An exact result for the behavior of Yang-Mills Green functions in the deep infrared region

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Based on

- K.-I. Kondo, Kugo-Ojima color confinement criterion and Gribov-Zwanziger horizon condition, arXiv:0904.4897[hep-th]. Phys.Lett.B. 678, 322-330 (2009).
- K.-I. Kondo, A nilpotent "BRST" symmetry for the Gribov-Zwanziger theory arXiv:0905.1899[hep-th]. Phys.Lett.B, submitted.
- K.-I. Kondo, in preparation.

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§ Introduction

We consider the quantum Yang-Mills theory (in the manifestly covariant gauge)

$$Z_{\rm YM} := \int [d\mathscr{A}]\delta(\partial\mathscr{A})\det(-\partial D[\mathscr{A}]))\exp\{-S_{YM}[\mathscr{A}]\}. \tag{1}$$

The gauge fixing condition $\partial\mathscr{A} = 0$ can not fix the gauge uniquely. Each gauge orbit intersects the gauge fixing hypersurface $\Gamma := \{\mathscr{A}; \partial\mathscr{A} = 0\}$ many times. There are Gribov copies.

Gribov (1978) proposed to restrict the functional integral to the (1st) Gribov region Ω to avoid the Gribov copies. (Note that $-\partial D[\mathscr{A} = 0] = -\partial\partial > 0$)

$$\Omega := \{\mathscr{A} \in \Gamma; -\partial D[\mathscr{A}] > 0\}, \quad \{\mathscr{A} = 0\} \in \Omega. \tag{2}$$

The boundary of Ω is called the Gribov horizon:

$$\partial\Omega := \{\mathscr{A} \in \Gamma; -\partial D[\mathscr{A}] = 0\}. \tag{3}$$

He predicted that as a result the gluon and ghost propagators exhibit unexpected behavior in the deep infrared (IR) region and that they play the essential role in confinement.

The gluon 2-point function (propagator)

$$D_{\mu\nu}^{AB}(k) = \delta^{AB}\left[\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) D_T(k^2) + \frac{\alpha}{k^2}\frac{k_\mu k_\nu}{k^2}\right]. \quad (4)$$

The ghost propagator

$$G^{AB}(k) = -\delta^{AB} G(k^2). \quad (5)$$

In the free case

$$D_T(k^2) = \frac{1}{k^2}, \quad G(k^2) = \frac{1}{k^2}. \quad (6)$$

The Gribov prediction in the IR region $k^2 \ll 1$

$$D_T(k^2) \cong \frac{k^2}{(k^2)^2 + M^4} \downarrow 0, \quad G(k^2) \cong \frac{M^2}{(k^2)^2} \uparrow \infty \quad (k^2 \downarrow 0). \quad (7)$$

The gluon propagator vanishes in the IR limit $k^2 \downarrow 0$, while the ghost propagator becomes more singular than the free case in the IR region. This power like behavior should be compared with the UV behavior with the logarithmic corrections. ...

Until 2006, it seemed that this prediction has been confirmed by numerical simulations on lattice, the scaling solution of the Schwinger-Dyson equation, and the functional renormalization group equation. ghost dominance

This result was considered to be reasonable to explain color confinement. Due to Kugo-Ojima (1977-1978), all color non-singlet objects can not be observed or confined, in other words, only color singlet objects are observed, if $u(0) = -1$ in the Lorentz covariant gauge. This is a sufficient condition for color confinement.

In the Landau gague, Kugo-Ojima criterion for color confinement $u(0) = -1$ is equivalent to the divergent ghost dressing function $F(0) = \infty$, since $F(k) = [1 + u(k^2)]^{-1}$ in the Landau gauge. $F(k^2) := k^2 G(k^2)$ or $G(k^2) = F(k^2)/k^2$

[Note that the Kugo-Ojima theory is based on the usual BRST formulation and does not take into account the Gribov problem.]

However, ...

These results are questioned by the Orsay group from 2007. By careful analyses of the Schwinger-Dyson equation, socalled the decoupling solution was discovered and it was found that the ghost dressing function must be finite.

After these works, reexaminations of numerical simulations on lattices, functional renormalization group equation seem to converge the result:

The gluon propagator goes to the non-zero finite constant in the IR limit, while the ghost propagator behaves like free (i.e., the ghost dressing function is non-zero finite).

§ Main results of this talk

Within the Gribov-Zwanziger theory for the D-dimensional SU(N) Yang-Mills theory in the Landau gauge, I consider how the restriction of the integration region to the (1st) Gribov region constrains the possible value for the Kugo-Ojima parameter for color confinement and the ghost dressing function.

(1) I prove that the ghost dressing function $F(k^2)$ is non-zero finite in the limit $k \to 0$ and hence the ghost propagator behaves like free in the deep infrared regime.

With an input, $F(k^2) \to 3$ in the limit $k \to 0$ irrespective of the number of color.

$\Longrightarrow$ The Kugo-Ojima color confinement criterion u(0)=-1 is not satisfied in a naive form. Rather, I find the exact value for the KO parameter

u(0)=-2/3 irrespective of the number of color.

(2) I have found that a nilpotent "BRST" like symmetry exists in the Gribov-Zwanziger theory (restricted to the 1st Gribov region). $\boldsymbol{\delta} S_{\rm GZ} = \boldsymbol{\delta} \tilde{S}_\gamma \neq 0$
This is important to look for a modified color confinement criterion a la Kugo-Ojima.

These results are in harmony with recent numerical simulation results on huge lattices and decoupling solution of the Schwinger-Dyson equation and the functional renormalization group equation.

§ Gribov-Zwanziger theory and horizon condition

- Gribov-Zwanziger theory (in Euclidean space)

$$Z_\gamma := \int \mathcal{D}\mathscr{A}\,\delta(\partial^\mu \mathscr{A}_\mu) \det M \exp\{-S_{YM} + \gamma \int d^D x h(x)\}, \tag{1}$$

where S_{YM} is the Yang-Mills action, M is the Faddeev-Popov operator $M := -\partial_\mu D_\mu = -\partial_\mu(\partial_\mu + g\mathscr{A}_\mu \times)$ and $h(x) = h[\mathscr{A}](x)$ is the Zwanziger horizon function given by

$$h(x) := -\int d^D y g f^{ABC} \mathscr{A}_\mu^B(x)(M^{-1})^{CE}(x,y) g f^{AFE} \mathscr{A}_\mu^F(y). \tag{2}$$

Here the parameter γ called the Gribov parameter is determined by solving a gap equation, commonly called the horizon condition:

$$\langle h(x) \rangle^\gamma = (N^2 - 1)D. \tag{3}$$

The action corresponding to the partition function (1) contains the *non-local* horizon term:

$$\int d^D x h(x) := -\int d^D x \int d^D y g f^{ABC} \mathscr{A}_\mu^B(x)(M^{-1})^{CE}(x,y) g f^{AFE} \mathscr{A}_\mu^F(y). \tag{4}$$

§ A local Lagrangian form for Gribov-Zwanziger theory

$$e^{\gamma \int d^D x h(x)} = \int [d\xi][d\bar{\xi}][d\omega][d\bar{\omega}] \exp\left\{ -\tilde{S}_\gamma[\mathscr{A}, \xi, \bar{\xi}, \omega, \bar{\omega}] \right\}, \tag{1}$$

where

$$\begin{aligned} \tilde{S}_\gamma =: \int d^D x [& \bar{\xi}_\mu^{CA} K^{AB} \xi_\mu^{CB} - \bar{\omega}_\mu^{CA} K^{AB} \omega_\mu^{CB} \\ & + i\gamma^{1/2} g f^{ABC} \mathscr{A}_\mu^B \xi_\mu^{AC} + i\gamma^{1/2} g f^{ABC} \mathscr{A}_\mu^B \bar{\xi}_\mu^{AC}]. \end{aligned} \tag{2}$$

The localized action S_{GZ} for the Gribov-Zwanziger theory is obtained

$$\begin{aligned} S_{\mathrm{GZ}} = & S_{\mathrm{YM}}^{\mathrm{tot}}[\mathscr{A}, \mathscr{C}, \bar{\mathscr{C}}, \mathscr{B}] + \tilde{S}_\gamma[\mathscr{A}, \xi, \bar{\xi}, \omega, \bar{\omega}] \\ = & S_{\mathrm{YM}}[\mathscr{A}] + S_{\mathrm{GF+FP}}[\mathscr{A}, \mathscr{C}, \bar{\mathscr{C}}, \mathscr{B}] + \tilde{S}_\gamma[\mathscr{A}, \xi, \bar{\xi}, \omega, \bar{\omega}], \end{aligned} \tag{3}$$

where

$$\begin{aligned} \mathscr{L}_{\mathrm{GF+FP}} := & \int d^D x \left\{ \mathscr{B} \cdot \partial_\mu \mathscr{A}_\mu + \frac{\alpha}{2} \mathscr{B} \cdot \mathscr{B} + i\bar{\mathscr{C}} \cdot \partial_\mu D_\mu \mathscr{C} \right\} \\ = & - i\boldsymbol{\delta} \left[\bar{\mathscr{C}} \cdot \left(\partial_\mu \mathscr{A}_\mu + \frac{\alpha}{2} \mathscr{B} \right) \right] \quad (\alpha = 0). \end{aligned} \tag{4}$$

§ Horizon condition and ghost dressing function

I have shown a key identity that the average of the horizon function is written as

$$\begin{aligned}\langle h(0)\rangle =& V_D^{-1}\int d^Dx\langle h(x)\rangle = -\lim_{k^2\to 0}\langle (g\mathscr{A}_\mu \times \mathscr{C})^A (g\mathscr{A}_\mu \times \bar{\mathscr{C}})^A\rangle_k \\ =& -(N^2-1)\left\{Du(0)+w(0)-F(0)[u(0)+w(0)]^2\right\}, \end{aligned} \quad (1)$$

where $u(0)$ is the Kugo-Ojima parameter defined by the $k^2 \to 0$ limit of $u(k^2)$:

$$\langle (D_\mu \mathscr{C})^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B\rangle_k := \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\delta^{AB} u(k^2), \quad (2)$$

and $w(0)$ is the massless pole residue in

$$\langle (g\mathscr{A}_\mu \times \mathscr{C})^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B\rangle_k^{m1PI} = \left[g_{\mu\nu}u(k^2) + \frac{k_\mu k_\nu}{k^2} w(k^2)\right]\delta^{AB}. \quad (3)$$

The ghost dressing function in the Landau gauge satisfies

$$F(k^2)\delta^{AB} := -k^2\langle \mathscr{C}^A \bar{\mathscr{C}}^B\rangle_k = [1+u(k^2)+w(k^2)]^{-1}\delta^{AB}. \quad (4)$$

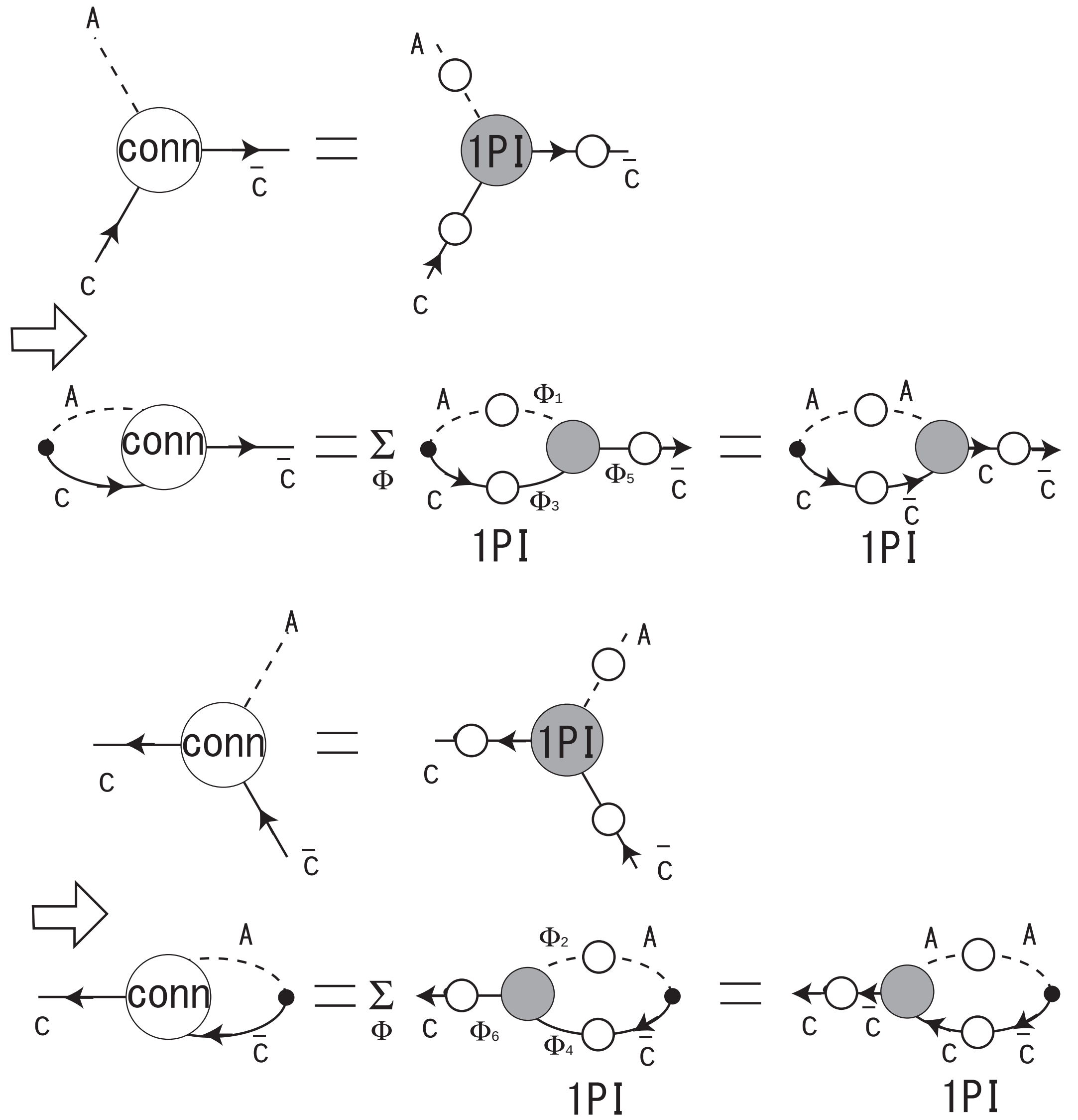

Figure 1: Diagrammatic representation of (a) $\langle (g\mathscr{A}_\mu \times \mathscr{C})^A \bar{\mathscr{C}}^B \rangle_k$ and $\langle (g\mathscr{A}_\mu \times \mathscr{C})^A \bar{\mathscr{C}}^B \rangle_k^{1\mathrm{PI}}$, (b) $\langle \mathscr{C}^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B \rangle_k$ and $\langle \mathscr{C}^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B \rangle_k^{1\mathrm{PI}}$.

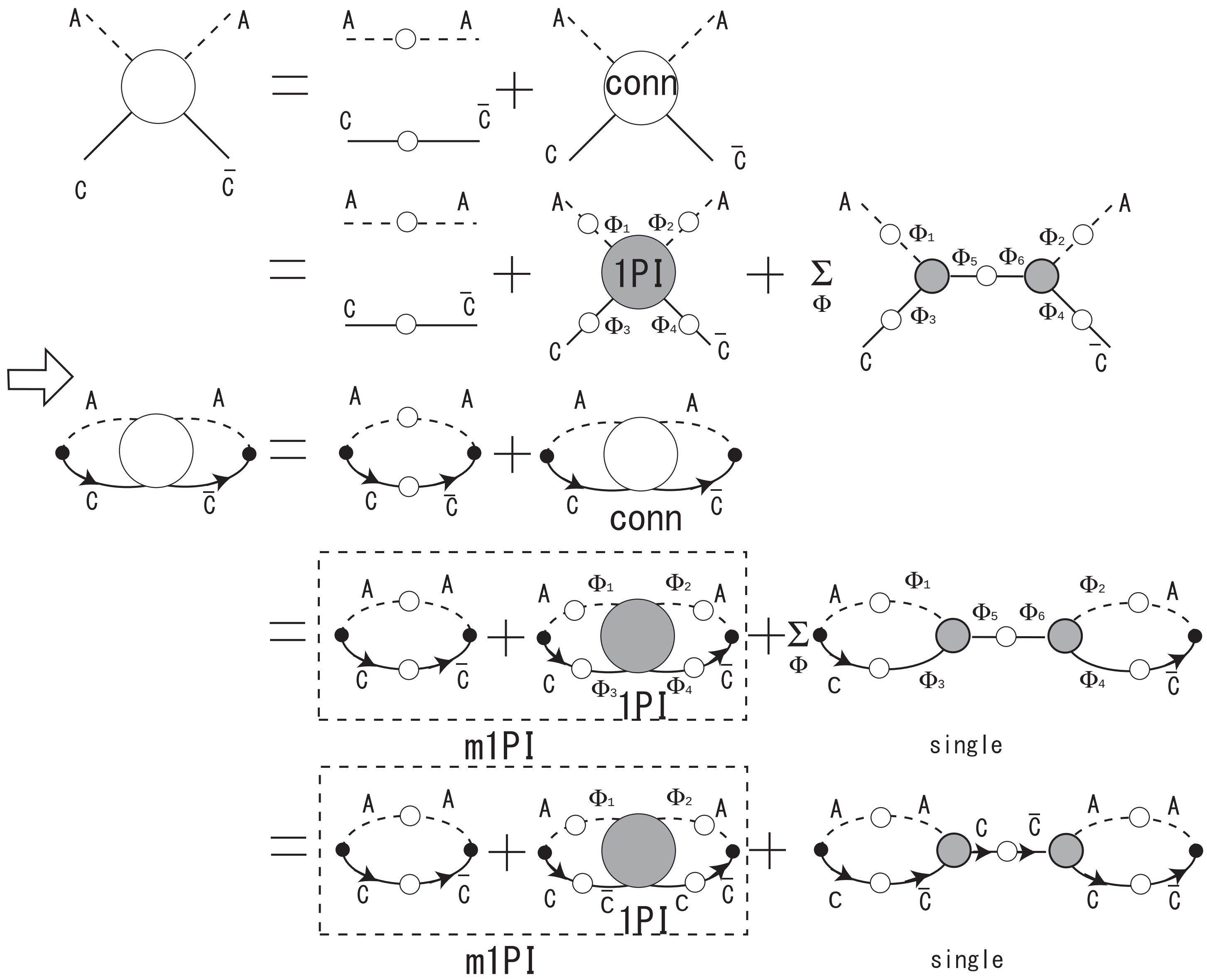

Figure 2: Diagrammatic representation of $\langle (g\mathscr{A}_\mu \times \mathscr{C})^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B \rangle_k$, $\langle (g\mathscr{A}_\mu \times \mathscr{C})^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B \rangle_k^{\rm conn}$, $\langle (g\mathscr{A}_\mu \times \mathscr{C})^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B \rangle_k^{\rm 1PI}$ and $\langle (g\mathscr{A}_\mu \times \mathscr{C})^A (g\mathscr{A}_\nu \times \bar{\mathscr{C}})^B \rangle_k^{\rm m1PI}$.

First, we consider the case of $w(0) = 0$.

- a new relationship $\langle h(0) \rangle = (N^2 - 1)\left\{ -Du(0) + \frac{u(0)^2}{1 + u(0)} \right\},$ (5)
- the horizon condition $\langle h(0) \rangle = (N^2 - 1)D,$ (6)

$$\rightarrow (D - 1)u(0)^2 + 2Du(0) + D = 0, \quad u(0) = (-D \pm \sqrt{D})/(D - 1) \tag{7}$$

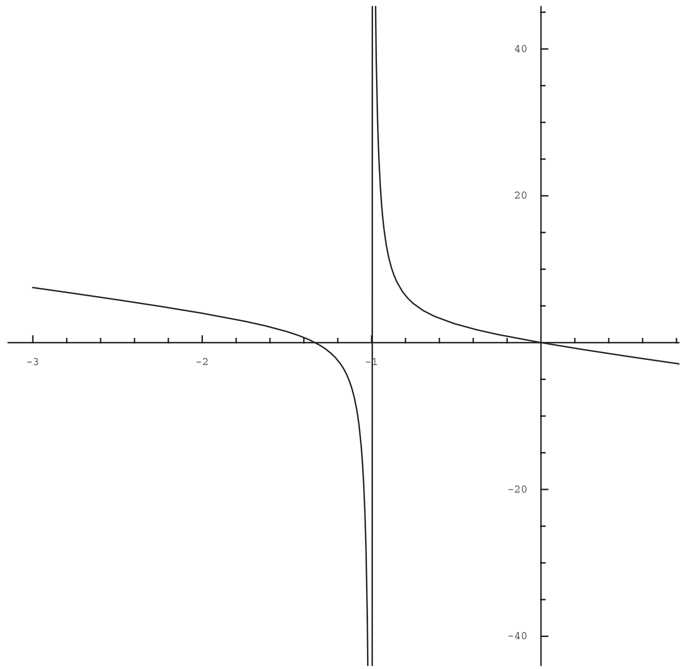

Figure 3: The plot of $\langle h(0) \rangle$ versus $u(0)$ for $D = 4$.

For $D = 4$, $3u(0)^2 + 8u(0) + 4 = 0$ has solutions $u(0) = -2/3, -2$. We obtain irrespective of the number of color N

$$u(0) = -\frac{2}{3}, \quad F(0) = [1 + u(0)]^{-1} = 3. \tag{8}$$

The ghost dressing function $F(k^2)$ is finite even in the deep infrared limit $k^2 \to 0$.

This differs from the Gribov result. Is it possible to reconcile this result with the old Gribov result? Yes. See the following.

Recall that the Gribov result was obtained by taking into account the $O(g^2)$ terms.

The formal power series expansion in $u(0)$ yields the horizon condition

$$\langle h(0) \rangle = (N^2 - 1) \left\{ -Du(0) + u(0)^2 - u(0)^3 + \cdots \right\} = (N^2 - 1)D. \tag{9}$$

If we took into account only a linear term in $u(0) = O(g^2)$ on the left-hand side, then the horizon condition would lead to the Kugo-Ojima criterion $u(0) = -1$ and the ghost dressing function $F(0) = [1 + u(0)]^{-1}$ would diverge.

In this way we can reproduce the Gribov approximate (wrong) result.

- Second, we consider the (unlikely) case of $w(0) \neq 0$. [See numerical results]

The horizon condition alone is not sufficient to determine both $u(0)$ and $w(0)$. Suppose the Kugo-Ojima confinement criterion is satisfied $u(0) = -1$. Then the horizon condition is

$$\langle h(0) \rangle^{\alpha=0} = -(N^2-1)\left\{-D+1+\frac{-1+w(0)}{w(0)}\right\} \cong (N^2-1)D. \tag{10}$$

This leads to the value of $w(0)$ irrespective of the spacetime dimension D and the number of color N:

$$w(0) = 1/2 \quad \text{for any} \quad D. \tag{11}$$

Even if $u(0) = -1$, therefore, the ghost propagator behaves like free $1/k^2$ at $k = 0$, no more singular than $1/k^2$: irrespective of the spacetime dimension D and the number of color N

$$\lim_{k^2 \to 0} [-k^2 \langle \mathscr{C}^A \bar{\mathscr{C}}^B \rangle_k]^{-1} = \delta^{AB} w(0) = \frac{1}{2}\delta^{AB} \neq 0, \quad F(0) = 2. \tag{12}$$

Thus, the ghost propagator behaves like free at low momenta, while the gluon propagator is non-vanishing at low momenta The original Gribov prediction is wrong?