A pure geometric Model For A Main Sequence Star

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1- **Introduction**

*All the parameters of the model are geometric objects.

*No phenomenological matter tensor

*No equation of state imposed

*Physical meanings are attributed to different geometric objects.

2. Underlying Geometry:

AP-structure of Riemann-Cartan type

(1)

$$\underset{i}{\lambda}_{\mu}{}_{+|\nu} = 0 \quad \Rightarrow \quad \underset{i}{\lambda}^{\mu}\underset{i}{\lambda}_{\nu} = \delta^{\mu}_{\nu}.$$

$$\Gamma^{\alpha}_{.\,\mu\nu} = \underset{i}{\lambda}^{\alpha}\underset{i}{\lambda}_{\mu,\nu}.$$

$$g_{\mu\nu} \stackrel{\text{def}}{=} \underset{i}{\lambda}_{\mu}\underset{i}{\lambda}_{\nu}, \quad g^{\mu\nu} \stackrel{\text{def}}{=} \underset{i}{\lambda}^{\mu}\underset{i}{\lambda}^{\nu} \quad \Rightarrow \quad g^{\mu\alpha}g_{\alpha\nu} = \delta^{\mu}_{\nu}.$$

$$\left\{ {\alpha \atop \mu\nu} \right\} \quad \Rightarrow \quad g_{\mu\nu;\delta} = 0$$

(1) Wanas, M.I. (2001), Stud. Cercet. Stiin. Ser. Mat.Univ.Bacau **10**, 297-309; gr-qc/0209050

- $$\Gamma^{\alpha}_{\cdot\ \mu\nu} \quad , \quad \tilde{\Gamma}^{\alpha}_{\cdot\ \mu\nu} (\stackrel{\text{def}}{=} \Gamma^{\alpha}_{\cdot\ \nu\mu}) \quad , \quad \Gamma^{\alpha}_{\cdot\ (\mu\nu)} \ , \left\{ {\alpha \atop \mu\nu} \right\}$$
- 4-Linear C $$\Lambda^{\alpha}_{\cdot\ \mu\nu} \stackrel{\text{def}}{=} \Gamma^{\alpha}_{\cdot\ \mu\nu} - \Gamma^{\alpha}_{\cdot\ \nu\mu} = \tilde{\Gamma}^{\alpha}_{\cdot\ \nu\mu} - \tilde{\Gamma}^{\alpha}_{\cdot\ \mu\nu}$$
-

$$M^{\alpha}_{\cdot\ \mu\nu\sigma} \stackrel{\text{def}}{=} \Gamma^{\alpha}_{\cdot\ \mu\sigma,\nu} - \Gamma^{\alpha}_{\cdot\ \mu\nu,\sigma} + \Gamma^{\epsilon}_{\cdot\ \mu\sigma}\Gamma^{\alpha}_{\cdot\ \epsilon\nu} - \Gamma^{\epsilon}_{\cdot\ \mu\nu}\Gamma^{\alpha}_{\cdot\ \epsilon\sigma},$$

$$\tilde{M}^{\alpha}_{\cdot\ \mu\nu\sigma} \stackrel{\text{def}}{=} \tilde{\Gamma}^{\alpha}_{\cdot\ \mu\sigma,\nu} - \tilde{\Gamma}^{\alpha}_{\cdot\ \mu\nu,\sigma} + \tilde{\Gamma}^{\epsilon}_{\cdot\ \mu\sigma}\tilde{\Gamma}^{\alpha}_{\cdot\ \epsilon\nu} - \tilde{\Gamma}^{\epsilon}_{\cdot\ \mu\nu}\tilde{\Gamma}^{\alpha}_{\cdot\ \epsilon\sigma},$$

$$\bar{M}^{\alpha}_{\cdot\ \mu\nu\sigma} \stackrel{\text{def}}{=} \Gamma^{\alpha}_{\cdot\ (\mu\sigma),\nu} - \Gamma^{\alpha}_{\cdot\ (\mu\nu),\sigma} + \Gamma^{\epsilon}_{\cdot\ (\mu\sigma)}\Gamma^{\alpha}_{\cdot\ (\epsilon\nu)} - \Gamma^{\epsilon}_{\cdot\ (\mu\nu)}\Gamma^{\alpha}_{\cdot\ (\epsilon\sigma)}.$$

$$R^{\alpha}_{\cdot\ \mu\nu\sigma} \stackrel{\text{def}}{=} \left\{ {\alpha \atop \mu\sigma} \right\}_{,\nu} - \left\{ {\alpha \atop \mu\nu} \right\}_{,\sigma} + \left\{ {\epsilon \atop \mu\sigma} \right\}\left\{ {\alpha \atop \epsilon\nu} \right\} - \left\{ {\epsilon \atop \mu\nu} \right\}\left\{ {\alpha \atop \epsilon\sigma} \right\}$$

- Contorsion $$\gamma^{\alpha}_{\cdot\ \mu\nu} \stackrel{\text{def}}{=} \underset{i}{\lambda}^{\alpha}\ \underset{i}{\lambda}_{\mu;\nu}$$
- Basic Vector $$C_{\mu} \stackrel{\text{def}}{=} \Lambda^{\alpha}_{\cdot\ \mu\alpha} = \gamma^{\alpha}_{\cdot\ \mu\alpha}$$

$$\gamma_{\mu\alpha\nu} = \frac{1}{2}(\Lambda_{\mu\alpha\nu} - \Lambda_{\nu\mu\alpha} - \Lambda_{\alpha\mu\nu})$$

$$\Delta^{\alpha}_{\cdot\ \mu\nu} \stackrel{\text{def}}{=} \gamma^{\alpha}_{\cdot\ \mu\nu} + \gamma^{\alpha}_{\cdot\ \nu\mu}.$$

$$\gamma^{\alpha}_{\cdot\ (\mu\nu)} = \frac{1}{2}\Delta^{\alpha}_{\cdot\ \mu\nu}, \quad \gamma^{\alpha}_{\cdot\ [\mu\nu]} = \Gamma^{\alpha}_{\cdot\ [\mu\nu]} = \frac{1}{2}\Lambda^{\alpha}_{\cdot\ \mu\nu}$$

2^{nd} Order Tensors

Skew-Symmetric Tensors	Symmetric Tensors
$\xi_{\mu\nu} \overset{\text{def}}{=} \gamma^{\ \ \ \alpha}_{\mu\nu.\mid\overset{\alpha}{+}}$ $\zeta_{\mu\nu} \overset{\text{def}}{=} C_{\alpha}\ \gamma^{\ \ \ \alpha}_{\mu\nu.}$	
$\eta_{\mu\nu} \overset{\text{def}}{=} C_{\alpha}\ \Lambda^{\alpha}_{.\mu\nu}$ $\chi_{\mu\nu} \overset{\text{def}}{=} \Lambda^{\alpha}_{.\mu\nu\mid\overset{\alpha}{+}}$ $\varepsilon_{\mu\nu} \overset{\text{def}}{=} C_{\mu\mid\overset{\nu}{+}} - C_{\nu\mid\overset{\mu}{+}}$ $\kappa_{\mu\nu} \overset{\text{def}}{=} \gamma^{\alpha}_{.\mu\epsilon}\gamma^{\epsilon}_{.\alpha\nu} - \gamma^{\alpha}_{.\nu\epsilon}\gamma^{\epsilon}_{.\alpha\mu}$	$\phi_{\mu\nu} \overset{\text{def}}{=} C_{\alpha}\ \Delta^{\alpha}_{.\mu\nu}$ $\psi_{\mu\nu} \overset{\text{def}}{=} \Delta^{\alpha}_{.\mu\nu\mid\overset{\alpha}{+}}$ $\theta_{\mu\nu} \overset{\text{def}}{=} C_{\mu\mid\overset{\nu}{+}} + C_{\nu\mid\overset{\mu}{+}}$ $\varpi_{\mu\nu} \overset{\text{def}}{=} \gamma^{\alpha}_{.\mu\epsilon}\gamma^{\epsilon}_{.\alpha\nu} + \gamma^{\alpha}_{.\nu\epsilon}\gamma^{\epsilon}_{.\alpha\mu}$
	$\omega_{\mu\nu} \overset{\text{def}}{=} \gamma^{\epsilon}_{.\mu\alpha}\gamma^{\alpha}_{.\nu\epsilon}$ $\sigma_{\mu\nu} \overset{\text{def}}{=} \gamma^{\epsilon}_{.\alpha\mu}\gamma^{\alpha}_{.\epsilon\nu}$ $\alpha_{\mu\nu} \overset{\text{def}}{=} C_{\mu}C_{\nu}$ $R_{\mu\nu} \overset{\text{def}}{=} \frac{1}{2}(\psi_{\mu\nu} - \phi_{\mu\nu} - \theta_{\mu\nu}) + \omega_{\mu\nu}$

(2) Mikhail, F.I. (1962) Ain Shams Sci. Bul. **6**, 87-111

3-Field Theory (3)

$$\lambda^* \qquad \tilde{M}_{\mu\nu} = \lambda^* g^{\mu\nu} \tilde{M}^{\alpha}_{.\ \mu\nu\alpha}$$

Field equations $\quad S_{\nu\sigma} = 0,$

$$S_{\nu\sigma} \stackrel{\text{def}}{=} -2G_{\nu\sigma} + N g_{\nu\sigma} - 2N_{\nu\sigma} + 2\gamma^{\gamma}_{.\ \nu\sigma|\overset{\gamma}{+}} + 2\gamma^{\epsilon}_{.\ \nu\mu}\gamma^{\mu}_{.\ \sigma\epsilon}$$
$$+\gamma^{\alpha}_{.\ \nu\gamma}\gamma^{\gamma}_{.\ \alpha\sigma} + \gamma^{\epsilon}_{.\ \mu\nu}\gamma^{\mu}_{.\ \sigma\epsilon} - 2C_{\alpha}\gamma^{\alpha}_{.\ \nu\sigma} = 0,$$

symmetric part $\quad S_{(\nu\sigma)} = 0, \rightarrow R_{\nu\sigma} - \frac{1}{2} g_{\nu\sigma} R = T^*_{\nu\sigma},$

$$T^*_{\nu\sigma} \stackrel{\text{def}}{=} \frac{1}{2}\psi_{\nu\sigma} - \frac{1}{2}\varphi_{\nu\sigma} + \omega_{\nu\sigma} - \frac{1}{2} g_{\nu\sigma}\omega.$$

Skew part $\quad S_{[\nu\sigma]} = 0, \rightarrow \epsilon_{\nu\sigma} = 0$

(3) Wanas, M.I. and Ammar, S.A. (2005), submitted for publication;gr-qc/0505092] **v1**.

4- Geometric Structure With Spherical Symmetry (4) :

$$\lambda_{i}^{\ \mu} = \begin{pmatrix} A & Dr & 0 & 0 \\ 0 & B\sin\theta\cos\varphi & \frac{B}{r}\cos\theta\cos\varphi & \frac{-B\sin\varphi}{r\sin\theta} \\ 0 & B\sin\theta\sin\varphi & \frac{B}{r}\cos\theta\sin\varphi & \frac{B\cos\varphi}{r\sin\theta} \\ 0 & B\cos\theta & \frac{-B}{r}\sin\theta & 0 \end{pmatrix},$$

A $\equiv$ A(r) , $\equiv$ B(r)

(4) Robertson, H.p. (1932), Ann. Math., Princeton (2), **33**,496-520.

5-Solution of the field equations

$$4\frac{B''}{B} - 4\frac{B'^2}{B^2} + 8\frac{B'}{Br} - 2\frac{A'B'}{AB} + 2\frac{A''}{A} - 3\frac{A'^2}{A^2} + 4\frac{A'}{Ar} = 0,$$

$$-4\frac{B'^2}{B^2} + 8\frac{B'}{Br} - 4\frac{B'A'}{BA} - \frac{A'^2}{A^2} + 4\frac{A'}{Ar} = 0,$$

$$4\frac{B''}{B} - 4\frac{B'^2}{B^2} + 4\frac{B'}{Br} - 2\frac{A'B'}{AB} + 2\frac{A''}{A} - 3\frac{A'^2}{A^2} + 2\frac{A'}{Ar} = 0.$$

$$\frac{A'}{A} = -2\frac{B'}{B}, \qquad A = \frac{C^*}{B^2}$$

5.1 Fixing the unkown functions:

$$T^{*1}_{\cdot\ 1} = B^2 \left[3\frac{B'^2}{B^2} - 2\frac{B'}{Br} \right],$$

$$T^{*2}_{\cdot 2} = B^2 \left[-\frac{B''}{B} - \frac{B'}{Br} - 3\frac{B'^2}{B^2} \right],$$

$$T^{*3}_{\cdot\ 3} = T^{*2}_{\cdot\ 2}$$

$$T^{*0}_{\cdot\ 0} = B^2 \left[-3\frac{B'^2}{B^2} + 2\frac{B''}{B} + 4\frac{B'}{Br} \right]$$

Assuming that $$T^{*1}_{\cdot\ 1} = T^{*2}_{\cdot\ 2} = T^{*3}_{\cdot\ 3}$$

$$A = \frac{C^*}{(7C\frac{r^2}{2} + 7C_1)^{\frac{2}{7}}}, \quad B = (7C\frac{r^2}{2} + 7C_1)^{\frac{1}{7}}$$

5.2 The metric of the associated Riemann-space:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$ds^2 = \frac{1}{(C^*)^2}\left(7C\frac{r^2}{2} + 7C_1\right)^{\frac{4}{7}} dt^2 + \frac{1}{\left(7C\frac{r^2}{2} + 7C_1\right)^{\frac{2}{7}}}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

Taking $(C^*)^2 = -C_2 \;,\;\; d\tau = i ds$

$$d\tau^2 = \frac{1}{C_2}\left(7C\frac{r^2}{2} + 7C_1\right)^{\frac{4}{7}} dt^2 - \frac{1}{\left(7C\frac{r^2}{2} + 7C_1\right)^{\frac{2}{7}}}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

5.3 Boundary conditions:

Geometric density $$\rho^*{}_0(r) \stackrel{\text{def}}{=} -T^{*0}{}_{.\,0}$$

Geometric pressure $$p^*{}_0(r) \stackrel{\text{def}}{=} T^{*1}{}_{.\,1} = T^{*2}{}_{.\,2} = T^{*3}{}_{.\,3}.$$

$$p_0^*(r) = 0 \text{ at } r = a \qquad C = -\frac{7C_1}{2a^2}$$

Schwartzchild exterior solution

$$d\tau^2 = \frac{(1-\frac{m}{2r})^2}{(1+\frac{m}{2r})^2}dt^2 - (1+\frac{m}{2r})^4(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2),$$

$$7C_1 = (-\frac{4}{3})\frac{1}{(1+\frac{m}{2a})^{14}}\ , \qquad C_2 = \frac{1}{(1-\frac{m}{2a})^2(1+\frac{m}{2a})^6}\ .$$

6-Sumary

*Gravitational potential

$$A^{-2} = b_1\left(1 - \frac{7}{4a^2}r^2\right)^{\frac{4}{7}}$$

$$B^{-2} = \frac{b_2}{\left(1 - \frac{7r^2}{4a^2}\right)^{\frac{2}{7}}}$$

*Geometric pressure

$$p_0^*(r) = \frac{1}{a^2 b_2}\frac{\left(1 - \frac{r^2}{a^2}\right)}{\left(1 - \frac{7}{4a^2}r^2\right)^{\frac{12}{7}}},$$

*geometric density

$$\rho_0^*(r) = 3\,\frac{1}{a^2 b_2}\frac{\left(1 - \frac{r^2}{2a^2}\right)}{\left(1 - \frac{7}{4a^2}r^2\right)^{\frac{12}{7}}}.$$

*parameters

$$b_1 \stackrel{\text{def}}{=} \left(\frac{-4}{3}\right)^{\frac{4}{7}}\frac{\left(1 - \frac{m}{2a}\right)^2}{\left(1 + \frac{m}{2a}\right)^2}, \qquad b_2 \stackrel{\text{def}}{=} \frac{\left(1 + \frac{m}{2a}\right)^4}{\left(\frac{-4}{3}\right)^{\frac{2}{7}}}.$$

7- Discussion

Corona at $r = a$

$$p_0^* = 0$$

$$\rho_0^* \; (\quad 0 , \quad \rho_0^* = 0 \quad \text{At} \quad r = \sqrt{2}\; a$$

Radiation Zone

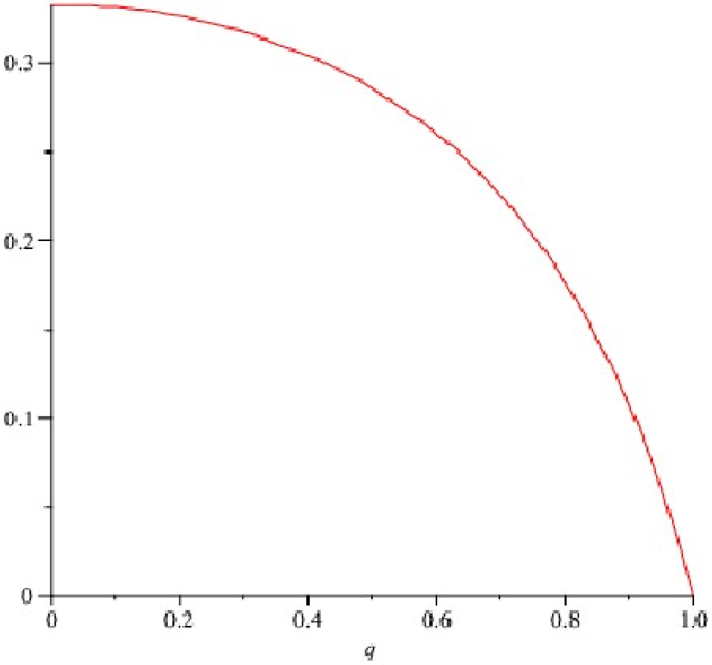
$$r \qquad 0$$

$$p_0^* Ł \; \frac{1}{3}\rho_0^*$$

General equation of state

$$q \stackrel{\text{def}}{=} \frac{r}{a} \; .$$

$$p_0^* = \frac{(1-q^2)}{3(1-\frac{q^2}{2})} \quad \rho_0^* \; .$$

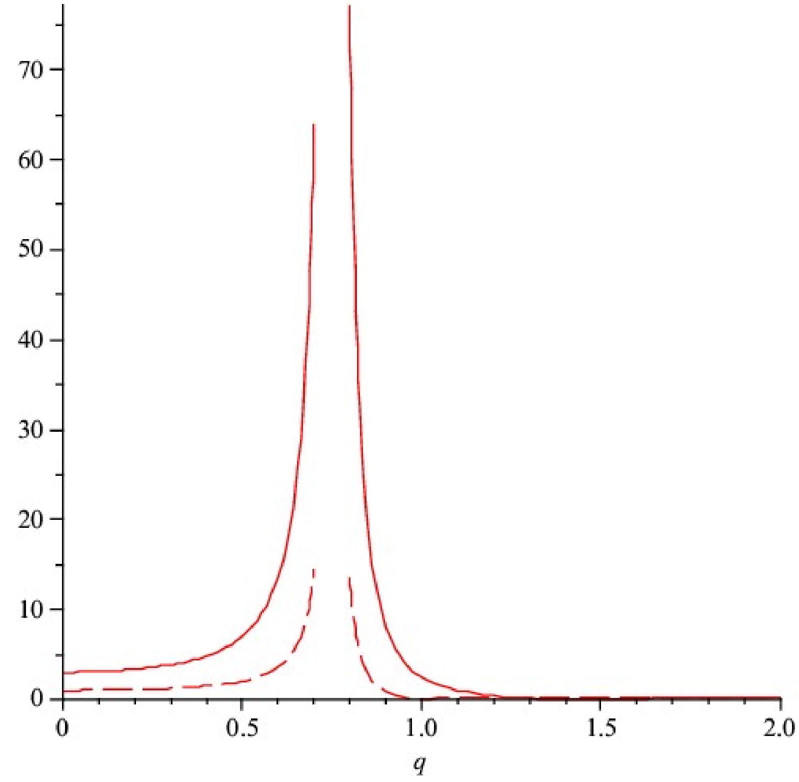


Singularity and Convection zone

$$A = \frac{const.}{(1 - \frac{7}{4}q^2)^{\frac{2}{7}}} \ ,$$

$$p_0^*(r) = const.\frac{(1 - q^2)}{(1 - \frac{7}{4}q^2)^{\frac{12}{7}}} \ ,$$

$$\rho_0^*(r) = const.\frac{(1 - \frac{q^2}{2})}{(1 - \frac{7}{4}q^2)^{\frac{12}{7}}} \ .$$



Thank You