Static Isotropic Spacetimes with Radially Imperfect Fluids

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Talk prepared for:

12th Marcel Grossmann Meeting
Paris
July 13, 2009

Based on: TK, arXiv:09.soon [gr-qc]

Outline

Background

Sources for static isotropic spacetimes.

A concrete example: modified Schwarzschild spacetime.

Phenomenology (connection with Dark Matter?).

Conclusion.

The equations of motion of General Relativity are

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

The left hand side is a function of the metric.

The right hand side is a function of matter fields.

Most often, the matter sources are specified, and the metric is solved for:

- Minkowski spacetime (zero source)
- Schwarzschild spacetime (zero source)
- (anti-)de Sitter spacetime (spacetime constant source)
- FRW cosmological models (zero and perfect fluid sources)

A different approach is to first specify a symmetry, and then solve.

Radially Imperfect Fluids

Any static isotropic spacetime has a line-element of the form

$$ds^2 = -f^2\,dt^2 + h^{-1}\,dr^2 + r^2\,d\Omega^2$$

The most general stress energy tensor consistent with the symmetry is

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p\,g_{\mu\nu} + qV_\mu V_\nu$$

This is a perfect fluid plus an additional component, the radial imperfect fluid component. (Also can be described in terms of viscous shear.)

Apart from the symmetry argument, explicit calculations of the stress-energy tensor of a collection of field modes of a free massless scalar field in a static isotropic spacetime also yield the form above.

D. Cox, P. D. Mannheim and J. O'Brien, arXiv:0903.4381 [hep-th]
P. D. Mannheim, arXiv:astro-ph/0505266

Equations of Motion

With the radial imperfect fluid component, the equations of motion become

$$G_{tt} = \rho f^2,$$
$$G_{rr} = (p + q)\, h^{-1},$$
$$G_{\theta\theta} = p\, r^2,$$
$$G_{\phi\phi} = p\, r^2 \sin^2\theta,$$

These are three independent equations.

There are five unspecified functions - $\rho, p, q, f,$ and h

There exist solutions with nonzero imperfect fluid component.

An idealized case involves setting the perfect fluid component to zero

$$\rho = p = 0$$

The equations can be solved for the metric and the imperfect fluid.

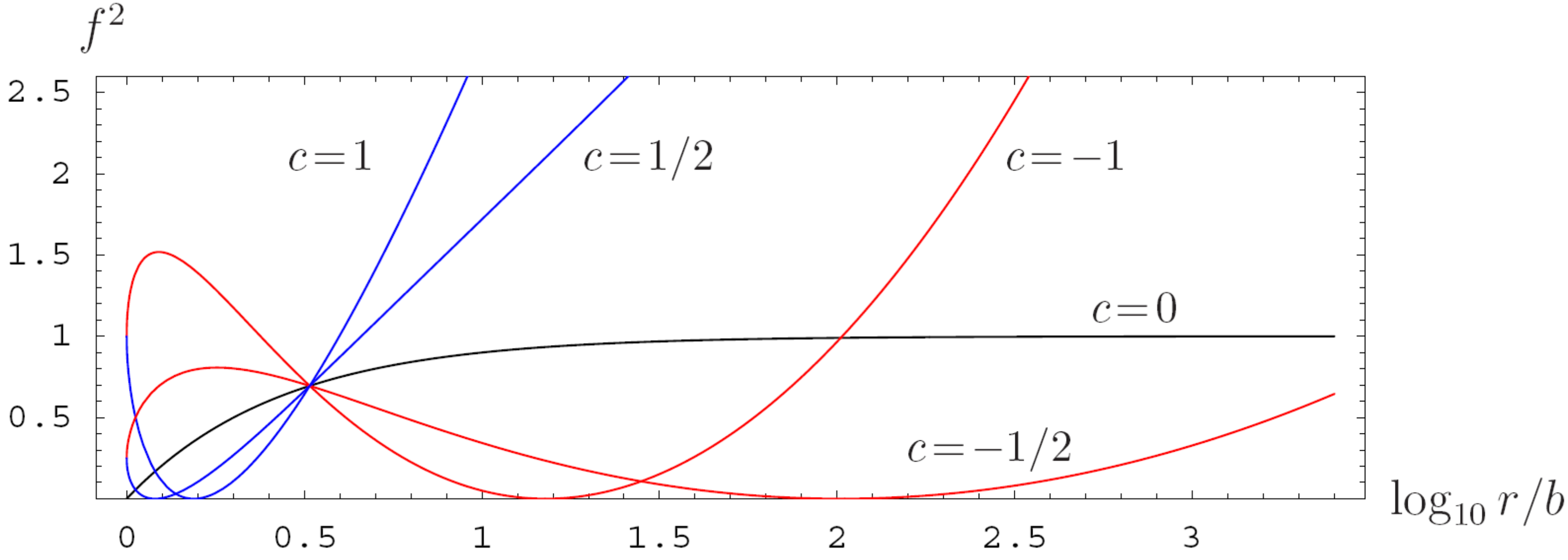
$$h = 1 - \frac{b}{r}$$

$$f = a\left(\sqrt{1-\frac{b}{r}}\right) - c\left(1 - \sqrt{1-\frac{b}{r}}\log\left(\frac{\sqrt{r}+\sqrt{r-b}}{\sqrt{d}}\right)\right)$$

$$q = \frac{c}{r^2 f}$$

Recall the line element ansatz is

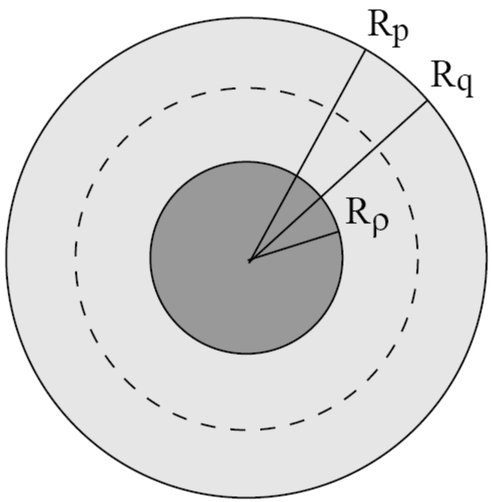
$$ds^2 = -f^2\, dt^2 + h^{-1}\, dr^2 + r^2\, d\Omega^2$$



The idealized modified Schwarzschild spacetime is pathological

- there are curvature singularity outside the Schwarzschild radius
- the spacetime is not asymptotically flat
- there are problems with signature

These issues can be resolved by considering a more complicated source, where each component of the stress-energy tensor has compact support.



$$\rho = \begin{cases} \rho(r) & \text{for } r < R_\rho \\ \Lambda & \text{for } r \geq R_\rho \end{cases}$$

$$p = \begin{cases} p(r) & \text{for } r < R_p \\ -\Lambda & \text{for } r \geq R_p \end{cases}$$

$$q = \begin{cases} q(r) & \text{for } r < R_q \\ 0 & \text{for } r \geq R_q \end{cases}$$

Despite the issues, the modified Schwarzschild space can be useful. It has a definite line-element and provides insight into how the radial imperfect fluid affects geometry.

c.f. P. D. Mannheim, arXiv:astro-ph/0505266

Phenomenology

Phenomenology of the modified black-hole spacetime can be discussed in terms of standardized tests of General Relativity, following Wald.

Orbiting test particles:

$$L^2 \sim \frac{cr^2}{2a} + \frac{br}{2} \quad \longrightarrow \quad r_\star = \left|\frac{ab}{c}\right| \quad \longrightarrow \quad |c_S| < 10^{-10}$$

Precession of a test particle:

$$\omega_p \sim \frac{3\sqrt{2}\, b^{3/2}}{4\, r^{5/2}} - c\frac{\sqrt{2}}{4\, a\sqrt{br}} \quad \longrightarrow \quad |c_S| < 10^{-19}$$

Gravitational redshift effect:

$$Y \sim -\frac{b\delta}{2r} - \frac{c\delta}{2a} \quad \longrightarrow \quad |c_E| < 10^{-15}$$

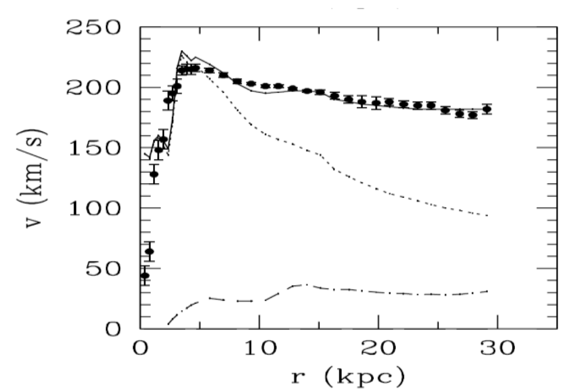
None of these tests significantly constrain the other parameter in the metric, *d*.

R. M. Wald, CUP 1984
V. Kagramanova, J. Kunz and C. Lammerzahl, arXiv:gr-qc/0602002
R. F. C. Vessor et. al., Phys. Rev. Lett. **45**, 2081 (1980)

Comparison to MOND

One of the effects of the imperfect fluid is to change the scaling of angular momentum (or orbiting velocity) for orbiting particles for large radius.

This is akin to what is observed in galaxies. Observations of galaxy rotation curves are often quoted as motivations for introducing dark-matter particles or modifying the dynamics of gravity (e.g. MOND)



In order to use the imperfect fluid component to describe galaxy rotation curves and be consistent with solar-system tests, the imperfect fluid parameter must actually be written as

$$c = \alpha \, b^{n_b} \, \Lambda^{n_\Lambda} \, \rho^{n_\rho} \qquad\qquad n_b = 1/2$$

The value and scaling of *c* should be calculable from first principles given a certain field (maybe a scalar field responsible for the cosmological constant?)

M. Milgrom, arXiv:0801.3133 [astro-ph]
R. H. Sanders and S. S. McGaugh, arXiv:astro-ph/0204521

Conclusion

Static isotropic spacetimes admit sources with a radial imperfect fluid component.

It is plausible that standard fields generate such a component.

Rough constraints can be found. Tight constraints must await better understanding of the imperfect fluid in multi-source situations.

Several questions arise as to whether standard fields (e.g. a scalar field responsible for the cosmological constant) can give rise to a sizable imperfect fluid component, and to whether it can have a connection with dark matter.

Thanks.

TK, arXiv:09.soon [gr-qc]