The Solution of Cosmological Constant Problem by Discrete Spacetime at Electroweak Scale

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Cosmological Constant Problem

• The vacuum energy behaves as the $\lambda$ term in the Einstein’s field equation

$$ R_{\mu\nu} - g_{\mu\nu}R/2 - g_{\mu\nu}\lambda = 8\pi G T_{\mu\nu} $$

which is known as the cosmological constant.

• The old cosmological constant problem – Why the observed value so many order smaller than that expected in quantum field theories?

• The new cosmological constant problem – Why it is of the same order of magnitude as the matter density at the present time?

Hypotheses

• Space-time behaves as the phase of Higgs condensate and is discrete in nature

• The fundamental unit (“the atom”) of the condensate is of Planck size

(Remark: The reason of using simplified hypotheses is that we do not want to restricted ourselves to any specific model of quantum gravity that is not necessary to the calculation of the cosmological constant in our theory.)
Solution of the Cosmological Constant Problem

• We have showed in the previous paper in 2001 (astro-ph/0105513) that if space-time behaves as the phase of Higgs condensate and is discrete in nature, the vacuum energy could be extremely large in microscopic scale but comparatively very small in macroscopic scale.

• It could be analogous to the situation that the mass density inside the atomic nucleus ($10^{18}$ kg/m$^3$) is extremely larger than the mass density of common material ($\sim 10^3$ kg/m$^3$) in the length scale of our daily life.

• The vacuum energy density due to the VEV of Higgs field is given by

$$\langle V \rangle = -gv^4/4 \quad (\text{where } V(\phi) = -\mu^2\phi^2 + g(\phi^+\phi)^2)$$

and could be interpreted as the binding energy density of the space-time condensate under our hypotheses.

• Let the average spacing between the constituents of the condensate is $L \sim \lambda$, then by using the relation $E = p = 2\pi/\lambda \sim 2\pi/L$. 
Solution of the Cosmological Constant Problem

Since we are living on the fundamental unit of spacetime ("the atom"), the microscopic vacuum energy density $<V>_{\text{micro}}$ (that is the binding energy density of each space-time condensate) should be averaged, or say projected, over the condensate in macroscopic scale to get the macroscopic vacuum energy density as

$$<V>_{\text{macro}} = m^4 \left( \frac{1}{LM_p} \right)^3 = m^4 \left( \frac{E}{2 \pi M_p} \right)^3,$$

if $E = m$ and therefore the cosmological constant can be found as

$$\lambda = 8 \pi G <V>_{\text{macro}} = m^7 / (2 \pi M_p^5) \sim (m_{\text{EW}})^7 / (M_p)^5$$

Put $m_{\text{EW}} = 100 \text{ GeV}$, the value of the cosmological constant is then equal to $10^{-52} [m]^2$ which corresponds to about 0.7 of the critical mass density of the universe.
Solution of the Cosmological Constant Problem

• This result is in excellent agreement with the recent cosmological observations.

• By using the above calculated cosmological constant, the age of the universe can be estimated to be 13-14 Gyrs.

• Based on the above results, the cosmological constant depends on the VEV of the Higgs condensate such that the cosmological constant was different in other vacuum phases of the early universe due to different VEV values.

• The cosmological constant for the Planck stage and GUT stage can be estimated as $(M_p)^2$ and $(m_{GUT})^7/(M_p)^5$ respectively. In GUT phase, it is 91 order of magnitude larger than the present one and thus it might be the origin of inflation.
Evolution of the Universe

• That means, in our theory, the inflation in early universe could have the same origin as the observed acceleration of universe at the present epoch.

• It is expected that, as inflation theory, vacuum energy was released to become mass energy and heated up the universe during the phase transition from one stage to the other.

• After the phase transition, the cosmological constant was then rapidly reduced to a value smaller than the matter density due to the change of VEV and the universe became matter dominated with decelerating expansion.

• However, the expansion of the universe continuously decreased the mass density that made the cosmological constant dominated again. The universe was then inflated, super-cooled and triggered another phase transition to the next stage.
Evolution of the Universe

- The gradual nature of acceleration in our theory also automatically handle the graceful exit problem and thus there is no need to modify the shape of the Higgs potential function to slow roll in inflation theory.

- The time of which the universe changed from deceleration to acceleration (i.e. the time of the universe dominated by the cosmological constant instead of the matter density) in the Planck, GUT and Electroweak stage can be found as $10^{-42}$s, $10^{-27}$s and 8 billion years of the universe respectively.

\[
\downarrow 10^{-42}s \quad \downarrow 10^{-27}s \quad \downarrow 8 \text{ Gyrs}
\]

\[
\left\{ \frac{(M_p)^2}{(m_{\text{GUT}})^7/(M_p)^5} \quad \frac{(m_{\text{EW}})^7/(M_p)^5}{\right\}
\]

Planck phase \quad GUT phase \quad Electroweak phase
Solution of the Cosmological Constant Problem

- The time calculated for the universe changed from deceleration to acceleration at about 8 billion years is in excellent agreement with the recent cosmological observations (Riess, A.G. et al 2004 ApJ) that the turnover from deceleration to acceleration of the universe occurred at about 5 billion years ago.

- The above mentioned relationship between matter density and cosmological constant also explains why the matter density of the present epoch is similar in order of magnitude as vacuum energy density. Therefore, the old and new cosmological constant problem can both be solved.
Approach of Vacuum Entropy and Holographic Principle

• Recently, the author further showed that if vacuum entropy is associated to the discrete space-time condensate (physics/0408060) and the constituent can be treated as information pixel that is defined as:

1) The vacant constituent of the condensate is in the “0” state;
2) The one occupied by matter is in the “1” state; irrespective of the different particle species for simplicity.

• Then the constraint of vacuum entropy by the Holographic Principle (HP) will give the relation

\[ S_v = \rho_n cV \]

where \( S_v \) is the vacuum entropy; \( \rho_n \) is the number density of the space-time constituent; \( c \) is the entropy contribution factor; \( V \) is the volume of the region.
Approach of Vacuum Entropy and Holographic Principle

• That means the information contained in a region is the interior number of constituents weighed by the entropy contribution factor $c$ which can be proved to be equal to

$$c = (m/M)^{1/2}$$

by HP where $m$ is the energy scale of the corresponding Higgs condensate and is the Planck mass.

• The vacuum entropy of the whole universe is then constrained by HP on its horizon as

$$A=4\pi R^2 M^2 > S_v = \mathcal{V} \rho_n c = (4\pi/3)(mR)^3 (m/M)^{1/2},$$

$A$ is the surface area of the horizon in Planck units; $R$ is the horizon size that the entropy contained in the bulk approach its maximum by HP.
Approach of Vacuum Entropy and Holographic Principle

• R can be found as $3R^{-1} > m^{7/2}/M^{5/2}$.

• As the horizon size continuously increases under the expansion of the universe, the holographic constraint will then be violated after attaining its maximum value because the volume bounded by the horizon was increased faster than the horizon area.

• Interestingly, if the cosmological constant exists, its contribution will gradually dominate the matter density and cause the acceleration of the universe.

• In a universe dominated by the cosmological constant (i.e. $\Omega_m \sim 0$), its horizon size is varied as $R(t) = (1-\exp(-Ht))/H$
Approach of Vacuum Entropy and Holographic Principle

• If \( t \) tends to infinity, the horizon size approaches its limiting value as

\[
R = \frac{1}{H} = \lambda^{-1/2}
\]

and the surface to volume ratio then becomes a constant.

• The HP will be protected and the situation can be rescued. The time required to change from a matter dominated universe to a \( \lambda \) dominated one can be estimated as \( t \sim \lambda^{-1/2} \)

• The horizon size at that time is as \( R \sim \lambda^{-1/2} \)

• By substituting the \( R \) value into the \( 3R^{-1} = \frac{m^{7/2}}{M^{5/2}} \) equation, we have \( \lambda \sim \frac{m^7}{M^5} \)

• The results are excellently agreed with the first approach mentioned before.
Approach of Vacuum Entropy and Holographic Principle

• The corresponding cosmic time for the three transition energy scale of the condensate, for attaining the maximum entropy of the universe, the Planck scale, GUT scale and the electroweak scale, could be found as at $10^{-42}$s, $10^{-27}$s and 8 billion years of the universe respectively which is the same as the times calculated above for the transitions from the decelerating to accelerating universe.

• If there is no further phase transition any more, the universe would then be dominated by the cosmological constant corresponding to the electroweak phase and the accelerated expansion would last forever.

• In view of the above results, it is expected that the ultimate theory of quantum gravity would automatically produce the cosmological constants values of the different phases of early universe in the associated field equations which is consistent with the Holographic Principle.
Holographic Noise in GEO600 Interferometer

• Recently, it has been proposed that GEO600 gravitational waves detection experiment would provide experimental evidences that the fabric space-time is grainy and made of tiny units like pixels in a scale much larger than the Planck scale (C.J. Hogan PRD 77 p104031).

• The predicted Holographic noise level due to the quantum fluctuation of space-time is comparable in magnitude to the currently measured excess detection noise in the frequency range from 100 to 600 Hz of the interferometer of GEO600.

• Therefore, the excess noise of GEO600 would already reveal the discreteness of the fabric space-time in electroweak length scale.
Holographic Noise in GEO600 Interferometer

• If such interpretation is substantiated, it provides an important support to the author’s proposed solution of cosmological constant by discreteness of space-time in electroweak scale.

• The detection of holographic noise does not require a long baseline and thus, a purposely built system can have far shorter arms with a smaller vacuum used.
Discussions and Conclusions

• Starting from the fundamental hypothesis that space-time behaves as the phase of Higgs condensate and discrete in nature, we have explained the cosmological constant problem by the averaged vacuum energy density of the space-time condensate which acts as the internal energy (or say binding energy) of the condensate (astro-ph/0105513).

• We also established the concept of vacuum entropy and the entropy contribution factor which helps us to determine the quantity of vacuum entropy from the number of space-time constituents (physics/0408060).

• The introduction of the vacuum entropy and HP, which both describe the thermodynamic behaviour of the vacuum, leads to the existence of the cosmological constant and also determines its value.
Discussions and Conclusions

- The cosmological constant values as given by the vacuum entropy/HP consideration at different phases of the universe are well agreed with those found in the approach as mentioned in the previous paper.

- Such agreement on both results is quite unexpected because they are based on different theoretical foundation.

- Therefore, it is expected that the ultimate theory of quantum gravity would automatically produce the cosmological constants values of the different phases of early universe in the associated field equations which is consistent with the Holographic Principle.

- If the presence of the holographic noise of the GEO600 interferometer is substantiated, it provides strong experimental support of the author’s proposed solution of cosmological constant by discreteness of space-time in electroweak scale.

- Combined with all the above results, it may reveal that our approach is on the right track and worth pursuing in that direction.