

# *Charged Membrane as a source for repulsive gravity*

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## Repulsive gravity: a simple example

- The first mentioning of the repulsive gravity due to the presence of electric field was made by

B. Hoffmann and L. Infeld. Phys. Rev. 51, 765, (1937).

- In Reissner-Nordstrom metric

$$-ds^2 = -f c^2 dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$f = 1 - \frac{2kM}{c^2 r} + \frac{kQ^2}{c^4 r^2}$$

gravity becomes repulsive for  $r < r_{rep} = Q^2/M$ ,

indeed a neutral test particle follows the radial eqn:

$$\frac{d^2 r}{ds^2} = -\frac{1}{2} \frac{df}{dr} = \frac{k}{c^4 r^2} \left( \frac{Q^2}{r} - M c^2 \right)$$

- When  $Q > M$  (naked singularity) the repulsive region is accessible.

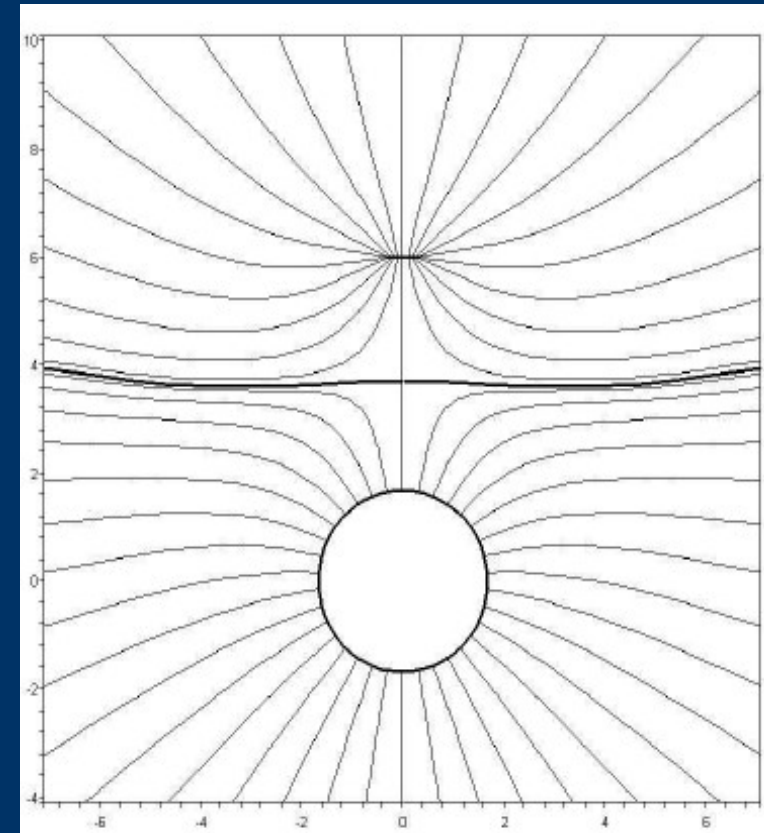
# *What is the problem*

- However we know that a RN metric is not physically acceptable until  $r=0$  for many different reasons (infinite curvature, infinite energy of the electric field, vacuum polarization etc.), so we need a matching solution with a physical source inside a certain radius  $r_0$
  - *Is it possible to find a stable membrane with a finite radius smaller than  $r_{rep}$ ?*
  - The answer we found is: yes.
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# Motivation – The Alekseev-Belinski solution

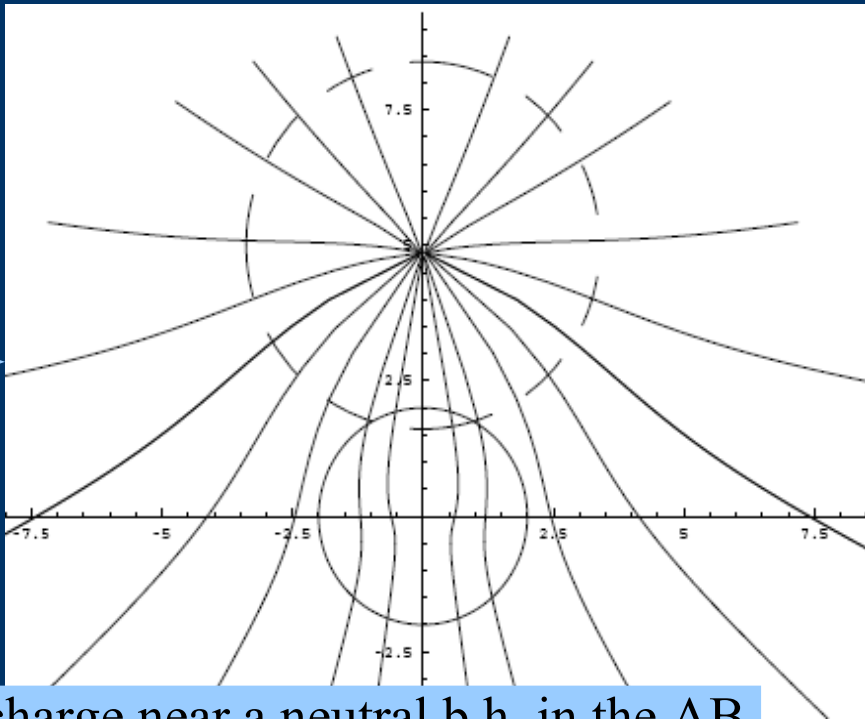
[G.A.Alekseev and V.A.Belinski. *Phys. Rev. D*, 76, 021501(R), 2007.]

- In the recently found Alekseev-Belinski solution it is described the equilibrium of two charged sources in GR.
- The equilibrium configurations are such that if one source is a black hole and the other is always a naked singularity.

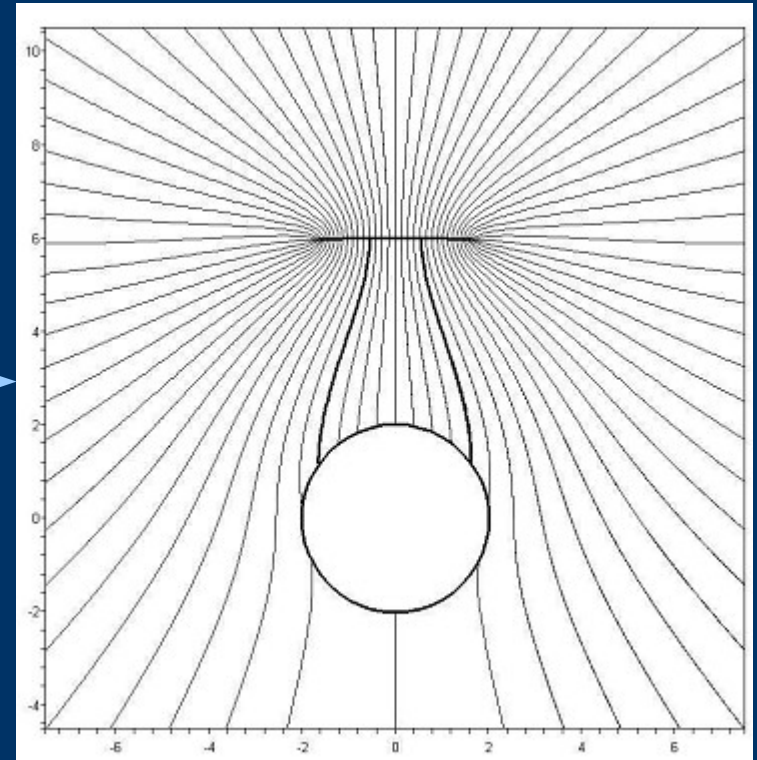


Plot of the electric force lines, [A. Paolino and M. Pizzi, *IJMPD*, Vol.17, No.8, pagg.1159–1177, (2008)].

- For nearby bodies the equilibrium condition differs from the Newtonian one due to the repulsive nature of the naked singularity.



A charge near a neutral b.h. in the AB solution (Pizzi,2008)



Two opposed-signed charges in equilibrium in the AB solution; up is the naked singularity. (Paolino&Pizzi,2008)

- Thus the question was:  
Is it possible to find a physical source producing repulsive gravity in the outside?

## *The model, $\varepsilon=\tau$*

More precisely we required that:

- 1) Inside the body: vacuum (i.e. flat spacetime).
- 2) Outside: the field of a RN naked singularity.
- 3) The radius of the body must be  $r < r_{rep} = Q^2/M$ .
- 4) No vacuum polarization.
- 5) Such stationary state is stable with respect to collapse or expansion.

→ We found a solution in the especially transparent case of a Nambu-Goto membrane with equation of state  $\varepsilon=\tau$ , where  $\tau$  is the tension ( $\tau = -p$ ).

# The technique used

$$R_i^k - \frac{1}{2}R\delta_i^k = \frac{8\pi k}{c^4}T_i^k$$
$$(F^{ik})_{;k} = \frac{4\pi}{c}\rho u^i$$

- We have to solve the E-M eqn

with  $T_i^k = \epsilon u_i u^k - (\delta_i^2 \delta_2^k + \delta_i^3 \delta_3^k)\tau + \frac{1}{4\pi}(F_{il}F^{kl} - \frac{1}{4}\delta_i^k F_{lm}F^{lm})$

- The equation of motion for a thin charged spherically symmetric fluid shell with tangential pressure moving in the RN field have been derived by ***J.E. Chase Nuovo Cimento B 67, 136, (1970)***. The corresponding dynamics for a charged elastic membrane with tension follows from his equation simply by the change of the sign of the pressure.
- We derived, however, the membrane's dynamics using  $\delta$ -shaped distributions.

- The starting point is:

$$- (ds^2)_{in} = -\Gamma^2(t)c^2 dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

$$- (ds^2)_{out} = -f(r)c^2 dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

$$- (ds^2)_{on} = -c^2 d\eta^2 + r_0^2(\eta)(d\theta^2 + \sin^2 \theta d\phi^2)$$

- Matching the intervals... one finds

$$\Gamma(t) = \frac{f(r_0) \sqrt{1 + c^{-2}(r_{0,\eta})^2}}{\sqrt{f(r_0) + c^{-2}(r_{0,\eta})^2}}$$

$$\frac{dt}{d\eta} = \frac{\sqrt{f(r_0) + c^{-2}(r_{0,\eta})^2}}{f(r_0)}$$

- Then, in order to find  $r_0(t)$ , one has to use the E-M eqns.
- We solved the E-M by direct integration, defining the sources as

$$\rho = \frac{cQ\delta[r - r_0(t)]}{4\pi r^2 u^0 \sqrt{-g_{00}g_{11}}}$$

$$\epsilon = \frac{\mu c^2 \delta[r - r_0(t)]}{4\pi r^2 u^0 \sqrt{-g_{00}g_{11}}}$$

and using the standard rules of distributions:

$$(1) \quad \frac{d}{dx}\theta(x) = \delta(x) ,$$

$$(2) \quad F(x)\delta(x) = \frac{1}{2}[F(-0) + F(+0)]\delta(x) ,$$

$$(3) \quad \frac{d}{dx}\theta^2(x) = 2\theta(x)\delta(x) = \delta(x) .$$

...The joint condition is:

$$Mc^2 = \mu(r_0)c^2 \sqrt{1 + \left(\frac{dr_0}{cd\eta}\right)^2} + \frac{Q^2}{2r_0} - \frac{k\mu^2(r_0)}{2r_0}$$

- Then, from  $T^j_{i;j} = 0$ , the energy density and the tension are found to be:

$$\epsilon_0 = \frac{\mu(r_0)c^2}{8\pi r_0^2} \left[ \frac{1}{\sqrt{1 + c^{-2}(r_{0,\eta})^2}} + \frac{f(r_0)}{\sqrt{f(r_0) + c^{-2}(r_{0,\eta})^2}} \right]$$
$$\tau_0(r_0) = \frac{d\mu(r_0)}{dr_0} \frac{r_0 \epsilon_0(r_0)}{2\mu(r_0)}$$

- $\mu(r_0)$  is unknown and to go further one has to specify the eqn of state:  $\epsilon = \tau$ .

- From the eqn of state we find:  $\mu = \gamma r_0^2$  , where  $\gamma$  is a positive constant (it is a surface density).
- Then it follows that the joint condition can be rewritten as

$$4 \left( \frac{dr_0}{c d\eta} \right)^2 - \left( \frac{k\gamma r_0}{c^2} + \frac{2M}{\gamma r_0^2} - \frac{Q^2}{c^2 \gamma r_0^3} \right)^2 = -4 .$$

- Thus the problem can be treated as the motion of a classical particle of “mass”=8 in the potential:

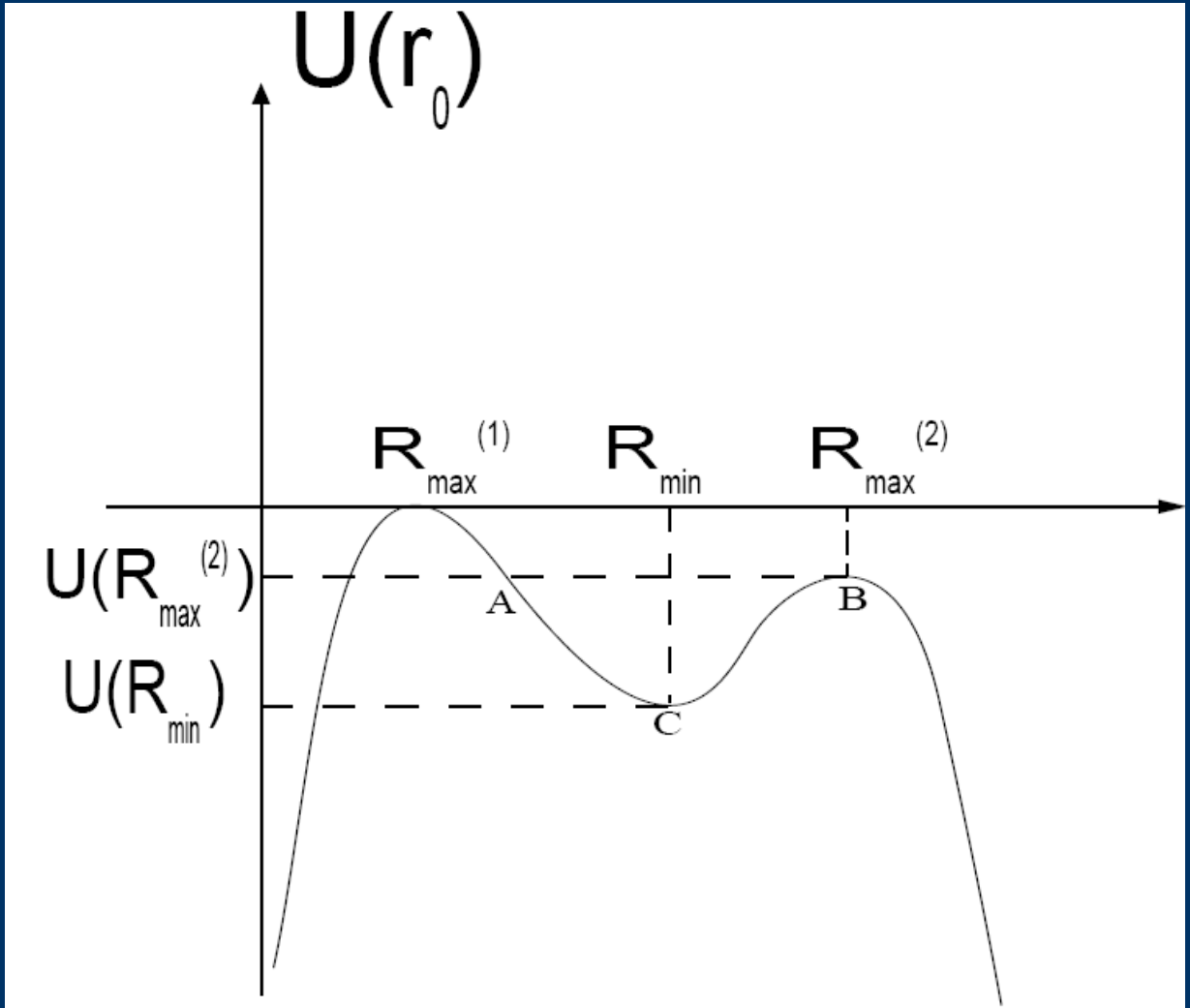
$$U(r_0) = - \left( \frac{k\gamma r_0}{c^2} + \frac{2M}{\gamma r_0^2} - \frac{Q^2}{c^2 \gamma r_0^3} \right)^2$$

$U(r_\theta)$  is the effective potential in which moves the shell, for values such that:

$$k\gamma^2 Q^6 < (Mc^2)^4$$

Only the *ABC* region is allowed, because it must be satisfied

$$Mc^2 - \frac{Q^2}{2r_0} - \frac{k\mu^2}{2r_0} > 0$$



- On  $R_{min}$  the shell is stable. We wish

$$R_{min} < \frac{Q^2}{Mc^2}$$

Note: there is no way for a membrane with positive effective rest mass  $\mu$  to collapse to the point  $r = 0$  leaving outside the field corresponding to the RN naked singularity solution. This conclusion is in agreement with the main result of the paper by D. Boulware. Phys. Rev. D8, 2363, (1973).

# *The last constraint: no vacuum polarization*

- The membrane's radius should be larger than the one at which the vacuum is polarized and the electron-pairs creation begins:

W. Heisenberg, H. Euler. *Z. Phys.* **38**, 714, (1936)  
J. Schwinger. *Phys. Rev.* **82**, 664., (1951).

$$\frac{Q}{R_{min}^2} \ll \mathcal{E}_{cr},$$

$$\mathcal{E}_{cr} = \frac{m_e^2 c^3}{e \hbar}$$

- This implies:

$$k\gamma^2 \ll \frac{x}{4 - 3x} \mathcal{E}_{cr}^2$$

where

$$\frac{k\gamma}{c^2} R_{min} = x$$

# The solution

- After having considered all the above constraints, finally one finds:

$x$  and  $\gamma$  are the only independent and arbitrary constants.

$$x < 1$$

$$M = \frac{c^4}{k^2 \gamma} (3x^2 - 2x^3)$$

$$Q^2 = \frac{c^8}{k^3 \gamma^2} (4x^3 - 3x^4)$$

$$k\gamma^2 \ll \frac{x}{4 - 3x} \mathcal{E}_{cr}^2$$

$$\epsilon = \frac{\gamma c^2}{8\pi} (1 + \sqrt{x^2 - 2x + 1}) \delta(r - R_{min}).$$

# *On the physical meaning of repulsive gravity*

- It is a typical non-linear effect of the E-M eqn.
- It is an effect not necessarily linked to the presence of charge: Repulsive gravitational forces arise also in neutral viscous fluids

J. Ponce de Leon. *J. Math. Phys.*, 28, 411, (1987)

and in the course of interaction between electrically-neutral topological gravitational-solitons.

V.A. Belinski. *Phys. Rev. D* 44, 3109, (1991).

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# Conclusions

- 1) We showed that exists a possibility to have a spherically charged membrane in a stable stationary state producing RN repulsive gravitational force outside its surface and having flat space inside.
  - 2) This stability should be understood in a very restrict sense. We do not know what will happen to this membrane after that an *arbitrary* perturbation will be given. (In general, arbitrary perturbations will change also the equation of state).
  - 3) A concrete realization of this membrane needs a very strong tension., this is the main physical problem of this model. However *in the realm of GR* there are no principal obstacles to construct such an object.
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# *Essential bibliography*

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