On the Crust of Neutron Stars

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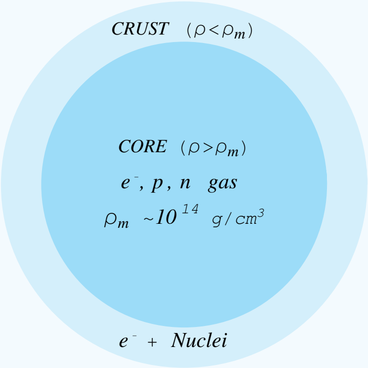
3$^{\text{th}}$ Stueckelberg Workshop on Relativistic Field Theories
July 8 - 18, Pescara

A Model for Neutron Stars in General Relativity

Neutron Stars have two different physical regions:

1. **Core**: it is composed by a relativistic degenerate plasma of electrons, protons and neutrons (hereafter e, p, n gas)
2. **Crust**: it is composed by nuclei and degenerate electrons (White Dwarf material).



Structure Equations

The General Relativistic equations describing the system are

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{1}$$

$$\frac{dP}{dr} = -\frac{G\left(\rho + \frac{P}{c^2}\right)\left(m + \frac{4\pi r^3 P}{c^2}\right)}{r^2\left(1 - \frac{2Gm}{rc^2}\right)}, \tag{2}$$

where m, ρ and P are the mass, the mass density and the pressure respectively.

Core Equations

$$P = \sum_{i=e,p,n} P_i = \sum_{i=e,p,n} k_i \, \phi_i, \qquad k_i = \frac{m_i c^2}{8\pi^2 \lambda_i^3} \tag{3}$$

$$\rho = \sum_{i=e,p,n} \rho_i = \frac{1}{c^2} \sum_{i=e,p,n} \epsilon_i = \frac{1}{c^2} \sum_{i=e,p,n} k_i \, \chi_i \tag{4}$$

where λ_i is the Compton wavelength of particles i and

$$\phi_i = \xi_i \left(\frac{2}{3} \xi_i^2 - 1 \right) \sqrt{\xi_i^2 - 1} + \log \left(\xi_i + \sqrt{\xi_i^2 - 1} \right) \tag{5}$$

$$\chi_i = \xi_i \left(2\xi_i^2 - 1 \right) \sqrt{\xi_i^2 - 1} - \log \left(\xi_i + \sqrt{\xi_i^2 - 1} \right) \tag{6}$$

with $\xi_i = \sqrt{1 + \left(\frac{p_i^F}{m_i c} \right)^2}$, p_i^F the Fermi momentum of particle i.

We assume local charge neutrality

$$n_e = n_p, \tag{7}$$

where n_e and n_p are the number densities of electrons and protons respectively, with

$$n_i = \frac{\left(p_i^F\right)^3}{3\pi^2\hbar^3}. \tag{8}$$

We also consider the β–equilibrium condition

$$\epsilon_e^F + \epsilon_p^F = \epsilon_n^F, \tag{9}$$

where $\epsilon_i^F = m_i c^2 \xi_i$ is the Fermi energy of particles i.

Crust Equations

We consider the Crust as a White Dwarf - like system, with the pressure given essentially by electrons and the density by nuclei

$$P \approx P_e = k_e \, \phi_e, \tag{10}$$

$$\rho \approx \rho_N \approx (\mu_e) \, m_n n_e, \tag{11}$$

where μ_e is the mean molecular weight per electron (for a completely ionized element of atomic weight A and number Z, $\mu_e = A/Z$).
In eq. (11) we have assumed the local charge neutrality of the system.

Numerical Integration Procedure

1. **Core**: we use the equations for the e, p, n gas and integrate eq. (1) and (2) by imposing the initial conditions $m(r = 0) = 0$, $\xi_e(r = 0) > 1$; we stop the integration at the Core radius R_c, that is the radius at which $\rho = \rho_m$.
2. **Crust**: we use the equations for the system of electrons and nuclei and integrate eq. (1) and (2) from $r = R_c$ by imposing the continuity of m and P_e in the transition between the Core and the Crust.

Results I - Pressure and Density

Example: $\rho_c = 5.5 \cdot 10^{15} g\, cm^{-3}$, $M = 0.68\, M_{\odot}$, $R = 7.8\, km$

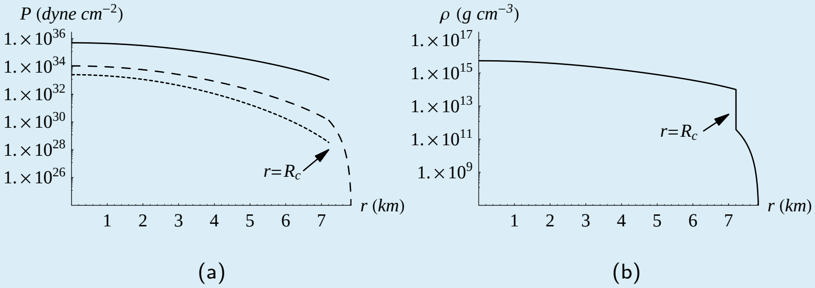


Figure: (a) Pressure of neutrons (solid line), protons (short–dashed line) and electrons (long–dashed line), (b) total mass density.

Results II - Mass and Thickness of the Crust

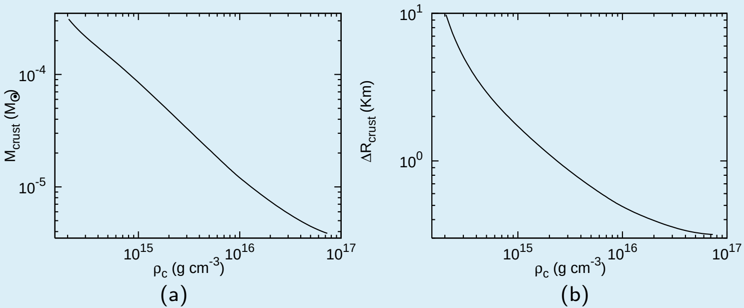


Figure: (a) Mass of the Crust as function of the central mass density (b) Thickness of the Crust as function of the central mass density.

The mass of the Core and the Crust

1. For the considered set of initial conditions we have obtained

$$3.6 \cdot 10^{-6} M_{\odot} \leq M_{crust} \leq 3.1 \cdot 10^{-4} M_{\odot}.$$

2. We have considered stars with local charge neutrality, that have a maximum mass of $\approx 0.7 M_{\odot}$ [10], but in nature Neutron Stars with greater masses exist (see, for example, PSR J0751+1807, having $M = 2.1 \pm 0.2 M_{\odot}$ [7]).
3. Theoretical Models predicting more massive Neutron Stars are, for example, the ones considering electrically charged stars, characterized by larger radii and masses [4].

A different determination of M_{crust} and ΔR_{crust}

We have integrated eq. (1) and (2) for the Crust using the following values for the mass and the radius of the Core:

$$10\,km \leq R_c \leq 20\,km,$$

$$1M_\odot \leq M_c \leq 2M_\odot$$

As an example, we show the results obtained fixing the value of initial pressure as:

$$P(R_c) = 1.6 \cdot 10^{30} dyne\,cm^{-2}.$$

Mass of the Crust

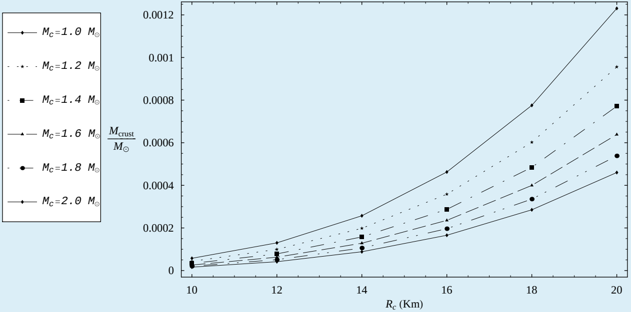


Figure: Mass of the Crust M_{crust} in units of solar masses, as function of the Core Radius R_c, for different values of Core mass M_c.

Thickness of the Crust

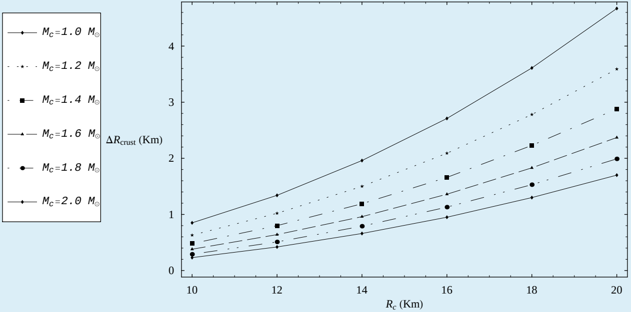


Figure: Thickness of the Crust ΔR_{crust} as function of the Core Radius R_c, for different values of Core mass M_c.

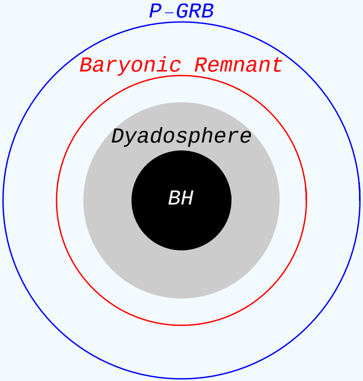
New values of M_{crust}

With the considered values of M_c and R_c we have obtained values of mass of the Crust in the range

$$1.60 \cdot 10^{-5} M_\odot \leq M_{crust} \leq 1.23 \cdot 10^{-3} M_\odot$$

1. We can compare these values with the mass of the Baryonic Remnant considered in the Fireshell Model of Gamma Ray Bursts;
2. We can also compare them with the values of M_{crust} obtained with other theoretical models.

The Fireshell Model of GRBs



The **baryon loading** is measured by the dimensionless quantity

$$B = \frac{M_B c^2}{E_{dya}}, \qquad (12)$$

where M_B is the mass of the baryonic remnant and E_{dya} is the energy of the dyadosphere [8]

Comparison with M_B

GRB	$M_B/M_\odot$
970228	5.0×10^{-3}
050315	4.3×10^{-3}
061007	1.3×10^{-3}
991216	7.3×10^{-4}
011121	9.4×10^{-5}
030329	5.7×10^{-5}
060614	4.6×10^{-6}
060218	1.3×10^{-6}

Table: Values of M_B obtained from eq. (12), with the values of B and E_{dya} used to reproduce the observed data of various GRBs [3]

A comparison with other models

We calculate M_{crust} and ΔR_{crust} for $M_c = 1.4 M_\odot$, $R_c = 12km$ and $P = 6.6 \cdot 10^{29} dyne\, cm^{-2}$ and compare them with the values obtained with other theoretical models [6].

Model	$M_{crust}(10^{-5} M_\odot)$	$\Delta R_{crust}(km)$
Our Model	3.240	0.63
BSk8	3.090	0.4509
SKm*	3.088	0.4408
$BSk8_2$	3.093	0.4666

Conclusions

1. We have calculated the pressure and the mass density of Neutron Stars, showing that the Core pressure is baryonic dominated and that ρ is approximately flat in the Core, rapidly decreasing in the Crust.
2. We have determined M_{crust} and ΔR_{crust}, finding that the Crust is lighter and smaller for stars with more compact Cores.
3. We have compared the range of values for M_{crust} with the values of M_B used to reproduce the observed data of some GRBs within the Fireshell Model and with the ones obtained with other theoretical models, finding that they are compatible.

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