Rotating acoustic black holes and superradiance in a nonlinear optical cavity

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A “photon fluid”

Light propagation in self-defocusing media:
\[ n(I) = n_0 - n_2 I, \quad I = |E|^2, \quad n_2 > 0 \]

Density and phase eqs. of a 2D BEC:
\[
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0
\]
\[
\partial_t \psi + \frac{1}{2} v^2 + \frac{c^2 n_2}{n_0^3} \rho - \frac{c^2}{2 k^2 n_0^2} \nabla^2 \rho^{1/2} = 0
\]
\[
P = \frac{c^2 n_2 \rho^2}{2 n_0^3}
\]

Optically-induced diverging lens
Repulsive interaction

$n$ locally drops

Wave nature of light: Diffraction
Quantum-pressure

Phased and density-space propagation

$E = \rho^{1/2} e^{i \phi}$

BEC:
self-defocusing nonlinearity
repulsive atomic interactions
Acoustic metric and hydrodynamic limit

\[ \partial_t \psi + \frac{1}{2} v^2 + \frac{c^2 n_2}{n_0^3} \rho - \frac{c^2}{2k^2 n_0^2} \nabla^2 \frac{\rho^{1/2}}{\rho^{1/2}} = 0 \]

\[ \rho = \rho_0 + \epsilon \rho_1 + O(\epsilon^2) \]

\[ \psi = \psi_0 + \epsilon \psi_1 + O(\epsilon^2) \]

\[ \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \psi_1 \right) = 0 \]

\[ g_{\mu\nu} = \left( \frac{\rho_0}{c_s} \right)^2 \left( \begin{array}{cc} -c_s^2 - v_0^2 & -v_0^T \\ -v_0 & I \end{array} \right) \]

\[ c_s^2 \equiv \frac{\partial P(\rho_0)}{\partial \rho} = \frac{c^2 n_2 \rho_0}{n_0^3} \]

\[ v_0^2 = v_r^2 + v_\theta^2 \]

\[ v_r = \partial_r \psi_0 \]

\[ v_\theta = \frac{1}{r} \partial_\theta \psi_0 \]

\[ \nu_0^2 > c_s^2, \text{Ergoregion} \]

\[ \nu_r^2 = c_s^2, \text{Event horizon} \]

\[ \rho_1, \psi_1 \text{ slowly varying amplitude plane waves} \]

\[ (\Omega - K \cdot v_0)^2 = \frac{c^2 n_2 \rho_0}{n_0^3} K^2 + \frac{c^2}{4k^2 n_0^2} K^4 \]

\[ \xi = \frac{\lambda}{2 \sqrt{n_0 n_2 \rho_0}} \]

\[ \Lambda \gg \xi \]

\[ g^{\mu\nu} K_\mu K_\nu = 0 \]

(hydrodynamic approximation)
Can we create a background similar to a rotating black hole? 

**NO!** Optical vortex solitons

\[ E_0 = \rho_0^{1/2}(r)e^{i\psi_0} \]

\[ \rho_0(0) = 0 \]

\[ \psi_0 = m\theta \]

**Ergosurface:**

\[ c_s(r) = v_\theta = \frac{cm}{kn_0 r} \]

No horizon!! \((v_r = 0)\)

1. Creation of the photon fluid (i.e. self-defocusing system)
2. Control of the background flow profile

**Self-defocusing Fabry-Perot optical cavity**

In an optical cavity the field phase profile is “pinned” by the external driving field (Transverse modes are degenerate)
Self-defocusing optical cavity

Damped-driven version of the NSE:  \( \Gamma = 0 \)

\[
\partial_t E = \frac{ic}{2kn_0} \nabla^2 E - i\omega \frac{n_2}{n_0} E|E|^2 + i\delta E - \Gamma (E - E_d)
\]

Validity range of acoustic metric is reduced
Within this range  \( E \approx E_d \)

\[
\xi \ll \Lambda \ll L_d
\]

\( L_d = c_s/\Gamma \)

\( g^{\mu\nu}K_\mu K_\nu = 0 \)


1. Validity range of acoustic metric is reduced
2. Within this range  \( E \approx E_d \)

\[
c_s \propto |E_d|^2, \quad v_0 \propto \nabla \psi_d
\]
10 cm-long cavity filled with $^{85}\text{Rb}$ vapor (self-defocusing medium). Typical $n_2I = 10^{-6}$ corresponding to $c_s = 3 \cdot 10^5 \text{ m/s}$, $\xi = 390 \mu\text{m}$, $L_d = 10 \text{ mm}$

$$E_d = \sqrt{\rho_d} \exp(i m \theta - 2i \pi \sqrt{\frac{r}{r_0}})$$

Phase-only-spatial-light modulator

Numerical integration of the intracavity field equation

$$r_H = 0.84 \text{ mm} \quad r_E = 1.12 \text{ mm}$$

$$c_s = \sqrt{\frac{c^2 n_2 \rho_d}{n_0^3}} \quad \nu_r = -\frac{c \pi}{kn_0 \sqrt{r_0 r}} \quad \nu_\theta = \frac{cm}{kn_0 r}$$

$$g_{\mu \nu} = \begin{pmatrix} -1 + \frac{\xi^2}{r_0 r} + \frac{m \xi}{\pi r} & 0 & -\frac{m \xi}{\pi} \\ 0 & (1 - \frac{\xi^2}{r_0 r})^{-1} & 0 \\ -\frac{m \xi}{\pi} & 0 & r^2 \end{pmatrix}$$

$$r_H = \frac{\xi^2}{r_0} \approx 0.77 \text{ mm} \quad r_E = (1/2) [r_H + (r_H + 4m^2r_Hr_0/\pi^2)^{1/2}] \approx 1.22 \text{ mm}$$
Superradiance

\[ \psi_1(t, r, \theta) = r^{-1/2} G(r^*) e^{i(\Omega t - n\theta)} \]

\[ dr^* = (1 - r_H/r)^{-1} dr \]

\[ r = r_H \text{ to } r^* = -\infty \quad r = +\infty \text{ to } r^* = +\infty \]

\[ \frac{d^2 G(r^*)}{dr^*2} + \left( \Omega - n \frac{m\xi}{\pi r^2} \right)^2 G(r^*) + \left[ \frac{1}{4r^2} \left( \frac{dr}{dr^*} \right)^2 - \frac{1}{r^2} \left( \frac{r_H}{2r} + n^2 \right) \left( \frac{dr}{dr^*} \right) \right] G(r^*) = 0 \]

\[ G(r^*) = \mathcal{T} e^{i(\Omega - n\Omega_H)r^*}, \quad r^* \to -\infty \]

\[ G(r^*) = e^{i\Omega r^*} + \mathcal{R} e^{-i\Omega r^*}, \quad r^* \to +\infty \]

No first derivative term \quad \implies \quad Wronskian of the solutions independent on \( r^* \)

\[ 1 - |\mathcal{R}|^2 = \left( \frac{\Omega - n\Omega_H}{\Omega} \right) |\mathcal{T}|^2 \]

\[ 0 < \Omega < n\Omega_H, \quad \mathcal{R} > 1 \]
Superradiance: Reflection coefficients

Numerical integration of radial KG-equation ($n=1,2; m=2,4,6,8$)

$R$ increases with $m$ and is equal to 1 at the critical frequency $\Omega = n\Omega_H$

Results based on the validity of acoustic metric: hydrodynamic limit, eikonal approximation

Real Experiment?
(2+1)-Intracavity field evolution equation
Conclusions

Sound-like wave propagation in a 2D-photon-fluid has been studied as an analog for field propagation in curved spacetime.

Light-matter interaction in self-defocusing media gives rise to an effective curved spacetime and the cavity configuration allows to control its geometry.

A rotating acoustic black hole can be realized by the simple injection of a suitable driving field profile.

Future Perspectives:

Superradiance in a nonlinear optical cavity (NO hydrodynamic approximation)