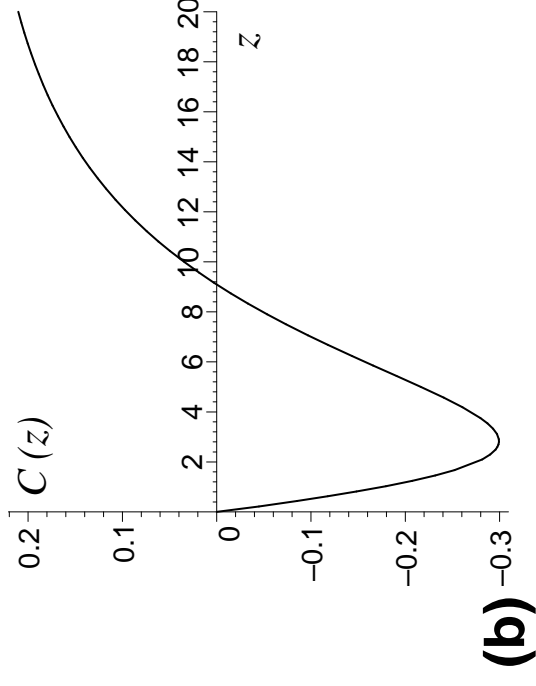
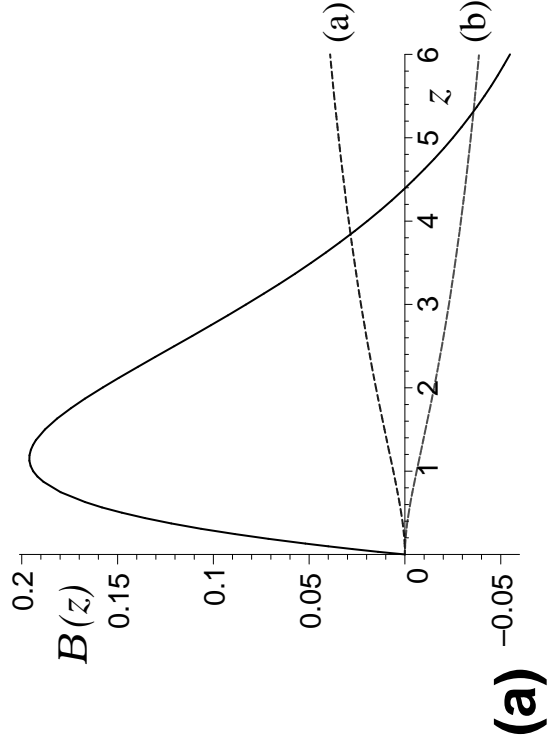
where $\mathcal{B}(z) \equiv [H(z)D'(z)]^2 - 1$. Thus

$$C(z) \equiv 1 + H^2(DD'' - D'^2) + HH'DD' = 0$$

for any homogeneous isotropic cosmology, irrespective of DE.

- Clarkson, Bassett and Lu [PRL 101 (2008) 011301] call this a “test of the Copernican principle”. However, it is really a test of Friedmann equation (cosmological principle).

Clarkson, Bassett and Lu homogeneity test



(a) $\mathcal{B} \equiv [H(z)D'(z)]^2 - 1$ for timescape model with $f_{v0} = 0.762$ (solid line) and two Λ CDM models (dashed lines): **(i)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda0} = 0.71$, $\Omega_{k0} = 0.01$; **(ii)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda0} = 0.73$, $\Omega_{k0} = -0.01$; **(b)** $\mathcal{C}(z)$.

- Will give a powerful test of FLRW assumption in future, with quantitatively different prediction for timescape model.

Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – *quasi-local gravitational energy*, of *gradients* in spatial curvature etc.
- “Timescape” model gives good fit to major independent tests of Λ CDM and may resolve significant puzzles – e.g., primordial lithium abundance anomaly
- Many tests can be done to distinguish from Λ CDM.
- It is crucial that Λ CDM assumptions such as Friedmann equation are not used in data reduction.
- Many details – averaging scheme etc – may change, but fundamental questions remain