Extremal Black Holes

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One of the successes of string theory has been an explanation of the Bekenstein-Hawking entropy of a class of supersymmetric black holes in terms of microscopic quantum states. Strominger, Vafa

$$S_{BH}(\vec{Q}) = \ln d_{micro}(\vec{Q})$$

$$S_{BH}(\vec{Q}) = A/4G_N$$

A= Area of event horizon of a black hole carrying a given set of charges $\vec{Q}$

G_N: Newton's constant

$d_{micro}(\vec{Q})$: degeneracy of microstates of charge $\vec{Q}$

$d_{\text{micro}}(\vec{Q})$ is usually calculated by considering a system of D-branes and other known objects in string theory carrying the same charges as the black hole, and then explicitly calculating the degeneracy of this system.

This calculation does not make any direct reference to black holes.

This formula is quite remarkable since it relates a geometric quantity in space-time to a counting problem.

However the Bekenstein-Hawking formula is an approximate formula that holds in classical general theory of relativity.

– works well only when the charges carried by the black hole are large and hence the curvature at the horizon is small.

In string theory there are stringy higher derivative corrections to the classical equations of motion of general relativity which modify the Bekenstein-Hawking formula.

These can be analyzed using a generalization of the Bekenstein-Hawking formula due to Wald.

There are also quantum corrections to the formula.

We would like to look for an exact formula for the black hole entropy taking into account stringy corrections and quantum corrections.

These are necessary if we want to compute the black hole entropy away from the large charge limit.

But this is only half the task.

Once we have found such a formula we would like to compare this with an exact formula for the microscopic entropy.

For this we shall need to find a precise expression – and not just the large charge limit – for $d_{micro}(\vec{Q})$.

In this talk I shall try to describe the progress on both these fronts.

We shall study these issues in the extremal, i.e. zero temperature limit.

Since in this limit the black holes cease to Hawking radiate, the notion of degeneracy is better defined for these black holes.

Often, but not always, these black holes preserve part of the supersymmetry, and hence are stable.

Our analysis on the black hole side will not directly make use of supersymmetry.

But while comparing with $\ln d_{micro}$ we shall work with supersymmetric examples.

Progess in microscopic counting

In a class of theories, known as N=4 supersymmetric string theories in four dimensions, one now has a complete understanding of the microscopic degeneracies of supersymmetric states.

Typically such theories have multiple gauge fields.

$\Rightarrow$ the black hole is characterized by multiple charges, collectively denoted by $\vec{Q}$.

The degeneracy is expressed as a function $d_{\text{micro}}(\vec{Q})$ of the charges.

Dijkgraaf, Verlinde, Verlinde; Cardoso, de Wit, Kapelli, Mohaupt; Shih, Strominger, Yin; Gaiotto; David, Jatkar, Sen; Dabholkar, Gaiotto, Nampuri; Cheng, Verlinde, $\cdots$

In these theories $d_{\text{micro}}(\vec{Q})$ is expressed as Fourier expansion coefficients of some well-known functions, *e.g.* 'inverse of Igusa cusp form'.

$\Rightarrow$ 'experimental data' to be explained by a 'theory of black holes'.

In the large charge limit these degeneracies agree with the exponential of the Bekenstein-Hawking entropy of black holes carrying the same set of charges

An example: Spectrum of a class of supersymmetric states in heterotic string theory compactified on a six dimensional torus.

charge^2	degeneracy $\mathbf{d_{micro}}$	ln $\mathbf{d_{micro}}$	$\mathbf{S_{BH}}$
2	50064	10.82	6.28
4	32861184	17.31	12.57
6	16193130552	23.51	18.85
8	7999169992704	29.71	25.13
10	4074192429737760	35.943	31.42

Note: In a systematic comparison we do not compare numbers, but compare the asymptotic expansions for large charges.

On the microscopic side we have a completely systematic algorithm for finding this asymptotic expansion for this special class of theories.

Our goal is to find a systematic approach to computing corrections to the black hole entropy.

AdS_2/CFT_1 correspondence will play a crucial role in this study.

What is AdS_2?

AdS_2 may be regarded as a two dimensional Lorentzian space embedded in a 3-dimensional space of signature (+,-,-) via the relation:

$$x^2 - y^2 - z^2 = -a^2$$

a: some constant giving the radius of AdS_2.

This space has an SO(2,1) isometry.

$$x^2 - y^2 - z^2 = -a^2$$

Introduce independent coordinates (η, t):

$$x = a \sinh\eta \cosh t, \quad y = a \cosh\eta, \quad z = a \sinh\eta \sinh t$$

$$dx^2 - dy^2 - dz^2 = a^2(d\eta^2 - \sinh^2\eta\, dt^2)$$

Define: $r = \cosh\eta$

$$ds^2 = a^2 \left[\frac{dr^2}{r^2 - 1} - (r^2 - 1)dt^2 \right], \quad r \geq 1$$

Why AdS_2?

All known black holes develop an AdS_2 factor in their near horizon geometry in the extremal limit.

– time translation symmetry gets enhanced to $SO(2,1)$ in the near horizon limit.

Reissner-Nordstrom solution in $D = 4$:

$$ds^2 = -(1-\rho_+/\rho)(1-\rho_-/\rho)d\tau^2 + \frac{d\rho^2}{(1-\rho_+/\rho)(1-\rho_-/\rho)} + \rho^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Define

$$2\lambda = \rho_+ - \rho_-, \quad t = \frac{\lambda\tau}{\rho_+^2}, \quad r = \frac{2\rho - \rho_+ - \rho_-}{2\lambda}$$

and take $\lambda \to 0$ limit keeping r, t fixed.

$$ds^2 = \rho_+^2\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right] + \rho_+^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$AdS_2 \qquad \times \qquad S^2$

Postulate: Any extremal black hole has an AdS_2 factor / $SO(2,1)$ isometry in the near horizon geometry.

– partially proved

Kunduri, Lucietti, Reall; Figueras, Kunduri, Lucietti, Rangamani

The presence of AdS_2 allows us to apply AdS_2/CFT_1 correspondence to this problem.

CFT_d: A d dimensional conformally invariant field theory.

CFT_1: A conformally invariant quantum mechanics.

Basic postulates of **AdS/CFT** correspondence (Euclidean version)

1. The boundary of euclidean AdS_{d+1} is a d-dimensional sphere S^d.

Given a string theory in euclidean $AdS_{d+1} \times K$ for some compact space K, there is an associated euclidean CFT_d on S^d such that

$$Z_{gravity} = Z_{CFT}$$

$Z_{gravity}$: result of path integral of the string theory on $AdS_{d+1} \times K$.

Z_{CFT}: result of the path integral over the CFT fields on S^d.

2. Often the $AdS_{d+1} \times K$ background arises as the near horizon geometry of an extremal black brane.

In this case there is a simple way to identify the dual CFT.

It is the theory obtained by taking the low energy limit of the quantum theory on the black brane.

We shall now apply this to AdS_2/CFT_1 correspondence.

Observations (special to AdS_2/CFT_1):

1. For all known black holes in string theory the microscopic system has an energy gap that separates the ground states from the first excited state.

For fixed values of charges $\vec{Q}$, only the ground states survive in the low energy limit.

Thus CFT_1 is a simple system with a finite dimensional Hilbert space, all states having zero energy, and the dimension of the Hilbert space given by $d_{micro}(\vec{Q})$.

2. For $d = 1$, S^d is a circle S^1 of some length L.

$$\Rightarrow \qquad Z_{CFT} = Tr(e^{-LH}) = d_{micro}$$

d_{micro}: ground state degeneracy

AdS_2/CFT_1 correspondence now implies:

$$Z_{gravity} = Z_{CFT} = d_{micro}$$

We declare $\ln Z_{gravity}$ to be the quantum generalization of the black hole entropy.

The equality between black hole entropy and $\ln d_{micro}$ now becomes a consequence of AdS_2/CFT_1 correspondence.

This gives the following prescription:

The exact degeneracy of an extremal black hole is given by the path integral of string theory over the near horizon $AdS_2 \times K$ geometry of the black hole.

Consistency check: In the classical limit this reduces to the exponential of the Wald entropy.

Given this exact formula for black hole entropy, we should be able to carry out systematic quantum corrections to the Wald's classical formula and compare these with the known microscopic results.

This is still underway.

However this formula also allows us to compute non-perturbative corrections to the black hole entropy by including new saddle points in the path integral whose asymptotic geometry coincide with that of $AdS_2 \times K$.

This has been used to explain some subtle dependence of $d_{micro}(\vec{Q})$ on the arithmetic properties of $\vec{Q}$, e.g. the gcd of certain components of $\vec{Q}$.

gcd: greatest common divisor

Summary

1. String theory offers the possibility of testing the correspondence between black hole entropy and microscopic degeneracies far beyond the leading order.

2. On the microscopic side we now have a complete understanding of the degeneracies for a class of states in a class of theories.

3. On the black hole side we have a complete expression for the degeneracy in terms path integral of string theory over the near horizon geometry.

4. Some checks have already been performed, but a complete comparison between the two sides is still underway.