An accelerating universe in a 5D non compact and non Ricci flat Kaluza-Klein Cosmology

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Abstract

In the framework of noncompact Kaluza-Klein theory, we investigate a (4 + 1)-dimensional universe consisting of a (4 + 1) dimensional Robertson-Walker type metric subject to a (4 + 1) dimensional energy-momentum tensor.

Key words: Dark pressure; inflation; accelerating universe; non compact Kaluza-Klein cosmology.

1 Introduction

The recent distance measurements from the light-curves of several hundred type Ia supernovae [1] suggest that our universe is currently undergoing a period of acceleration. The question of dark energy has been therefore the focus of a large amount of activities in recent years within Quintessence, Phantom and K-essence models [2].

In this chapter, based on the idea of noncompact higher dimension [3], a 5D cosmological model is introduced which is not Ricci flat and is coupled to a higher dimensional energy momentum tensor. It is shown that the higher dimensional sector of this model may induce a dark pressure in four dimensional universe so that for a flat universe under specific conditions one may have early inflation, subsequent deceleration and current acceleration.

2 The model

We start with the 5D line element\(^1\)

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) - \Phi^2 dl^2, \]

\(^1\)Here \(k\) takes the values +1, 0, −1 according to a close, flat or open universe, respectively, \(dl\) is the line element along the extra dimension and \(\Phi\) is an scalar field living in this direction.
subject to 5D non-vacuum Einstein equations

\[ G_{AB} = 8\pi G T_{AB}. \]  

The 5D Ricci tensor is given in terms of the 5D Christoffel symbols by

\[ R_{AB} = \partial_C \Gamma^C_{AB} - \partial_B \Gamma^C_{AC} + \Gamma^C_{AB} \Gamma^D_{CD} - \Gamma^C_{AD} \Gamma^D_{BC}. \]  

The geometric reduction from 5D to 4D leads to

\[ \hat{R}_{\alpha\beta} = R_{\alpha\beta} - \nabla_\alpha \nabla_\beta \Phi, \quad R_{44} = \Phi^2 \]  

We now consider the 5D energy-momentum tensor

\[ T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta - p g_{\alpha\beta}, \quad T_{44} = -\bar{p} g_{44} = -\bar{p} \Phi^2, \]  

where \( \bar{p} \) acts as a pressure living along the extra dimension. Substituting (6) in front of (5), respectively, we obtain the field equations

\[ G_{\alpha\beta} = 8\pi G [(\rho + p) u_\alpha u_\beta - \bar{p} g_{\alpha\beta}] + \frac{1}{\Phi} \left( \nabla_\alpha \nabla_\beta \Phi - \Box \Phi g_{\alpha\beta} \right), \quad R = 16\pi G \bar{p}. \]  

The \( g^{\alpha\beta} \) trace of the first equation combined with second one gives

\[ \Box \Phi = \frac{1}{3} (8\pi G(\rho - 3p) + 16\pi G\bar{p}) \]  

The following replacements for a non-vanishing scalar field

\[ \frac{1}{\Phi} \Box \Phi = \frac{1}{3} (8\pi G(\rho - 3p) + 16\pi G\bar{p}), \quad \frac{1}{\Phi} \nabla_\alpha \nabla_\beta \Phi = \frac{1}{3} (8\pi G(\rho - 3p) + 16\pi G\bar{p}) u_\alpha u_\beta, \]  

in the 4D Einstein equations lead to

\[ G_{\alpha\beta} = 8\pi G [(\rho + \bar{p}) u_\alpha u_\beta - \bar{p} g_{\alpha\beta}], \quad \bar{p} = \frac{1}{3} (\rho + 2\bar{p}). \]  

The 4D field equations lead to two independent equations

\[ 3 \ddot{a}^2 + k = \frac{8\pi G}{a^2}, \quad 2a\ddot{a} + \dot{a}^2 + k = -8\pi G \bar{p}. \]  

Combining these equations lead to the conservation and acceleration equations

\[ \frac{d}{dt} (\rho a^3) + \bar{p} \frac{d}{dt} (a^3) = 0, \quad \frac{\dot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3\bar{p}) = -\frac{8\pi G}{3} (\rho + \bar{p}). \]
The scalar field equation together with the extra dimensional equation read
\[ \ddot{\Phi} + 3\dot{\Phi} - \frac{8\pi G}{3}((\rho - 3p) + 2\bar{p})\Phi, \quad \frac{-6(k + \dot{a}^2 + \ddot{a}a)}{a^2} = 16\pi G\bar{p}. \] (13)
Taking \( a(t) = a_0 t^\alpha, \bar{p}(t) = \bar{p}_0 t^\beta, \) \( k = 0 \) we get \( \beta = -2. \) Assuming \( \Phi(t) = \Phi_0 t^\gamma \) and \( \rho(t) = \rho_0 t^\delta (\rho_0 > 0) \) together with the equations of state for matter pressure \( p = \omega \rho \) and dark pressure \( \bar{p} = \Omega \rho \) we rewrite the acceleration equation, scalar field equation and conservation equation \(^2\)
\[ \alpha(\alpha - 1) + \frac{8\pi G}{3}\rho_0 (1 + \Omega) = 0, \] (14)
\[ \gamma(\gamma - 1) + 3\alpha\gamma - \frac{8\pi G}{3}\rho_0 ((1 - 3\omega) + 2\Omega) = 0, \] (15)
\[ 2\rho_0 [(2 + \Omega)\alpha - 1] = 0, \] (16)
The demand for acceleration \( \ddot{\alpha} > 0 \) requires \( \rho_0 (1 + \Omega) < 0, \Omega < -1 \) or \( \alpha > 1. \) We may propose a typical relation between the parameters \( \Omega \) and \( \omega \)
\[ \Omega = -f(\omega) + \frac{3\omega}{1 + \omega}, \] (17)
where \( 1 < f(\omega) < \frac{7}{4} \) for the following values of \( \omega. \) Therefore, we have
\[ \left\{ \begin{array}{ll}
\Omega << -1 & \text{for } \omega \simeq -1 \\
-1 < \Omega < -\frac{1}{3} & \text{for } \omega = \frac{1}{3} \\
-\frac{7}{4} < \Omega < -1 & \text{for } \omega = 0.
\end{array} \right. \] (18)
The case \( \omega \simeq -1 (1 + \omega > 0) \) corresponds to the early universe and shows a very high acceleration \( \ddot{\alpha} >> 0 \) due to a large \( |\Omega| >> 1. \) In fact, one may assume a suitable value for \( \omega \) that leads to \( \alpha \approx 22 \) and triggers an inflation of \( 10^{43} \) order of magnitude during \( 10^{-36} - 10^{-34} \) seconds after the Big Bang. The case \( \omega = \frac{1}{3} \) corresponds to the radiation dominant era and shows a deceleration \( \ddot{\alpha} < 0. \) Finally, the case \( \omega = 0 \) corresponds to the matter dominant era and shows an acceleration \( \ddot{\alpha} > 0. \)

References

\(^2\delta = -2 \) has been used due to the consistency with the power law behavior \( t^{3\alpha - 3} \) in the conservation equation.